

Online Appendix for “Between Lives and Economy: COVID-19 Containment Policy in Open Economies”

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This online appendix details the equilibrium algorithm.

Given $L_{i,t}$ for each period t as a result of the epidemiological law of motion specified in Section 2, we solve the equilibrium period-by-period. Therefore, we drop the time subscript to keep the notation cleaner. There is an inner loop and an outer loop, of which the rounds of iteration are indexed by $r = 0, 1, 2, \dots$. For $\zeta = 0$, start with an initial guess of wages $\{w_i(0), r_i(0)\}$ such that the global nominal income adds up to one (as in [Alvarez and Lucas \(2007\)](#)), i.e.,

$$\sum_{i=1}^K [w_i(0)L_i + r_i(0)K_i] = 1.$$

The equilibrium is computed by the following algorithm.

1. **Inner loop to obtain price indices.** Let $\xi = 1, 2, \dots$ index the iteration of the inner loop. Given wages and rents $(w_i(\zeta), r_i(\zeta))$, start with an arbitrary initial guess of the price indices of intermediate bundles $\{P_i^{M,j}(0)\}$.

- a. With $\{P_i^{M,j}(\xi)\}$, the unit bundle costs are calculated via

$$c_i^j(\xi) = [w_i(\zeta)]^{\beta_i^{j,L}} [r_i(\zeta)]^{\beta_i^{j,K}} \left[P_i^{M,j}(\zeta) \right]^{1 - \beta_i^{j,L} - \beta_i^{j,K}}.$$

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b. With $\{P_i^{M,j}(\xi)\}$, trade shares and sectoral prices are computed by

$$\begin{aligned}\pi_{i,n}^j(\xi) &= \frac{T_i^j [c_i^j(\xi)\tau_{i,n}^j]^{-\theta}}{\sum_{k=1}^K T_k^j [c_k^j(\xi)\tau_{k,n}^j]^{-\theta}} \\ &= \frac{T_i^j [c_i^j(\xi)\tau_{i,n}^j]^{-\theta}}{\Phi_n^j(\xi)} \\ P_i^j(\xi) &= \Gamma \left(\frac{\theta + 1 - \kappa}{\theta} \right)^{\frac{1}{1-\kappa}} [\Phi_n^j(\xi)]^{-\frac{1}{\theta}}.\end{aligned}$$

c. Update the price index of the intermediate-input bundle:

$$P_i^{M,j}(\xi + 1) = \prod_{l=1}^J [P_i^l(\xi)]^{\gamma_i^{j,l}}.$$

d. Check convergence of $P_i^{M,j}(\cdot)$ by

$$\max_{j,i} \|P_i^{M,j}(\xi + 1) - P_i^{M,j}(\xi)\| < tolerance_{\text{inner loop}}.$$

If the above condition does not hold, go back to Step (a) and start from $P_i^{M,j}(\xi + 1)$. If it holds, then assign the following values to the outer loop:

$$\begin{aligned}\pi_{i,n}^j(\zeta) &= \pi_{i,n}^j(\xi) \\ P_i^j(\zeta) &= P_i^j(\xi) \\ P_i^{M,j}(\zeta) &= P_i^{M,j}(\xi + 1) \\ P_i(\zeta) &= \prod_{j=1}^J [P_i^j(\zeta)]^{\alpha_i^j}.\end{aligned}$$

2. Update the total nominal income in each country:

$$Y_i(\zeta) = w_i(\zeta)L_i + \varphi_i \sum_{n=1}^K r_n(\zeta)K_n.$$

3. By definition of X_i^j ,

$$X_i^j(\zeta) = \alpha_i^j Y_i(\zeta) + \sum_{l=1}^J \gamma_i^{l,j} (1 - \beta_i^l) \sum_{n=1}^K \pi_{i,n}^l(\zeta) X_n^l(\zeta),$$

which entails a linear system of equations written as

$$\begin{aligned}\mathbf{b}(\zeta) &= \mathbf{A}(\zeta) \times \mathbf{X}(\zeta) \\ JK \times 1 & \quad JK \times JK \quad JK \times 1 \\ &= [\tilde{\mathbf{A}}(\zeta) - \mathbf{I}] \times \mathbf{X}(\zeta),\end{aligned}$$

where

$$\begin{aligned} [\mathbf{b}(r)]_{(j,i)} &= -\alpha_i^j Y_i(\zeta) \\ [\tilde{\mathbf{A}}(\zeta)]_{(j,i),(l,n)} &= \gamma_i^{l,j} (1 - \beta_i^{L,l} - \beta_i^{K,l}) \pi_{i,n}^l(\zeta) \\ [\mathbf{X}(\zeta)]_{(j,i)} &= X_i^j(\zeta). \end{aligned}$$

Given $\{w_i(\zeta), r_i(\zeta)\}$ and $\{\pi_{i,n}^l(\zeta)\}$, solve $[\mathbf{X}(\zeta)]_{(j,i)}$.

4. Use the labor-market clearing condition to define excess demand $Z_i^L(\zeta)$ and $Z_i^K(\zeta)$ by

$$\begin{aligned} Z_i^L(\zeta) &\equiv \frac{1}{w_i(\zeta)} \left[\sum_{j=1}^J \sum_{n=1}^K \beta_i^{L,j} \pi_{i,n}^j(\zeta) X_n^j(\zeta) - w_i(\zeta) L_i \right] \\ Z_i^K(\zeta) &\equiv \frac{1}{r_i(\zeta)} \left[\sum_{j=1}^J \sum_{n=1}^K \beta_i^{K,j} \pi_{i,n}^j(\zeta) X_n^j(\zeta) - r_i(\zeta) K_i \right]. \end{aligned}$$

In a similar fashion to the approach in [Alvarez and Lucas \(2007\)](#), wages and rents are updated by

$$\begin{aligned} w_i(\zeta + 1) &= w_i(\zeta) \left[1 + \psi \frac{Z_i^L(\zeta)}{L_i} \right] \\ r_i(\zeta + 1) &= r_i(\zeta) \left[1 + \psi \frac{Z_i^K(\zeta)}{K_i} \right], \end{aligned}$$

where $\psi \in (0, 1)$ controls the speed of wage adjustment.

5. Stop iterations if

$$\max_i \{ \max\{|Z_i(\zeta)|, |Z_i^K(\zeta)|\} \} < \text{tolerance}.$$

Otherwise, go back to Step 1.

6. Compute the sectoral employment share by

$$\ell_i^j(\zeta) = \frac{\beta_i^{L,j} R_i^j(\zeta)}{\sum_{l=1}^J \beta_i^l R_i^l(\zeta)},$$

where

$$R_i^j(\zeta) = \sum_{n=1}^K \pi_{i,n}^{L,j}(\zeta) X_n^j(\zeta).$$

Finally, the equilibrium sectoral labor shares $\{\ell_i^j\}$ are used in the epidemiological law of motion to obtain the SIRD objects and L_i for the next period.

References

Alvarez, F. and Lucas, R. E. (2007). General Equilibrium Analysis of the Eaton–Kortum Model of International Trade. *Journal of Monetary Economics*, 54(6):1726–1768.