Online Appendix for "Between Lives and Economy: COVID-19 Containment Policy in Open Economies"

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This online appendix details the equilibrium algorithm.

Given $L_{i,t}$ for each period t as a result of the epidemiological law of motion specified in Section 2, we solve the equilibrium period-by-period. Therefore, we drop the time subscript to keep the notation cleaner. There is an inner loop and an outer loop, of which the rounds of iteration are indexed by $r = 0, 1, 2, ...$ For $\zeta = 0$, start with an initial guess of wages $\{w_i(0), r_i(0)\}$ such that the global nominal income adds up to one (as in Alvarez and Lucas (2007)), i.e.,

$$\sum_{i=1}^{K} [w_i(0)L_i + r_i(0)K_i] = 1.$$

The equilibrium is computed by the following algorithm.

1. **Inner loop to obtain price indices.** Let $\xi = 1, 2, ...$ index the iteration of the inner loop. Given wages and rents $(w_i(\zeta), r_i(\zeta))$, start with an arbitrary initial guess of the price indices of intermediate bundles $\{P_i^{M,j}(0)\}$.

   a. With $\{P_i^{M,j}(\xi)\}$, the unit bundle costs are calculated via

$$c_i^j(\xi) = [w_i(\zeta)]^{\beta_i^{L,j}} [r_i(\zeta)]^{\beta_i^{K,j}} \left[P_i^{M,j}(\zeta)\right]^{1-\beta_i^{L,j}-\beta^{K,j}}.$$

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b. With $\{P_i^{M,j}(\xi)\}$, trade shares and sectoral prices are computed by

$$\begin{aligned}\pi_{i,n}^j(\xi) =& \frac{T_i^j\left[c_i^j(\xi)\tau_{i,n}^j\right]^{-\theta}}{\sum_{k=1}^K T_k^j\left[c_k^j(\xi)\tau_{k,n}^j\right]^{-\theta}}\\ =& \frac{T_i^j\left[c_i^j(\xi)\tau_{i,n}^j\right]^{-\theta}}{\Phi_n^j(\xi)}\\ P_i^j(\xi) =& \Gamma\left(\frac{\theta+1-\kappa}{\theta}\right)^{\frac{1}{1-\kappa}}[\Phi_n^j(\xi)]^{-\frac{1}{\theta}}.\end{aligned}$$

c. Update the price index of the intermediate-input bundle:

$$P_i^{M,j}(\xi+1) = \prod_{l=1}^J\left[P_i^l(\xi)\right]^{\gamma_i^{j,l}}.$$

d. Check convergence of $P_i^{M,j}(.)$ by

$$\max_{j,i}||P_i^{M,j}(\xi+1) - P_i^{M,j}(\xi)|| < tolerance_{\text{inner loop}}.$$

If the above condition does not hold, go back to Step (a) and start from $P_i^{M,j}(\xi+1)$. If it holds, then assign the following values to the outer loop:

$$\begin{aligned}\pi_{i,n}^j(\zeta) &= \pi_{i,n}^j(\xi)\\ P_i^j(\zeta) &= P_i^j(\xi)\\ P_i^{M,j}(\zeta) &= P_i^{M,j}(\xi+1)\\ P_i(\zeta) &= \prod_{j=1}^J\left[P_i^j(\zeta)\right]^{\alpha_i^j}.\end{aligned}$$

2. Update the total nominal income in each country:

$$Y_i(\zeta) = w_i(\zeta)L_i + \varphi_i\sum_{n=1}^K r_n(\zeta)K_n.$$

3. By definition of X_i^j,

$$X_i^j(\zeta) = \alpha_i^j Y_i(\zeta) + \sum_{l=1}^J \gamma_i^{l,j}(1-\beta_i^l)\sum_{n=1}^K \pi_{i,n}^l(\zeta)X_n^l(\zeta),$$

which entails a linear system of equations written as

$$\begin{aligned}\underset{JK\times 1}{\mathbf{b}(\zeta)} =& \underset{JK\times JK}{\mathbf{A}(\zeta)} \times \underset{JK\times 1}{\mathbf{X}(\zeta)}\\ =& [\widetilde{\mathbf{A}}(\zeta) - \mathbf{I}] \times \mathbf{X}(\zeta),\end{aligned}$$

where

$$[\mathbf{b}(r)]_{(j,i)} = -\alpha_i^j Y_i(\zeta)$$
$$[\widetilde{\mathbf{A}}(\zeta)]_{(j,i),(l,n)} = \gamma_i^{l,j}(1-\beta_i^{L,l}-\beta_i^{K,l})\pi_{i,n}^l(\zeta)$$
$$[\mathbf{X}(\zeta)]_{(j,i)} = X_i^j(\zeta).$$

Given $\{w_i(\zeta), r_i(\zeta)\}$ and $\{\pi_{i,n}^l(\zeta)\}$, solve $[\mathbf{X}(\zeta)]_{(j,i)}$.

4. Use the labor-market clearing condition to define excess demand $Z_i^L(\zeta)$ and $Z_i^K(\zeta)$ by

$$Z_i^L(\zeta) \equiv \frac{1}{w_i(\zeta)}\left[\sum_{j=1}^J \sum_{n=1}^K \beta_i^{L,j}\pi_{i,n}^j(\zeta)X_n^j(\zeta) - w_i(\zeta)L_i\right]$$
$$Z_i^K(\zeta) \equiv \frac{1}{r_i(\zeta)}\left[\sum_{j=1}^J \sum_{n=1}^K \beta_i^{K,j}\pi_{i,n}^j(\zeta)X_n^j(\zeta) - r_i(\zeta)K_i\right].$$

In a similar fashion to the approach in Alvarez and Lucas (2007), wages and rents are updated by

$$w_i(\zeta+1) = w_i(\zeta)\left[1+\psi\frac{Z_i^L(\zeta)}{L_i}\right]$$
$$r_i(\zeta+1) = r_i(\zeta)\left[1+\psi\frac{Z_i^K(\zeta)}{K_i}\right],$$

where $\psi \in (0,1)$ controls the speed of wage adjustment.

5. Stop iterations if

$$\max_i\{\max\{|Z_i(\zeta)|, |Z_i^K(\zeta)|\}\} < \text{tolerance}.$$

Otherwise, go back to Step 1.

6. Compute the sectoral employment share by

$$\ell_i^j(\zeta) = \frac{\beta_i^{L,j}R_i^j(\zeta)}{\sum_{l=1}^J \beta_i^l R_i^l(r)},$$

where

$$R_i^j(\zeta) = \sum_{n=1}^K \pi_{i,n}^{L,j}(\zeta)X_n^j(\zeta).$$

Finally, the equilibrium sectoral labor shares $\{\ell_i^j\}$ are used in the epidemiological law of motion to obtain the SIRD objects and L_i for the next period.

References

Alvarez, F. and Lucas, R. E. (2007). General Equilibrium Analysis of the Eaton–Kortum Model of International Trade. *Journal of Monetary Economics*, 54(6):1726–1768.