

Comparative Advantage and Optimal Trade Policy with Strategic Interactions*

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Abstract

This paper introduces strategic interactions into a Ricardian model where two countries optimize trade policies. In a Nash Equilibrium, we find that optimal import tariffs are zero, while export taxes rise with comparative advantage, aligning with but extending the unilateral model of [Costinot et al. \(2015\)](#) by allowing both countries to act strategically. Our analysis reveals that welfare gains from trade policy diminish when both countries pursue optimal policies. Surprisingly, a smaller country can benefit from Nash Equilibrium through higher export taxes in goods with technological advantages. Applying the model to agriculture, we assess welfare under autarky, free trade, Nash Equilibrium, and unilateral policy, showing that small, competitive nations can outperform free trade through strategic policy, providing insights into optimal trade policy across goods.

Keywords: Strategic Interactions; Comparative Advantage; Optimal Trade Policy; Industrial Policy; Agricultural Policy

JEL Classification: F10; F11; F13; L52; Q17

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1 Introduction

Over two centuries have passed since David Ricardo proposed the groundbreaking concept of comparative advantage, a theory that has profoundly shaped international trade. Comparative advantage theory has since been expanded, refined, and successfully applied to address numerous questions in trade economics (See, for example, [Dornbusch et al., 1977](#); [Eaton and Kortum, 2002](#); [Costinot et al., 2012](#); [Caliendo and Parro, 2015](#)). Yet, classical studies on optimal trade policies often concentrate on one or a limited number of goods, leaving open the question of how optimal trade policies might relate to comparative advantages across a broad array of goods. In their seminal work, [Costinot et al. \(2015\)](#) study a country's optimal trade policy in a canonical two-country Ricardian model *à la* [Dornbusch et al. \(1977\)](#). Focusing exclusively on the Home country's trade policy and assuming that the Foreign country remains passive (imposing no import tariffs or export taxes), they conclude that the optimal welfare-maximizing policy for the Home country involves uniform import tariffs and export taxes that are weakly monotonic in relation to its comparative advantage.

Building on the framework of [Costinot et al. \(2015\)](#), this paper introduces strategic interactions into the Ricardian model, allowing both countries to set optimal trade policies. In this model, a continuum of goods is produced using constant-returns-to-scale technology in a perfectly competitive market. Differences in productivity across goods generate comparative advantages, and each country's planner determines optimal import/export taxes or subsidies for each good, taking the trade policy of the other country's planner as given. This setup enables us to analyze a Nash Equilibrium of optimal trade policies—an extension from [Costinot et al. \(2015\)](#), who focused solely on unilateral trade policies. To achieve an analytical characterization of Nash Equilibrium properties, we assume quasilinear preferences, which simplifies the model structure relative to the original framework.

Given the differences in the setup, our model requires a nuanced interpretation. While a continuum of goods is often seen as representing distinct sectors within an economy, here it is more accurately viewed as representing individual products within a single sector. Consequently, our model adopts a partial equilibrium perspective, similar to approaches in the Industrial Organization literature. In reality, import tariffs vary across products within a sector, and industrial policies frequently impose differential taxes or subsidies on these products to influence export and import dynamics. Our model provides a valuable framework for examining these policy variations. For our quantitative application, we focus on the agricultural sector, where government intervention is extensive, making these considerations particularly relevant.

We demonstrate that, in a Nash Equilibrium, a country's optimal import tariffs are zero, while the optimal export taxes increase weakly with respect to its comparative advantage.

Consequently, the structure of optimal trade policies with respect to comparative advantages in this model aligns with that of [Costinot et al. \(2015\)](#). However, the welfare gains derived from these optimal trade policies are lower in our framework. The reasoning is as follows. For goods in which Home exhibits strong comparative disadvantages, a passive Foreign would lead Home to choose free trade, allowing production to shift to lower-cost firms in Foreign. In this scenario, the world prices—and thus Home’s domestic prices—are determined solely by Foreign’s marginal costs, enabling Home to capture a substantial consumer surplus. However, when Foreign is also strategic, it imposes strictly positive export taxes on these goods, resulting in increased world and domestic prices for Home. Consequently, Home’s consumer surplus diminishes. Indeed, [Costinot et al.](#) suggest that Home’s welfare could be quantitatively smaller in a Nash Equilibrium,¹ and our findings confirm this conjecture.

In terms of welfare implications, previous literature generally posits that a small country would fare worse under a Nash Equilibrium of optimal trade policies compared to a scenario of free trade (across all goods). However, our model suggests that this outcome may be reversed. Specifically, a small country can experience welfare gains under a Nash Equilibrium of optimal trade policies by imposing high export taxes on goods in which it has a significant technological edge over the other country. By doing so, the small country reaps substantial tax revenues, effectively capturing a portion of Foreign’s consumer surplus.

In our quantitative analysis, we examine the welfare implications across four trade situations—autarky, free trade, Nash Equilibrium, and unilateral optimal trade policy (when the other country is passive)—using the agricultural sector as a case study. By setting the United States and China as focal countries in different scenarios, we calculate welfare, tax revenue, and consumer surplus, normalizing each country’s welfare to 100% under autarky for comparative clarity. Our results reveal that while both countries experience welfare gains when moving from autarky to free trade, unilateral optimal policies yield the highest welfare for each country when they alone control trade taxes. Interestingly, under Nash Equilibrium, the welfare of smaller, technologically superior country, such as the US relative to China, improves compared with the free trade scenario due to high tax revenues in sectors where they hold strong competitive advantages. This echoes the aforementioned theoretical prediction.

Our research connects to multiple strands of literature, primarily focusing on optimal trade policy within international trade models. Trade policy, as a specialized form of industrial policies, has long been a central topic in trade research ([Ossa, 2011](#); [Felbermayr et al., 2015](#); [Haaland and Venables, 2016](#); [Costinot et al., 2020](#)). Previous literature frequently focuses on optimal trade policies restricted to a single or limited set of goods or examines trade policies that apply uniformly across sectors ([Itoh and Kiyono, 1987](#); [Opp, 2010](#); [Ossa, 2014](#); [Bagwell and Lee, 2020](#)). **Our paper introduces strategic interactions into a Ricardian**

¹See their footnote 8.

model *à la* Costinot et al. (2015), analyzing both unilateral optimal and Nash Equilibrium trade policies across goods. A notable theoretical prediction—supported by quantitative simulations—is that a small country may prevail under a Nash Equilibrium of optimal trade policies compared to free trade, a result that stands in contrast to previous literature (Kennan and Riezman, 1988; Syropoulos, 2002; Ossa, 2014). Our paper also closely relates to Beshkar and Lashkaripour (2017), who develop a general equilibrium, multi-industry model that illustrates the interdependencies between import tariffs, export taxes, and non-revenue trade barriers. Notably, the Beshkar and Lashkaripour (2017) model converges to the framework of Costinot et al. (2015) when trade elasticity is uniform and approaches zero, which implies a Cobb–Douglas utility aggregator. By contrast, our model incorporates uniform and finite trade elasticity alongside a Constant Elasticity of Substitution (CES) utility aggregator, thus complementing the framework presented in Beshkar and Lashkaripour (2017).

Additionally, this paper contributes to the literature on quantitative approaches to industrial policies, with a focus on the agricultural sector. Many studies on industrial policies highlight the importance of scale effects (Bartelme et al., 2025). For example, Ju et al. (2024) quantify trade wars and industrial policy competition between China and the US by extending the Caliendo and Parro (2015) model to incorporate sectoral external economies of scale. The positive aggregate effects of industrial policy on a production network through input-output linkages are demonstrated by Liu (2019). The economic impacts of industrial policies are substantial, especially when considering specific industries within certain economies (Kalouptsi, 2018; Itskhoki and Moll, 2019; Barwick et al., 2024b); notable examples include China’s auto industry (Bai et al., 2020), the shipbuilding industry (Barwick et al., 2025), and the vehicle market (Barwick et al., 2024a). Our paper discusses optimal trade policies specifically within the agricultural sector, an arena historically significant in international trade negotiations and policy disputes (Ossa, 2014; Bagwell et al., 2016). Unlike the industries extensively studied in the aforementioned literature, agriculture is characterized by a Ricardian structure (Costinot and Donaldson, 2016), which warrants a focused investigation. We show that the optimal agricultural policy is to protect weaker sectors and provide a quantitative assessment of the welfare implications under various policy scenarios. Our numerical simulations suggest that even a small yet technologically competitive country may benefit more from strategic industry policies than from free trade, enriching our understanding of industrial policy effectiveness across a broader array of sectors.

Noticeably, our work is closely related to Lashkaripour and Lugovskyy (2023), who analyze optimal industrial and trade policies within a class of quantitative trade models featuring scale effects and profit-generating markups, showing that industrial policies must operate in conjunction with trade policies to address sectoral misallocation. The unilateral optimal policies under the small open economy assumption closely align with the results presented here.

However, our research differs from Lashkaripour and Lugovskyy (2023) in several key respects. We focus on Nash policies in a two-country Ricardian model *à la* Dornbusch et al. (1977), where both countries can impose trade policies, rather than only one country choosing unilateral optimal policies as in Lashkaripour and Lugovskyy (2023). Due to the non-smooth property of Dornbusch et al.’s world production possibility frontier (Costinot et al., 2015), we cannot apply optimization methods such as Lagrangian techniques or gradient-based approaches that are typically used in the literature (Beshkar and Lashkaripour, 2017; Lashkaripour and Lugovskyy, 2023; Wang et al., 2024; Bartelme et al., 2025). Accordingly, we adopt quasilinear preferences to eliminate the general-equilibrium wage effect, thereby simplifying the derivation of Nash policies and allowing for a clear analytical solution. Although Lashkaripour and Lugovskyy (2023) accommodate a multi-country setting, the uniqueness of equilibrium and the validity of policy formulas in their framework require restrictive assumptions. For instance, they assume that a country’s policy cannot influence aggregate relative wages in the rest of the world—an assumption that holds trivially in the two-country case but would require strong internal coordination otherwise. In addition, the preferences in Lashkaripour and Lugovskyy (2023) must be homothetic in a multi-country environment, in contrast to the simpler quasilinear specification employed here. Finally, owing to the Krugman setup, the unilateral optimal policies in Lashkaripour and Lugovskyy (2023) are sector-specific, i.e., uniform taxes within a sector. By contrast, the optimal trade policies in this paper are variety-specific and can vary across goods within a sector, making our framework a powerful toolkit for the analysis of industrial policy.

This paper is organized as follows. Section 2 introduces the theoretical model and formulates the government’s welfare optimization problem. In Section 3, we derive each country’s unilateral optimal trade policy, given the trade taxes set by the other country, addressing the best response problem. Section 4 characterizes the Nash Equilibrium in which two strategic countries set their optimal trade policies noncooperatively. Section 5 discusses the welfare implications of different trade policies. Section 6 applies the theoretical predictions to real-world scenarios. Finally, Section 7 concludes the paper.

2 Model

2.1 Setup

We incorporate strategic interactions into a Ricardian model *à la* Costinot et al. (2015). In this framework, the world economy consists of two countries, Home and Foreign. The Home country has a population mass of L , while Foreign has a mass of L^* . For ease of reference, variables associated with the foreign country are marked with an asterisk. Consumer preferences span a continuum of non-numeraire goods, indexed by $i \in \Omega$, as well as a numeraire good

priced at $p_0 = 1$. Preferences are represented by the following quasilinear utility function:

$$\mathcal{U}^l(c^l) \equiv \int_{i \in \Omega} u_i^l(c_i^l) di + c_0^l = \int_{i \in \Omega} \beta_i \frac{(c_i^l)^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} di + c_0^l,$$

where $c^l \equiv ((c_i^l)_{i \in \Omega}, c_0^l) \geq 0$ represents the individual consumption bundle, and $u_i^l(c_i^l) \equiv \beta_i \frac{(c_i^l)^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}$ denotes the CES utility function for each good $i \in \Omega$. Here, $\sigma \geq 1$ is the elasticity of substitution between non-numeraire goods, and (β_i) are exogenous preference parameters, normalized such that $\int_i \beta_i di = 1$.

We characterize preferences using a quasilinear utility function for three main reasons. First, most theoretical studies on optimal or Nash trade policies adopt quasilinear utility.² This literature generally employs quasilinear preferences to obtain analytical solutions by eliminating general-equilibrium income effects and cross-good price effects, thereby simplifying derivations. While one could dispense with the numeraire by assuming CES preferences, such a specification would preclude closed-form Nash policies and necessitate numerical simulations of welfare outcomes, which are less transparent and less analytically insightful (Costinot et al., 2015; Costinot and Werning, 2019). Second, from the perspective of empirical relevance and policy evaluation, our focus is on agricultural policies. Grouping all other sectors into the numeraire approximates actual economic activity reasonably well, particularly given the small share of agriculture in national income and the relatively limited impact of agricultural policies on aggregate wages. Moreover, the agricultural sector features easily measurable productivity, owing to stable natural conditions and accumulated agronomic knowledge, as well as relatively low heterogeneity across origins of agricultural products (Costinot and Donaldson, 2016; Costinot et al., 2016). Hence, combining quasilinear preferences with Ricardian technology provides a suitable and tractable framework for analyzing agricultural policies and their welfare consequences. Third, incorporating general-equilibrium wage effects would not overturn our quantitative conclusions. As demonstrated by Costinot et al. (2015), the Lagrange multipliers associated with foreign resource constraints—which capture general-equilibrium considerations—are quantitatively

²For example, see Mayer (1984) on the direct democracy model; Grossman and Helpman (1994) on the protection-for-sale framework; Grossman and Helpman (1995) on trade wars and trade talks; Bagwell and Staiger (1999, 2004) on World Trade Organization (WTO) theory within a neoclassical trade model; Ossa (2011) on WTO theory within Krugman’s “new trade” model; Bagwell and Lee (2020) on unilateral, efficient, and Nash trade policies in a symmetric, two-country version of the Melitz and Ottaviano (2008) model; Grossman and Helpman (2021) on identity politics and trade policy; Becko et al. (2025) on optimal tariffs with geopolitical alignment; and Blanchard et al. (2025) on global value chains (GVCs) and trade policy. Maggi (2014) provides a comprehensive survey of the trade policy and trade agreement literature in the Handbook of International Economics, where a freely traded numeraire good is typically adopted in the theoretical analysis. One exception is Lashkaripour and Lugovskyy (2023), which studies unilateral optimal policy in a different framework, and we have provided a detailed comparison with their work in the introduction.

close to zero, even for a country as large as the United States. This suggests that most countries are effectively “small” in the sense that they cannot materially affect factor market clearing conditions in the rest of the world. Consequently, general-equilibrium considerations have negligible effects on optimal taxes in practice.

Labor is the sole factor of production, with a linear production technology across all sectors. The unit labor requirements for producing each good $i \in \Omega$ are constant, denoted by a_i for Home and a_i^* for Foreign. Firms operate under perfect competition, with labor perfectly mobile across sectors within each country but immobile internationally. Each consumer is assumed to be endowed with one unit of labor, resulting in labor endowments of L and L^* at Home and Foreign, respectively. The numeraire good is produced using a one-to-one technology in both countries, ensuring that it is always produced and consumed in each. Consequently, the wage rates are normalized to one: $w = w^* = 1$, which presupposes sufficiently large labor endowments. Additionally, the numeraire good is freely traded, with no taxes or subsidies imposed by either government.

We introduce strategic interactions between the two countries, with the domestic government imposing ad-valorem trade taxes or subsidies, denoted by $t \equiv (t_i)$, and the foreign government responding with its own ad-valorem trade taxes or subsidies, $t^* \equiv (t_i^*)$. When good i is imported, $t_i, t_i^* \geq 0$ represents an import tariff, while $t_i, t_i^* \leq 0$ represents an import subsidy. Conversely, if good i is exported, $t_i, t_i^* \geq 0$ indicates an export subsidy, and $t_i, t_i^* \leq 0$ an export tax. Tax revenues are rebated evenly to domestic consumers through lump-sum transfers, T and T^* , respectively.

In a competitive equilibrium with trade taxes for Home, utility optimization by domestic consumers, profit maximization by domestic firms, a balanced budget by the domestic government, and the clearing of the domestic labor market imply that:

$$c^l \in \operatorname{argmax}_{\tilde{c}^l \geq 0} \left\{ \int_i u_i^l(\tilde{c}_i^l) di + \tilde{c}_0^l \mid \int_i p_i(1 + t_i) \tilde{c}_i^l di + \tilde{c}_0^l \leq 1 + T^l \right\}, \quad (1)$$

$$q_i \in \operatorname{argmax}_{\tilde{q}_i \geq 0} \{ p_i(1 + t_i) \tilde{q}_i - a_i \tilde{q}_i \}, \quad (2)$$

$$T = T^l L = \int_i p_i t_i (c_i - q_i) di = \int_i p_i t_i (c_i^l L - q_i) di, \quad (3)$$

$$L = \int_i a_i q_i di + q_0, \quad (4)$$

where $p \equiv (p_i) \geq 0$, $q \equiv (q_i) \geq 0$, and $c \equiv (c_i) = (c_i^l L)$ represent the schedule of world prices, domestic output, and aggregate consumption in Home (H), respectively.

Similarly, the equilibrium conditions for Foreign imply:

$$c^{l*} \in \operatorname{argmax}_{\tilde{c}^l \geq 0} \{ \int_i u_i^{l*}(\tilde{c}_i^l) di + \tilde{c}_0^l \mid \int_i p_i(1 + t_i^*) \tilde{c}_i^l di + \tilde{c}_0^l \leq 1 + T^{l*} \}, \quad (5)$$

$$q_i^* \in \operatorname{argmax}_{\tilde{q}_i \geq 0} \{ p_i(1 + t_i^*) \tilde{q}_i - a_i^* \tilde{q}_i \}, \quad (6)$$

$$T^* = T^{l*} L^* = \int_i p_i t_i^* (c_i^* - q_i^*) di = \int_i p_i t_i^* (c_i^{l*} L^* - q_i^*) di, \quad (7)$$

$$L^* = \int_i a_i^* q_i^* di + q_0^*, \quad (8)$$

where $q^* \equiv (q_i^*) \geq 0$ denotes the foreign output, and $c^* \equiv (c_i^*) = (c_i^{l*} L^*)$ represents the aggregate consumption in Foreign (F).

Finally, good market clearing conditions imply:

$$q_0 + q_0^* = c_0 + c_0^* = c_0^l L + c_0^{l*} L^*, \quad (9)$$

$$q_i + q_i^* = c_i + c_i^* = c_i^l L + c_i^{l*} L^*, \forall i. \quad (10)$$

2.2 Consumers' Problem

As previously mentioned, we assume that labor endowments are sufficiently large to ensure that the production and consumption of the numeraire good remain active. In a quasilinear utility setting, this implies positive consumption for any non-numeraire good. The consumers' optimization problem can then be expressed as:

$$\max_{c^l > 0} \mathcal{U}^l(c^l) = \int_{i \in \Omega} \beta_i \frac{(c_i^l)^{1 - \frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} di + c_0^l$$

subject to

$$\int_i p_i(1 + t_i) c_i^l di + c_0^l = 1 + T^l \equiv I^l.$$

Solving this optimization problem yields the individual consumption of the non-numeraire good i :

$$c_i^l = \left[\frac{p_i(1 + t_i)}{\beta_i} \right]^{-\sigma}. \quad (11)$$

The resulting individual indirect utility function is given by:

$$\begin{aligned}\mathcal{U}^l &= 1 + T^l + \int_{i \in \Omega} \left[\beta_i \frac{(c_i^l)^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} - p_i(1+t_i)c_i^l \right] di \\ &= \underbrace{1}_{\text{wage}} + \underbrace{T^l}_{\text{tax revenue}} + \underbrace{\frac{1}{\sigma-1} \int_{i \in \Omega} \left\{ [p_i(1+t_i)]^{1-\sigma} \beta_i^\sigma - \beta_i \sigma \right\} di}_{\text{consumer surplus}}.\end{aligned}\quad (12)$$

The welfare decomposition in (12) shows that, under quasilinear utility, consumers' gross utility consists of the consumer surplus from the consumption of non-numeraire goods, the producer surplus (which is zero under perfect competition), tax revenue, and wage income from one unit of endowed labor, as discussed in [Furusawa and Konishi \(2004\)](#). Given our assumption that the numeraire good is always consumed, this implies that $I^l \equiv 1 + T^l > \int_{i \in \Omega} [p_i(1+t_i)]^{1-\sigma} \beta_i^\sigma di$.

2.3 Government's Problem

Departing from the setup in [Costinot et al. \(2015\)](#), we assume that both Home and Foreign are strategic countries. Each government aims to maximize the aggregate utility of its domestic consumers by choosing an optimal trade policy t . Thus, the domestic government's optimization problem is defined as

$$\max_t \mathcal{U}(c) = \mathcal{U}^l(c^l)L = -\frac{\sigma L}{\sigma-1} + L + \int_{i \in \Omega} p_i t_i (c_i - q_i) di + \frac{L}{\sigma-1} \int_{i \in \Omega} [p_i(1+t_i)]^{1-\sigma} \beta_i^\sigma di, \quad (13)$$

where $q_i \in \operatorname{argmax}_{\tilde{q}_i \geq 0} \{p_i(1+t_i)\tilde{q}_i - a_i \tilde{q}_i\}$, as shown in (2).

Note that the government's problem consists of two parts: the first is a constant, while the second is additively separable in i . This allows us to solve the government's problem on a good-by-good basis. Thus, the good-specific objective function in Home (H) is defined as:

$$\mathcal{W}_i = p_i t_i (c_i - q_i) + \frac{L}{\sigma-1} [p_i(1+t_i)]^{1-\sigma} \beta_i^\sigma. \quad (14)$$

Similarly, the foreign government's problem can be written as:

$$\max_{t^*} \mathcal{U}^*(c^*) = -\frac{\sigma^* L^*}{\sigma^*-1} + L^* + \int_{i \in \Omega} p_i t_i^* (c_i^* - q_i^*) di + \frac{L^*}{\sigma^*-1} \int_{i \in \Omega} [p_i(1+t_i^*)]^{1-\sigma^*} (\beta_i^*)^{\sigma^*} di, \quad (15)$$

where $q_i^* \in \operatorname{argmax}_{\tilde{q}_i \geq 0} \{p_i(1+t_i^*)\tilde{q}_i - a_i^* \tilde{q}_i\}$. The good-specific objective function in Foreign (F) is then:

$$\mathcal{W}_i^* = p_i t_i^* (c_i^* - q_i^*) + \frac{L^*}{\sigma^*-1} [p_i(1+t_i^*)]^{1-\sigma^*} (\beta_i^*)^{\sigma^*}. \quad (16)$$

3 Unilateral Optimal Trade Policy

As long as Home produces, perfect competition implies $p_i(1 + t_i) = a_i$. Home's aggregate consumption is then $c_i = Lc_i^I = L(\frac{a_i}{\beta_i})^{-\sigma}$ from (11). We can thus express world prices, Home's output, and consumption as functions of its net imports, $m \equiv c - q$, and taxes as follows:

$$p_i(m_i, t_i, t_i^*) \equiv \min\left\{\frac{a_i}{1 + t_i}, u_i'(m_i/L)/(1 + t_i)\right\}, \quad (17)$$

$$q_i(m_i, t_i, t_i^*) \equiv \max\{L(a_i/\beta_i)^{-\sigma} - m_i, 0\}, \quad (18)$$

$$c_i(m_i, t_i, t_i^*) \equiv \max\{L(a_i/\beta_i)^{-\sigma}, m_i\}. \quad (19)$$

From (17)-(19), when Home's net imports are sufficiently high such that $L(\frac{a_i}{\beta_i})^{-\sigma} - m_i < 0$, Home's firms do not produce good i . In this case, Home's consumption equals its net imports, m_i , and the world price is determined by Home's marginal utility and taxes: $p_i(m_i, t_i, t_i^*) = u_i'(m_i/L)/(1 + t_i)$.

By similar reasoning, we find that foreign aggregate consumption is $c_i^* = L^*c_i^{I*} = L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}$, with the world price satisfying $p_i(1 + t_i^*) = a_i^*$ when Foreign produces. Consequently, world prices, Foreign's output, and consumption can be expressed as functions of Home's net imports and the respective taxes:

$$p_i(m_i, t_i, t_i^*) \equiv \min\{u_i^{*'}(-m_i/L^*)/(1 + t_i^*), a_i^*/(1 + t_i^*)\}, \quad (20)$$

$$q_i^*(m_i, t_i, t_i^*) \equiv \max\{0, L^*(a_i^*/\beta_i^*)^{-\sigma^*} + m_i\}, \quad (21)$$

$$c_i^*(m_i, t_i, t_i^*) \equiv \max\{-m_i, L^*(a_i^*/\beta_i^*)^{-\sigma^*}\}. \quad (22)$$

From (20)-(22), when Home's net imports are very low, i.e., $L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*} + m_i < 0$, Foreign's firms do not produce good i . In this case, Foreign's consumption equals Home's net exports, $-m_i$. The world price is then determined by Foreign's marginal utility and taxes: $p_i(m_i, t_i, t_i^*) = u_i^{*'}(-m_i/L^*)/(1 + t_i^*)$.

From the analysis above, we identify two cut-off points for Home's net imports: $M_i^I \equiv -L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}$, above which Foreign must produce good i , and $M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$, below which Home must produce good i . Accordingly, we can divide Home's net imports of good i into three regions: Region I with $m_i \leq M_i^I \equiv -L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}$, Region II with $M_i^I \leq m_i \leq M_i^{II}$, and Region III with $m_i \geq M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$. Our objective in this section is to determine Home's optimal trade taxes (t_i), given Foreign's trade taxes (t_i^*), $\forall i \in \Omega$. In this scenario, the domestic government aims to $\max_{t_i} \mathcal{W}_i$, as defined in (14), subject to the conditions in (17)-(22). We will now solve the domestic government's welfare optimization problem region by region.

3.1 Region I: $m_i \leq M_i^I \equiv -L^* \left(\frac{a_i^*}{\beta_i^*} \right)^{-\sigma^*}$

In the first region, where $m_i \leq M_i^I \equiv -L^* \left(\frac{a_i^*}{\beta_i^*} \right)^{-\sigma^*}$, only Home produces good i . Under these conditions, equations (17)–(22) can be simplified as follows: $m_i \leq -L^* \left(\frac{a_i^*}{\beta_i^*} \right)^{-\sigma^*}$, and $p_i = \frac{a_i}{1+t_i} = \frac{\beta_i^*}{1+t_i^*} \left(\frac{-m_i}{L^*} \right)^{-1/\sigma^*} \leq \frac{a_i^*}{1+t_i^*}$. Home's net imports can thus be expressed as $m_i = -L^* \left(\frac{a_i^*}{\beta_i^*} \frac{1+t_i^*}{1+t_i} \right)^{-\sigma^*}$. Accordingly, the domestic government's objective function in (14) can be rewritten as:

$$\begin{aligned} \mathcal{W}_i(m_i, t_i; t_i^*) &= p_i t_i (c_i - q_i) + \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma \\ &= a_i m_i - p_i m_i + \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma \\ &= a_i m_i - \frac{\beta_i^*}{1+t_i^*} \left(\frac{-m_i}{L^*} \right)^{-1/\sigma^*} m_i + \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma. \end{aligned} \quad (23)$$

The function $\mathcal{W}_i(m_i, t_i; t_i^*)$ is strictly increasing for $m_i \in (-\infty, m_i^I)$ and strictly decreasing for $m_i \in (m_i^I, M_i^I)$, where $m_i^I \equiv -L^* \left[\frac{a_i}{\beta_i^*} \frac{\sigma^*}{\sigma^*-1} (1+t_i^*) \right]^{-\sigma^*}$. Consequently, the domestic government will select the optimized net imports of good i , denoted as $m_i^{Io} = m_i^I$, yielding a world price of $p_i = \frac{\beta_i^*}{1+t_i^*} \left(\frac{-m_i^I}{L^*} \right)^{-1/\sigma^*} = \frac{\sigma^*}{\sigma^*-1} a_i$. Home's optimal trade tax for good i satisfies $1+t_i = a_i/p_i = \frac{\sigma^*-1}{\sigma^*}$, and the optimized welfare for Home is: $\mathcal{W}_i^{Io} = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + \frac{L^* a_i}{\sigma^*-1} \left[\frac{a_i}{\beta_i^*} \frac{\sigma^*}{\sigma^*-1} (1+t_i^*) \right]^{-\sigma^*}$. Recalling that $M_i^I \equiv -L^* \left(\frac{a_i^*}{\beta_i^*} \right)^{-\sigma^*}$, the interval $[m_i^I, M_i^I]$ is non-empty if $\frac{a_i}{a_i^*} \leq \frac{\sigma^*-1}{\sigma^*} \frac{1}{1+t_i^*} \equiv A_i^I$, or equivalently, $1+t_i^* \leq \frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} \equiv B_i^I$.

However, if $m_i^I \geq M_i^I$, i.e., $1+t_i^* \geq \frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i}$, then $\mathcal{W}_i(m_i, t_i; t_i^*)$ is maximized at $m_i^{Io} = M_i^I$. From the condition $m_i = -L^* \left(\frac{a_i^*}{\beta_i^*} \frac{1+t_i^*}{1+t_i} \right)^{-\sigma^*} = M_i^I = -L^* \left(\frac{a_i^*}{\beta_i^*} \right)^{-\sigma^*}$, Home's optimal tax becomes $1+t_i = \frac{a_i}{a_i^*} (1+t_i^*) \geq \frac{\sigma^*-1}{\sigma^*}$. As a result, the world price satisfies $p_i = \frac{a_i}{1+t_i} = \frac{a_i^*}{1+t_i^*}$. Home's welfare, in this case, becomes $\mathcal{W}_i^{Io} = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + (a_i - \frac{a_i^*}{1+t_i^*}) \left[-L^* \left(\frac{a_i^*}{\beta_i^*} \right)^{-\sigma^*} \right]$.

The intuition behind the varying values of $(1+t_i)$ in response to $(1+t_i^*)$ is as follows. In this region, only Home produces good i . When $\frac{a_i}{a_i^*} \leq \frac{\sigma^*-1}{\sigma^*} \frac{1}{1+t_i^*} \equiv A_i^I$, or equivalently, $1+t_i^* \leq \frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} \equiv B_i^I$, Foreign's import tariff is relatively low, giving Home significant flexibility to influence the world price. In this case, Home sets a constant monopoly markup over its marginal cost, leading to a world price of $p_i = \frac{\sigma^*}{\sigma^*-1} a_i$. However, if $1+t_i^* \geq \frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i}$ —indicating that Foreign has raised its import tariff to shield its domestic producers—Home must lower its export tax to adjust to Foreign's tariff, ensuring that Foreign is indifferent between producing or not producing good i . As a result, the world price becomes $p_i = \frac{a_i}{1+t_i} = \frac{a_i^*}{1+t_i^*}$, reflecting a limit-pricing strategy by Home, as described by Costinot et al. (2015). If Foreign's tariff becomes exceedingly high, the world price could fall too low. To sustain domestic production and exports, the Home government might shift from an export tax to a positive export

subsidy, with $1 + t_i > 1$. However, it's essential to note that this analysis assumes that only Home is producing. The trade taxes (t_i) determined here may not necessarily be the optimal ones overall, given the known values (a_i, a_i^*, t_i^*) .

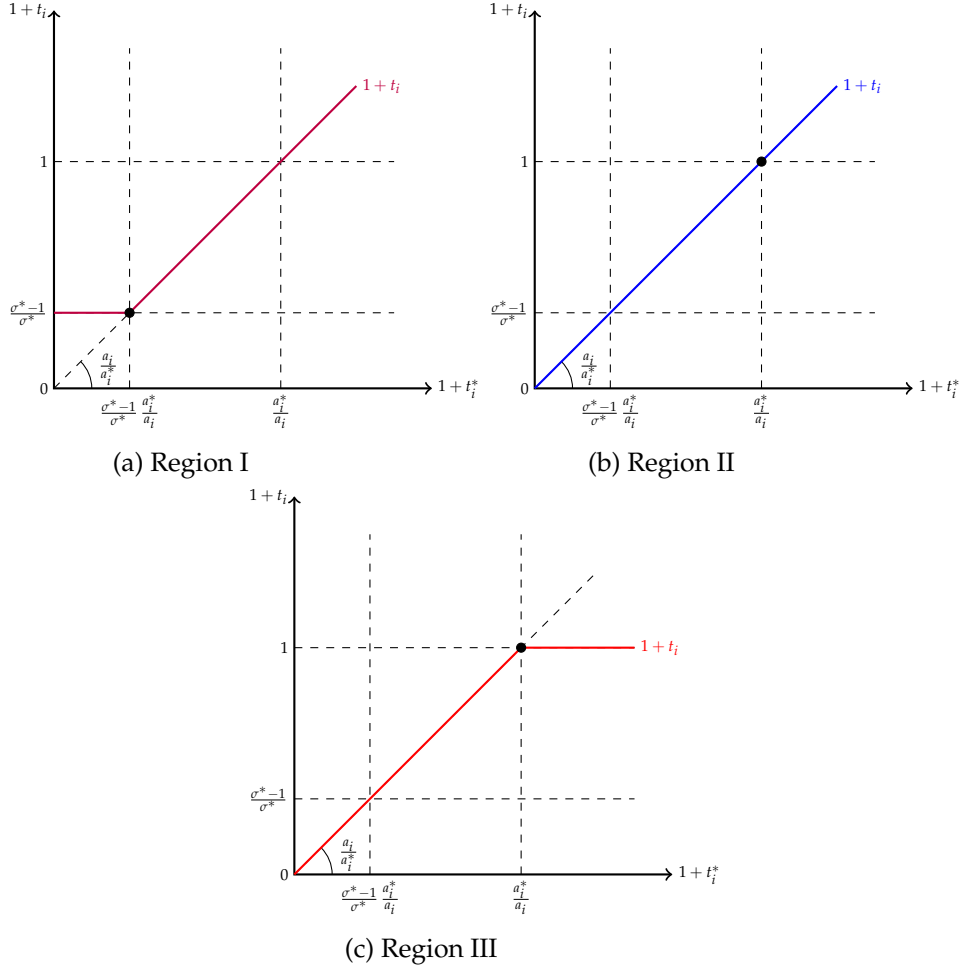


Figure 1: Best Response in Region I, II, and III

3.2 Region II: $-L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*} \equiv M_i^I \leq m_i \leq M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$

In the middle region, both countries are able to produce good i . Under these conditions, equations (17)–(22) can be simplified as follows: $-L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*} \equiv M_i^I \leq m_i \leq M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$, and $p_i = \frac{a_i}{1+t_i} = \frac{a_i^*}{1+t_i^*}$. Accordingly, the domestic government's objective function in (14) can be rewritten as:

$$\mathcal{W}_i(m_i, t_i; t_i^*) = (a_i - p_i)m_i + \frac{L}{\sigma - 1} a_i^{1-\sigma} \beta_i^\sigma. \quad (24)$$

If $a_i \leq \frac{a_i^*}{1+t_i^*} = p_i$, then $\mathcal{W}_i(m_i, t_i; t_i^*)$ is decreasing in m_i . Therefore, $\mathcal{W}_i(m_i, t_i; t_i^*)$ is maxi-

mized at $m_i^{IIo} = M_i^I$, where only Home produces. The optimal tax satisfies $1 + t_i = a_i/p_i = \frac{a_i}{a_i^*}(1 + t_i^*) \leq 1$. Home's welfare in this case is: $\mathcal{W}_i^{IIo} = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + (a_i - \frac{a_i^*}{1+t_i^*}) \left[-L^* (\frac{a_i^*}{\beta_i^*})^{-\sigma^*} \right]$. We define this as Region IIA, where $\frac{a_i}{a_i^*} \leq \frac{1}{1+t_i^*} \equiv A_i^{II}$ and $1 + t_i^* \leq \frac{a_i^*}{a_i} \equiv B_i^{II}$.

In contrast, if $a_i \geq \frac{a_i^*}{1+t_i^*} = p_i$, then $\mathcal{W}_i(m_i, t_i; t_i^*)$ is increasing in m_i . Consequently, $\mathcal{W}_i(m_i, t_i; t_i^*)$ is maximized at $m_i^{IIo} = M_i^{II}$, where only Foreign produces. The optimal tax satisfies $1 + t_i = a_i/p_i = \frac{a_i}{a_i^*}(1 + t_i^*) \geq 1$. Home's welfare in this scenario is: $\mathcal{W}_i^{IIo} = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + (a_i - \frac{a_i^*}{1+t_i^*}) \left[L (\frac{a_i}{\beta_i})^{-\sigma} \right]$. This region is defined as Region IIB, where $\frac{a_i}{a_i^*} \geq \frac{1}{1+t_i^*} \equiv A_i^{II}$ and $1 + t_i^* \geq \frac{a_i^*}{a_i} \equiv B_i^{II}$.

As noted, in Region II, both Home and Foreign are able to produce, but Home's capacity to influence world prices is significantly constrained by Foreign. Therefore, a limit-pricing strategy still applies in this region. In Region IIA, where $1 + t_i^* \leq \frac{a_i^*}{a_i}$, Foreign's chosen trade taxes are relatively low, keeping world prices above Home's marginal production cost. This allows Home to take over the entire production of good i , export to Foreign, and impose positive export taxes. In contrast, when $1 + t_i^* \geq \frac{a_i^*}{a_i}$, Foreign's trade taxes are high enough to squeeze world prices below Home's marginal cost. Consequently, Home ceases production of these goods, relying instead on imports from Foreign and simultaneously imposing positive import tariffs.

3.3 Region III: $m_i \geq M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$

The argument presented for Region I similarly applies to Region III. In this third region, where $m_i \geq M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$, only Foreign produces good i . Under these conditions, equations (17)–(22) can be simplified as: $m_i \geq M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$, and $p_i = \frac{a_i^*}{1+t_i^*} = \frac{\beta_i}{1+t_i} (\frac{m_i}{L})^{-1/\sigma} \leq \frac{a_i}{1+t_i}$. From this, we obtain Home's net imports as $m_i = L \left[\frac{a_i^*(1+t_i)}{\beta_i(1+t_i^*)} \right]^{-\sigma}$ and the relationship $p_i(1 + t_i) = \beta_i (\frac{m_i}{L})^{-1/\sigma}$. Accordingly, the domestic government's objective function in (14) can be rewritten as:

$$\begin{aligned} \mathcal{W}_i(m_i, t_i; t_i^*) &= p_i t_i (c_i - q_i) + \frac{L}{\sigma-1} [p_i(1 + t_i)]^{1-\sigma} \beta_i^\sigma \\ &= \left[\beta_i \left(\frac{m_i}{L} \right)^{-1/\sigma} - \frac{a_i^*}{1+t_i^*} \right] m_i + \frac{L}{\sigma-1} \left[\beta_i \left(\frac{m_i}{L} \right)^{-1/\sigma} \right]^{1-\sigma} \beta_i^\sigma \\ &= \frac{\sigma \beta_i}{\sigma-1} L^{1/\sigma} m_i^{1-1/\sigma} - \frac{a_i^*}{1+t_i^*} m_i \end{aligned} \quad (25)$$

which is strictly increasing if $m_i \in (M_i^{II}, m_i^{III})$, and strictly decreasing if $m_i \in (m_i^{III}, +\infty)$, where $m_i^{III} \equiv L \left[\frac{a_i^*}{\beta_i} \frac{1}{1+t_i^*} \right]^{-\sigma}$. Consequently, the domestic government will choose the optimized net imports of good i , $m_i^{IIo} = m_i^{III}$, resulting in a world price of $p_i = \frac{\beta_i}{1+t_i} (\frac{m_i}{L})^{-1/\sigma} =$

$\frac{1}{1+t_i} \frac{a_i^*}{1+t_i^*} = \frac{a_i^*}{1+t_i^*}$. Thus, Home's optimal trade tax for good i must satisfy $1+t_i = 1$. Home's optimized welfare is then: $\mathcal{W}_i^{IIo} = \frac{L}{\sigma-1} \left[\frac{a_i^*}{1+t_i^*} \right]^{1-\sigma} \beta_i^\sigma$. Recalling that $M_i^{II} \equiv L \left(\frac{a_i}{\beta_i} \right)^{-\sigma}$, the interval $[M_i^{II}, m_i^{III}]$ is non-empty if $\frac{a_i}{a_i^*} \geq \frac{1}{1+t_i^*} \equiv A_i^{III} = A_i^{II}$ or $1+t_i^* \geq \frac{a_i^*}{a_i} \equiv B_i^{III} = B_i^{II}$.

However, if $m_i^{III} \leq M_i^{II}$, i.e., $0 < 1+t_i^* \leq \frac{a_i^*}{a_i}$, then $\mathcal{W}_i(m_i, t_i; t_i^*)$ is maximized at $m_i^{IIIo} = M_i^{II}$. Given $m_i = L \left[\frac{a_i^*(1+t_i)}{\beta_i(1+t_i^*)} \right]^{-\sigma} = M_i^{II} \equiv L \left(\frac{a_i}{\beta_i} \right)^{-\sigma}$, Home's optimal tax becomes $1+t_i = \frac{a_i}{a_i^*} (1+t_i^*) \leq 1$. As a result, the world price is $p_i = \frac{a_i^*}{1+t_i^*} = \frac{a_i}{1+t_i}$. Home's welfare in this case becomes: $\mathcal{W}_i^{IIIo} = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + (a_i - \frac{a_i^*}{1+t_i^*}) \left[-L^* \left(\frac{a_i^*}{\beta_i^*} \right)^{-\sigma^*} \right]$.

In this region, only Foreign produces good i . When $\frac{a_i}{a_i^*} \geq \frac{1}{1+t_i^*}$, or equivalently, $1+t_i^* \geq \frac{a_i^*}{a_i}$, Foreign implements a high export subsidy, enabling Home to import from Foreign at a low world price. In this scenario, Home effectively embraces free trade, and domestic firms cease production. However, when $0 < 1+t_i^* \leq \frac{a_i^*}{a_i}$ —meaning that Foreign reduces its export subsidy for good i or even imposes a positive export tax, leading to a higher world price—Home must impose a positive import subsidy to ensure that its domestic firms remain indifferent to producing or not.

3.4 Best Response

We have determined Home's best responses in each region, as illustrated in Figure 1. According to the definition of optimal trade policy, (t_i) represents the domestic government's optimal choice given the foreign taxes (t_i^*) and the technologies (a_i, a_i^*) across all three regions. In other words,

$$1+t_i \equiv \operatorname{argmax}_{(t_i^*, a_i, a_i^*)} \left\{ \mathcal{W}_i^{Io}(t_i^*; a_i, a_i^*), \mathcal{W}_i^{IIo}(t_i^*; a_i, a_i^*), \mathcal{W}_i^{IIIo}(t_i^*; a_i, a_i^*) \right\}. \quad (26)$$

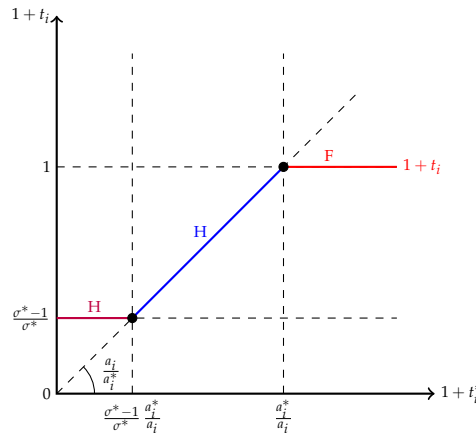


Figure 2: Home's Unilateral Optimal Trade Policy

Note that there are two cut-off values for $(1 + t_i^*)$: $B_i^I \equiv \frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i}$ and $B_i^{II} \equiv \frac{a_i^*}{a_i} \equiv B_i^{III}$. Accordingly, Home's optimal trade policy $(1 + t_i)$ can be divided into five scenarios: Scenario 1 with $0 < 1 + t_i^* < \frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i}$, Scenario 2 with $1 + t_i^* = \frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i}$, Scenario 3 with $\frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} < 1 + t_i^* < \frac{a_i^*}{a_i}$, Scenario 4 with $1 + t_i^* = \frac{a_i^*}{a_i}$, and Scenario 5 with $1 + t_i^* > \frac{a_i^*}{a_i}$. In each scenario, Home selects the trade policy that maximizes welfare among $\mathcal{W}_i^{Io}(t_i^*; a_i, a_i^*)$, $\mathcal{W}_i^{IIo}(t_i^*; a_i, a_i^*)$, and $\mathcal{W}_i^{IIIo}(t_i^*; a_i, a_i^*)$.

After comparing the optimized welfare across the three regions in each of the five scenarios, we can summarize Home's unilateral optimal trade policy in the following proposition, as shown in Figure 2.

Proposition 1 *If the two strategic countries set their optimal trade policies noncooperatively, given the foreign trade taxes (t_i^*) and technology parameters (a_i, a_i^*) , Home's unilateral optimal trade policy is characterized as follows. The producer, optimal trade taxes (t_i) , world prices (p_i) , Home's net imports (m_i) , and Home's optimized welfare $(\mathcal{W}_i)$ satisfy:*

(a) *only Home produces, $1 + t_i = \frac{\sigma^*-1}{\sigma^*}$, $p_i = \frac{\sigma^*}{\sigma^*-1} a_i$, $m_i = m_i^I$, and $\mathcal{W}_i = \mathcal{W}_i^{Io}$, if $1 + t_i^* \in (0, B_i^I)$;*

(b) *only Home produces, $1 + t_i = \frac{\sigma^*-1}{\sigma^*} = \frac{a_i}{a_i^*} (1 + t_i^*)$, $p_i = \frac{\sigma^*}{\sigma^*-1} a_i = \frac{a_i^*}{1+t_i^*} = \frac{a_i}{1+t_i}$, $m_i = m_i^I = M_i^I$, and $\mathcal{W}_i = \mathcal{W}_i^{Io} = \mathcal{W}_i^{IIo}$, if $1 + t_i^* = B_i^I$;*

(c) *only Home produces, $1 + t_i = \frac{a_i}{a_i^*} (1 + t_i^*)$, $p_i = \frac{a_i}{1+t_i} = \frac{a_i^*}{1+t_i^*}$, $m_i = M_i^I$, and $\mathcal{W}_i = \mathcal{W}_i^{IIo}$, if $1 + t_i^* \in (B_i^I, B_i^{II})$;*

(d) *either Home or Foreign produces, $1 + t_i = 1$, $p_i = a_i = \frac{a_i^*}{1+t_i^*}$, $m_i \in [M_i^I, M_i^{II}]$, and $\mathcal{W}_i = \mathcal{W}_i^{IIo} = \mathcal{W}_i^{IIIo}$, if $1 + t_i^* = B_i^{II}$;*

(e) *only Foreign produces, $1 + t_i = 1$, $p_i = \frac{a_i^*}{1+t_i^*}$, $m_i = m_i^{III}$, and $\mathcal{W}_i = \mathcal{W}_i^{IIIo}$, if $1 + t_i^* \in (B_i^{II}, +\infty)$; where $B_i^I \equiv \frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i}$, $B_i^{II} \equiv \frac{a_i^*}{a_i} \equiv B_i^{III}$; $M_i^I \equiv -L^* \left(\frac{a_i^*}{\beta_i^*} \right)^{-\sigma^*}$, $M_i^{II} \equiv L \left(\frac{a_i}{\beta_i} \right)^{-\sigma}$; $m_i^I \equiv -L^* \left[\frac{a_i}{\beta_i^*} \frac{\sigma^*}{\sigma^*-1} (1 + t_i^*) \right]^{-\sigma^*}$, $m_i^{III} \equiv L \left[\frac{a_i^*}{\beta_i^*} \frac{1}{1+t_i^*} \right]^{-\sigma}$; $\mathcal{W}_i^{Io} \equiv \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + \frac{L^* a_i}{\sigma^*-1} \left[\frac{a_i}{\beta_i^*} \frac{\sigma^*}{\sigma^*-1} (1 + t_i^*) \right]^{-\sigma^*}$, $\mathcal{W}_i^{IIo} \equiv \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + (a_i - \frac{a_i^*}{1+t_i^*}) \left[-L^* \left(\frac{a_i^*}{\beta_i^*} \right)^{-\sigma^*} \right]$, $\mathcal{W}_i^{IIIo} \equiv \frac{L}{\sigma-1} \left[\frac{a_i^*}{1+t_i^*} \right]^{1-\sigma} \beta_i^\sigma$.*

Proposition 1 underscores the significant influence of fundamental comparative advantage $(\frac{a_i^*}{a_i})$ and Foreign's trade policy (t_i^*) on production decisions, optimal import levels, and the imposition of optimal trade taxes, with strategic interactions accounted for. If Foreign sets taxes $(1 + t_i^*)$ above its fundamental comparative advantage $(\frac{a_i^*}{a_i})$, thereby overly protecting its domestic firms, it becomes optimal for Home to adopt a free trade policy and rely solely on imports from Foreign. Conversely, if Foreign's production is not excessively protected, i.e., $1 + t_i^* < \frac{a_i^*}{a_i}$, Home can take over the entire production of good i and impose a positive export tax. However, Home must employ a limit-pricing strategy that leaves foreign firms indifferent to producing good i . If the foreign tariff is exceptionally low—meaning Home has a strong comparative advantage in producing good i —then Home can apply a monopoly

markup over its marginal cost, effectively behaving like a monopolist.

An interesting case arises when Foreign sets $t_i^* = 0$.³ From the preceding discussion, a specialization cut-off exists at $\frac{a_i^*}{a_i} = 1$, which is related to absolute advantage. This cut-off stems from the equalized wage rates in both countries, a result of quasilinear preferences. As long as Home has an absolute advantage over Foreign in producing good i , it will take over the entire production and can impose a positive export tax. Otherwise, Home will adopt a free trade policy without producing good i , reallocating its labor toward the numeraire good.

The unilateral optimal trade policy can also be examined in the presence of exogenous iceberg trade costs. In Appendix A, we demonstrate that the unilateral optimal trade policy with trade costs remains qualitatively similar to the case without trade costs, with the primary difference being the emergence of a range of nontraded goods.

4 Nash Equilibrium

Based on Home's best response above, we can similarly characterize Foreign's best response, as depicted in Figure 3. Recall that Home's optimal trade policy introduced two cut-offs for $(1 + t_i^*)$: $\frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i}$ and $\frac{a_i^*}{a_i}$. Now, Foreign's best response adds two additional cut-offs: $\frac{\sigma-1}{\sigma}$ and 1. Combining these four cut-offs, we identify seven possible cases for $\frac{a_i}{a_i^*}$: Case 1 with $0 < \frac{a_i}{a_i^*} < \frac{\sigma^*-1}{\sigma^*}$, Case 2 with $\frac{a_i}{a_i^*} = \frac{\sigma^*-1}{\sigma^*}$, Case 3 with $\frac{\sigma^*-1}{\sigma^*} < \frac{a_i}{a_i^*} < 1$, Case 4 with $\frac{a_i}{a_i^*} = 1$, Case 5 with $1 < \frac{a_i}{a_i^*} < \frac{\sigma}{\sigma-1}$, Case 6 with $\frac{a_i}{a_i^*} = \frac{\sigma}{\sigma-1}$, and Case 7 with $\frac{a_i}{a_i^*} > \frac{\sigma}{\sigma-1}$. We will discuss each of these cases individually. Figure 4 illustrates the Nash Equilibrium in each scenario, with the blue curve representing Home's best response and the red curve representing Foreign's.

Case 1: $0 < \frac{a_i}{a_i^*} < \frac{\sigma^*-1}{\sigma^*}$. In this case, the ranking of the four cut-offs is $\frac{\sigma-1}{\sigma} < 1 < \frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} < \frac{a_i^*}{a_i}$. Based on this ranking, we can plot the best responses of both countries together, as shown in Sub-figure 4a of Figure 4. The Nash Equilibrium policy is $1 + t_i = \frac{\sigma^*-1}{\sigma^*}$ and $1 + t_i^* = 1$. In equilibrium, only Home produces good i , and the world price is $p_i = \frac{\sigma^*}{\sigma^*-1} a_i$. Home's optimal imports are $m_i^o = -L^* \left[\frac{a_i}{\beta_i^* \sigma^*-1} \right]^{-\sigma^*}$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + \frac{L^* a_i}{\sigma^*-1} \left[\frac{a_i}{\beta_i^* \sigma^*-1} \right]^{-\sigma^*}$ and domestic price $p_i^d = p_i(1 + t_i) = a_i$.

Case 2: $\frac{a_i}{a_i^*} = \frac{\sigma^*-1}{\sigma^*}$. In this case, the ranking of the four cut-offs is $\frac{\sigma-1}{\sigma} < 1 = \frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} < \frac{a_i^*}{a_i}$. Based on this ranking, we can plot both countries' best responses together, as shown in Sub-figure 4b of Figure 4. The Nash Equilibrium policy is $1 + t_i = \frac{\sigma^*-1}{\sigma^*}$ and $1 + t_i^* = 1$. In

³Note that Home's best response in the case of $t_i^* = 0$ coincides with the unilateral optimal trade policy in Costinot et al. (2015) when import tariffs are excluded, and with the unilateral optimal trade policy in Lashkaripour and Lugovskyy (2023) when scale economies are absent. This correspondence is natural, as our analysis considers Nash policies, while their focus is restricted to unilateral optimal trade policy.

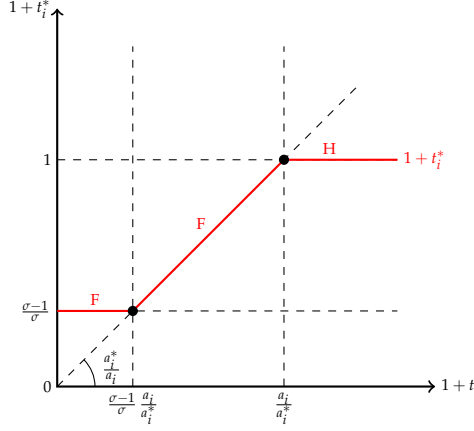


Figure 3: Foreign's Unilateral Optimal Trade Policy

equilibrium, only Home produces good i , and the world price is $p_i = \frac{\sigma^*}{\sigma^*-1} a_i = a_i^*$. Home's optimal imports are $m_i^o = -L^* \left[\frac{a_i^*}{\beta_i^*} \frac{\sigma^*}{\sigma^*-1} \right]^{-\sigma^*} = -L^* \left[\frac{a_i^*}{\beta_i^*} \right]^{-\sigma^*} = M_i^I$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + \frac{L^* a_i}{\sigma^*-1} \left[\frac{a_i^*}{\beta_i^*} \frac{\sigma^*}{\sigma^*-1} \right]^{-\sigma^*} = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + (a_i^* - a_i) L^* \left[\frac{a_i^*}{\beta_i^*} \right]^{-\sigma^*}$ and domestic price $p_i^d = a_i$.

Case 3: $\frac{\sigma^*-1}{\sigma^*} < \frac{a_i}{a_i^*} < 1$. In this case, the ranking of the four cut-offs is $\frac{\sigma-1}{\sigma} < 1 < \frac{a_i^*}{a_i}$ and $\frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} < 1 < \frac{a_i^*}{a_i}$. Based on this ranking, we can plot both countries' best responses together, as shown in Sub-figure 4c of Figure 4. The Nash Equilibrium policy is $1 + t_i = \frac{a_i}{a_i^*}$ and $1 + t_i^* = 1$. In equilibrium, only Home produces good i , and the world price is $p_i = a_i^*$. Home's optimal imports are $m_i^o = -L^* \left[\frac{a_i^*}{\beta_i^*} \right]^{-\sigma^*} = M_i^I$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + (a_i^* - a_i) L^* \left[\frac{a_i^*}{\beta_i^*} \right]^{-\sigma^*}$ and domestic price $p_i^d = a_i$.

Case 4: $\frac{a_i}{a_i^*} = 1$. In this case, the ranking of the four cut-offs is $\frac{\sigma-1}{\sigma} < 1 = \frac{a_i^*}{a_i}$ and $\frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} < 1 = \frac{a_i^*}{a_i}$. Based on this ranking, we can plot both countries' best responses together, as shown in Sub-figure 4d of Figure 4. The Nash Equilibrium policy is $1 + t_i = 1$ and $1 + t_i^* = 1$. In equilibrium, either Home or Foreign can produce good i , and the world price is $p_i = a_i^* = a_i$. Home's optimal imports m_i^o can take any value in $[M_i^I, M_i^{II}]$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma$ and domestic price $p_i^d = a_i = a_i^*$.

Case 5: $1 < \frac{a_i}{a_i^*} < \frac{\sigma}{\sigma-1}$. In this case, the ranking of the four cut-offs is $\frac{\sigma-1}{\sigma} < \frac{a_i^*}{a_i} < 1$ and $\frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} < \frac{a_i^*}{a_i} < 1$. Based on this ranking, we can plot both countries' best responses together, as shown in Sub-figure 4e of Figure 4. The Nash Equilibrium policy is $1 + t_i = 1$ and $1 + t_i^* = \frac{a_i^*}{a_i}$.

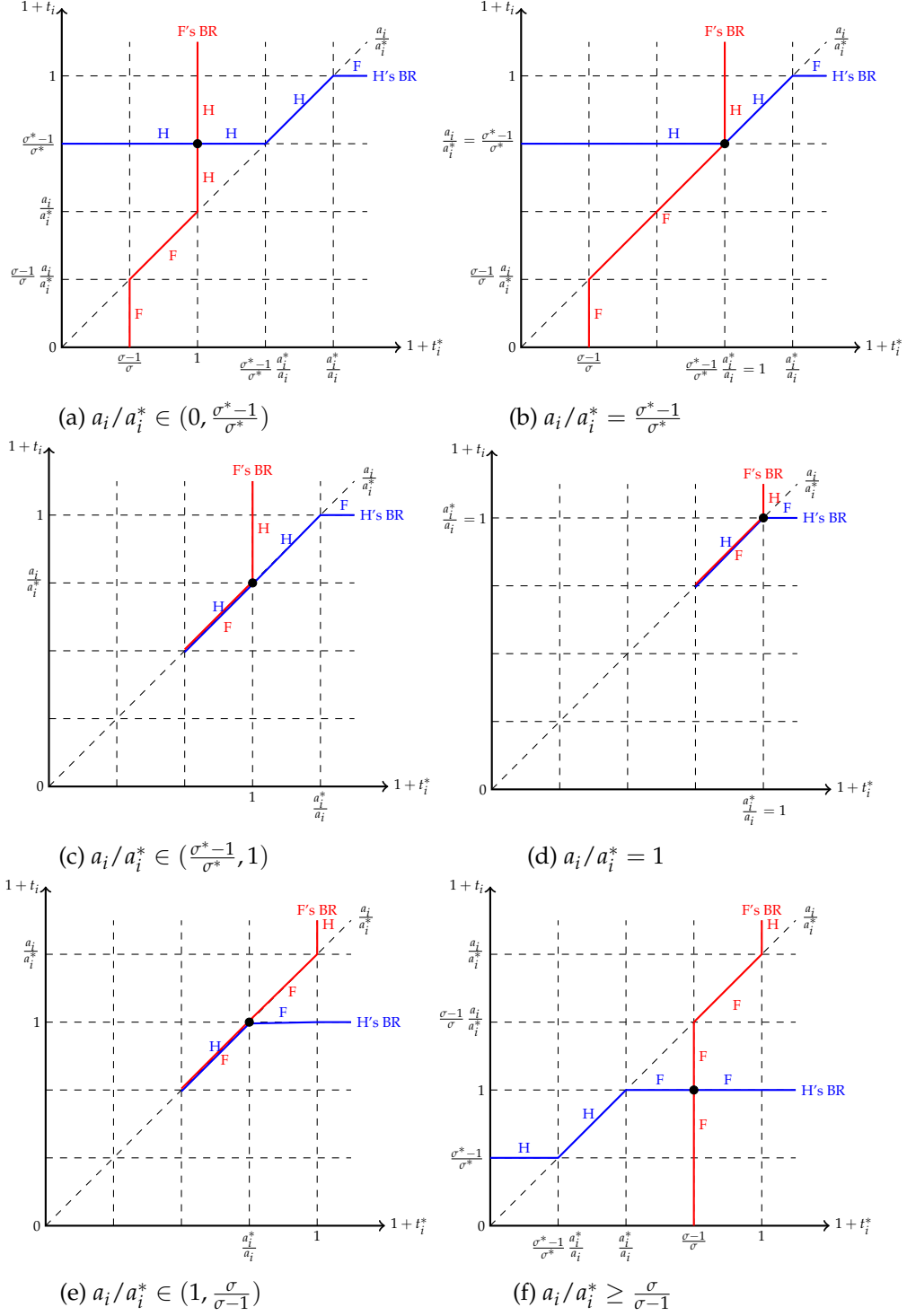


Figure 4: Nash Equilibrium in Each Case

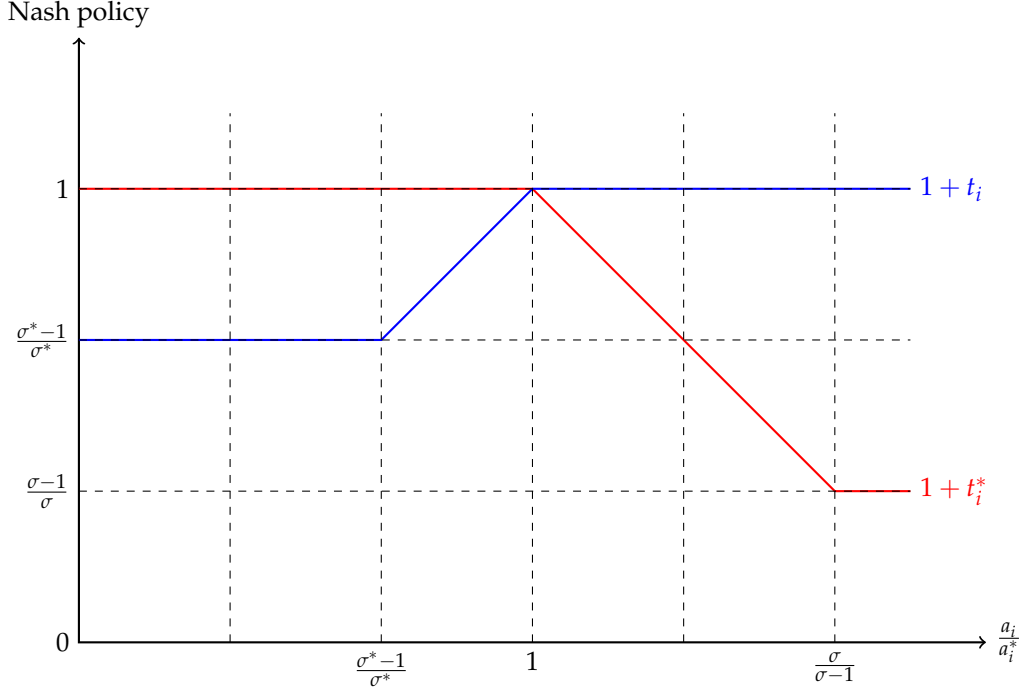


Figure 5: Nash Equilibrium in Noncooperative Trade Policy

In equilibrium, only Foreign produces good i , and the world price is $p_i = a_i$. Home's optimal imports are $m_i^o = L \left[\frac{a_i}{\beta_i} \right]^{-\sigma} = M_i^{II}$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma$ and domestic price $p_i^d = a_i$.

Case 6: $\frac{a_i}{a_i^*} = \frac{\sigma}{\sigma-1}$. In this case, the ranking of the four cut-offs is $\frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} < \frac{a_i^*}{a_i} = \frac{\sigma-1}{\sigma} < 1$. The Nash Equilibrium policy is $1+t_i = 1$ and $1+t_i^* = \frac{a_i^*}{a_i} = \frac{\sigma-1}{\sigma}$. In equilibrium, only Foreign produces good i , and the world price is $p_i = a_i = \frac{\sigma}{\sigma-1} a_i^*$. Home's optimal imports are $m_i^o = M_i^{II} = L \left[\frac{a_i}{\beta_i} \right]^{-\sigma} = L \left[\frac{a_i^*}{\beta_i} \frac{\sigma}{\sigma-1} \right]^{-\sigma}$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma = \frac{L}{\sigma-1} \left[a_i^* \frac{\sigma}{\sigma-1} \right]^{1-\sigma} \beta_i^\sigma$ and domestic price $p_i^d = a_i = \frac{\sigma}{\sigma-1} a_i^*$.

Case 7: $\frac{a_i}{a_i^*} > \frac{\sigma}{\sigma-1}$. In this case, the ranking of the four cut-offs is $\frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} < \frac{a_i^*}{a_i} < \frac{\sigma-1}{\sigma} < 1$. Based on this ranking, we can plot both countries' best responses together, as shown in Sub-figure 4f of Figure 4. The Nash Equilibrium policy is $1+t_i = 1$ and $1+t_i^* = \frac{\sigma-1}{\sigma}$. In equilibrium, only Foreign produces good i , and the world price is $p_i = \frac{\sigma}{\sigma-1} a_i^*$. Home's optimal imports are $m_i^o = L \left[\frac{a_i^*}{\beta_i} \frac{\sigma}{\sigma-1} \right]^{-\sigma}$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1} \left[a_i^* \frac{\sigma}{\sigma-1} \right]^{1-\sigma} \beta_i^\sigma$ and domestic price $p_i^d = \frac{\sigma}{\sigma-1} a_i^*$.

Combining the previous discussion, we summarize the Nash Equilibrium in noncoop-

erative trade policy with respect to each country's comparative advantage in the following proposition, as illustrated in Figure 5.

Proposition 2 *If the two strategic countries set their optimal trade policy noncooperatively, the Nash Equilibrium taxes are (a) $1 + t_i = \frac{\sigma^*-1}{\sigma^*}$ and $1 + t_i^* = 1$, if $\frac{a_i}{a_i^*} \leq \frac{\sigma^*-1}{\sigma^*}$; (b) $1 + t_i = \frac{a_i}{a_i^*}$ and $1 + t_i^* = 1$, if $\frac{a_i}{a_i^*} \in (\frac{\sigma^*-1}{\sigma^*}, 1]$; (c) $1 + t_i = 1$ and $1 + t_i^* = \frac{a_i^*}{a_i}$, if $\frac{a_i}{a_i^*} \in (1, \frac{\sigma}{\sigma-1}]$; (d) $1 + t_i = 1$ and $1 + t_i^* = \frac{\sigma-1}{\sigma}$, if $\frac{a_i}{a_i^*} > \frac{\sigma}{\sigma-1}$.*

Proposition 2 underscores the significant role of comparative advantage in determining the Nash trade taxes, especially with the introduction of strategic interactions. As noted in Costinot et al. (2015), when a country has a very strong comparative advantage in producing good i , it can manipulate world prices by charging a monopoly markup over its marginal cost. In contrast, the other country, with much lower productivity in producing this good, is forced to adopt free trade, unable to constrain its partner from exploiting its monopoly power.

In the intermediate range, where neither country's technology dominates the other's, each country's ability to manipulate the world price is constrained by its partner's unit labor requirement. In equilibrium, a specialization cut-off for comparative advantage, $\frac{a_i}{a_i^*} = 1$, emerges, closely linked to absolute advantage. If Home has an absolute disadvantage in producing good i compared to Foreign (i.e., $a_i > a_i^*$), Home will opt for free trade and cease production of good i . In this scenario, Foreign takes over the entire production of good i and imposes a positive export tax that increases with its comparative advantage. This pricing strategy, known as limit-pricing as described by Costinot et al. (2015), is designed to make Home's firms exactly indifferent to producing or not.

Notably, there is never a situation where an import tariff arises. As emphasized, the quasi-linear preferences imply equalized wages between the two countries.⁴ If a country has an absolute disadvantage in producing good i , it will abandon production of that good entirely and adopt free trade, reallocating the labor previously employed in producing good i to the numeraire good. Alternatively, if the country were to subsidize domestic firms to produce this good, it would increase the domestic price, thereby reducing consumer surplus, regardless of whether positive or negative tax revenue is generated.

Let's compare Home's policy in the Nash Equilibrium with its trade policy when Foreign is passive (i.e., $t_i^* = 0$). The structure of trade taxes in relation to comparative advantage remains identical in both scenarios. However, Home is strictly worse off in the Nash Equilibrium, where Foreign is also strategic. The reasoning is straightforward: for goods in which Home has an absolute disadvantage, if Foreign is passive, Home will adopt free trade, allow-

⁴Differentiated wages across countries could easily be introduced by assigning different unit labor requirements for the numeraire good, which would yield different comparative advantage cut-offs in the Nash Equilibrium characterization. However, the conclusions regarding Nash policies remain qualitatively unchanged.

ing production to be taken over by Foreign's lower-cost firms. Consequently, the world price and Home's domestic price are determined solely by Foreign's marginal cost, enabling Home to gain a large consumer surplus. In contrast, if Foreign is strategic, it imposes strictly positive export taxes on these goods, resulting in higher world and domestic prices for Home. This reduces Home's consumer surplus. This finding provides theoretical support for [Costinot et al.](#)'s conjecture that Home's welfare may be substantially lower in the Nash Equilibrium.

5 Welfare Implications

Given the Nash policies described above, we can now compare the welfare implications under four trade settings: autarky, free trade, unilateral optimal trade taxes, and Nash Equilibrium with optimal trade taxes. Recall that under a quasilinear utility function, the consumer's objective is to maximize $\mathcal{U}^l(c^l) \equiv \int_{i \in \Omega} u_i^l(c_i^l) di + c_0^l = \int_{i \in \Omega} \beta_i \frac{(c_i^l)^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} di + c_0^l$, subject to the budget constraint $\int_i p_i^d c_i^l di + c_0^l = I^l = 1 + T^l$, where the individual's total income I^l consists of wage income and tax revenue T^l , and p_i^d represents domestic prices. The maximized aggregate utility, or domestic welfare, can be expressed as:

$$\mathcal{W} = -\frac{\sigma L}{\sigma-1} + \underbrace{L}_{\text{wage}} + \underbrace{T}_{\text{tax revenue}} + \underbrace{\frac{L}{\sigma-1} \int_{i \in \Omega} (p_i^d)^{1-\sigma} \beta_i^\sigma di}_{\text{consumer surplus}}. \quad (27)$$

5.1 Autarky

In autarky, all goods are produced domestically with no ad-valorem taxes or subsidies, so $p_i^d = a_i, \forall i$. From (27), the autarky welfare at home is given by:

$$\mathcal{W}^{Autarky} = -\frac{\sigma L}{\sigma-1} + \underbrace{L}_{\text{wage}} + \underbrace{0}_{\text{tax revenue}} + \underbrace{\frac{L}{\sigma-1} \int_{i \in \Omega} a_i^{1-\sigma} \beta_i^\sigma di}_{\text{consumer surplus}}. \quad (28)$$

5.2 Free Trade

If both countries adopt a free trade policy, then $p_i^d = p_i = \min\{a_i, a_i^*\}$, as shown in [Dornbusch et al. \(1977\)](#). From (27), the domestic welfare under free trade is:

$$\mathcal{W}^{FreeTrade} = -\frac{\sigma L}{\sigma-1} + \underbrace{L}_{\text{wage}} + \underbrace{0}_{\text{tax revenue}} + \underbrace{\frac{L}{\sigma-1} \int_{i \in \{a_i \leq a_i^*\}} a_i^{1-\sigma} \beta_i^\sigma di + \frac{L}{\sigma-1} \int_{i \in \{a_i > a_i^*\}} (a_i^*)^{1-\sigma} \beta_i^\sigma di}_{\text{consumer surplus}}. \quad (29)$$

It is evident that both countries are better off under free trade compared to autarky, as shown by comparing (28) with (29):

$$\mathcal{W}^{FreeTrade} - \mathcal{W}^{Autarky} = \underbrace{\frac{L}{\sigma-1} \int_{i \in \{a_i > a_i^*\}} [(a_i^*)^{1-\sigma} - a_i^{1-\sigma}] \beta_i^\sigma di}_{\text{positive consumer surplus}} \quad (30)$$

Each country gains from trade by specializing in sectors where it has a comparative advantage, generating higher consumer surplus in sectors where it has a comparative disadvantage by consuming imports from the other country.

5.3 Unilateral Optimal

If only the Home country sets trade policies while Foreign remains passive (i.e., $t_i^* = 0$), then $p_i^d = p_i(1 + t_i) = \min\{a_i, a_i^*\}$. From (27), the domestic welfare under the unilateral optimal policy is:

$$\begin{aligned} \mathcal{W}^{Unilateral} = & -\frac{\sigma L}{\sigma-1} + \underbrace{\frac{L}{\sigma-1}}_{\text{wage}} + \underbrace{\frac{L}{\sigma-1} \int_{i \in \{a_i \leq a_i^*\}} a_i^{1-\sigma} \beta_i^\sigma di + \frac{L}{\sigma-1} \int_{i \in \{a_i > a_i^*\}} (a_i^*)^{1-\sigma} \beta_i^\sigma di}_{\text{consumer surplus}} \quad (31) \\ & + \underbrace{\frac{L^*}{\sigma^*-1} \int_{i \in \{\frac{a_i}{a_i^*} < \frac{\sigma^*-1}{\sigma^*}\}} \left(\frac{\sigma^*-1}{\sigma^*}\right)^{\sigma^*} a_i^{1-\sigma^*} (\beta_i^*)^{\sigma^*} di + \int_{i \in \{\frac{\sigma^*-1}{\sigma^*} \leq \frac{a_i}{a_i^*} < 1\}} L^* \left(\frac{a_i^*}{\beta_i^*}\right)^{-\sigma^*} (a_i^* - a_i) di}_{\text{tax revenue}}. \end{aligned}$$

Comparing (29) with (31), we find that the Home country is strictly better off under the unilateral optimal policy than under free trade:

$$\mathcal{W}^{Unilateral} - \mathcal{W}^{FreeTrade} = \underbrace{\frac{L^*}{\sigma^*-1} \int_{i \in \{\frac{a_i}{a_i^*} < \frac{\sigma^*-1}{\sigma^*}\}} \left(\frac{\sigma^*-1}{\sigma^*}\right)^{\sigma^*} a_i^{1-\sigma^*} (\beta_i^*)^{\sigma^*} di + \int_{i \in \{\frac{\sigma^*-1}{\sigma^*} \leq \frac{a_i}{a_i^*} < 1\}} L^* \left(\frac{a_i^*}{\beta_i^*}\right)^{-\sigma^*} (a_i^* - a_i) di}_{\text{positive tax revenue}}.$$

The intuition here is that, for goods in which Home has an absolute advantage, it can generate positive tax revenue by imposing strictly positive export taxes on these products.

5.4 Nash Equilibrium

When both countries set trade policies noncooperatively, the world prices p_i and domestic optimal trade taxes t_i are determined in Nash Equilibrium, as shown in Proposition 2. Thus, Home's prices are:

$$p_i^d = p_i(1 + t_i) = \begin{cases} a_i, & \text{otherwise;} \\ \frac{\sigma}{\sigma-1} a_i^*, & \text{if } \frac{a_i}{a_i^*} > \frac{\sigma}{\sigma-1}. \end{cases} \quad (32)$$

Using (32), (27), and Proposition 2, we can express Home's welfare under noncooperative trade policies as follows:

$$\begin{aligned} \mathcal{W}^{NE} = & -\frac{\sigma L}{\sigma-1} + \underbrace{\frac{L}{\sigma-1} \int_{i \in \{\frac{a_i}{a_i^*} \leq \frac{\sigma}{\sigma-1}\}} a_i^{1-\sigma} \beta_i^\sigma di + \frac{L}{\sigma-1} \int_{i \in \{\frac{a_i}{a_i^*} > \frac{\sigma}{\sigma-1}\}} \left(\frac{\sigma}{\sigma-1} a_i^*\right)^{1-\sigma} \beta_i^\sigma di}_{\text{consumer surplus}} \quad (33) \\ & + \underbrace{\frac{L^*}{\sigma^*-1} \int_{i \in \{\frac{a_i}{a_i^*} < \frac{\sigma^*-1}{\sigma^*}\}} \left(\frac{\sigma^*-1}{\sigma^*}\right)^{\sigma^*} a_i^{1-\sigma^*} (\beta_i^*)^{\sigma^*} di + \int_{i \in \{\frac{\sigma^*-1}{\sigma^*} \leq \frac{a_i}{a_i^*} < 1\}} L^* \left(\frac{a_i^*}{\beta_i^*}\right)^{-\sigma^*} (a_i^* - a_i) di}_{\text{tax revenue}}. \end{aligned}$$

We compare Home's welfare under Nash Equilibrium with that under autarky:

$$\begin{aligned} \mathcal{W}^{NE} - \mathcal{W}^{Autarky} = & \underbrace{\frac{L}{\sigma-1} \int_{i \in \{\frac{a_i}{a_i^*} > \frac{\sigma}{\sigma-1}\}} \left[\left(\frac{\sigma}{\sigma-1} a_i^*\right)^{1-\sigma} - a_i^{1-\sigma} \right] \beta_i^\sigma di}_{\text{positive consumer surplus}} \quad (34) \\ & + \underbrace{\frac{L^*}{\sigma^*-1} \int_{i \in \{\frac{a_i}{a_i^*} < \frac{\sigma^*-1}{\sigma^*}\}} \left(\frac{\sigma^*-1}{\sigma^*}\right)^{\sigma^*} a_i^{1-\sigma^*} (\beta_i^*)^{\sigma^*} di + \int_{i \in \{\frac{\sigma^*-1}{\sigma^*} \leq \frac{a_i}{a_i^*} < 1\}} L^* \left(\frac{a_i^*}{\beta_i^*}\right)^{-\sigma^*} (a_i^* - a_i) di}_{\text{positive tax revenue}}. \end{aligned}$$

We find that the Home country is strictly better off under noncooperative trade policies than under autarky. This is due to two reasons. First, Home can generate larger consumer surplus by opting for free trade on its least productive goods, despite Foreign imposing a constant monopoly export tax. Second, strictly positive tax revenue can be obtained from goods where Home has a comparative advantage in production.

Next, we compare Home's welfare under Nash Equilibrium with its welfare under a unilateral optimal trade policy:

$$\mathcal{W}^{NE} - \mathcal{W}^{Unilateral} = \underbrace{\frac{L}{\sigma-1} \int_{i \in \{\frac{\sigma}{\sigma-1} < \frac{a_i}{a_i^*} \leq 1\}} \left[\left(\frac{\sigma}{\sigma-1} a_i^*\right)^{1-\sigma} - a_i^{1-\sigma} \right] \beta_i^\sigma di + \frac{L}{\sigma-1} \int_{i \in \{\frac{a_i}{a_i^*} > 1\}} \left[\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} - 1 \right] (a_i^*)^{1-\sigma} \beta_i^\sigma di}_{\text{negative consumer surplus}}.$$

We explicitly demonstrate that Home's welfare is lower under Nash Equilibrium than under the unilateral optimal trade policy. In the unilateral optimal trade policy setting, where Foreign remains passive, Home can obtain higher consumer surplus for goods in which it has an absolute disadvantage while still generating the same tax revenue as it would under Nash Equilibrium.

Furthermore, we compare Home's welfare under Nash Equilibrium with that under free

trade:

$$\begin{aligned}
\mathcal{W}^{NE} - \mathcal{W}^{FreeTrade} &= \underbrace{\frac{L}{\sigma-1} \int_{i \in \{1 < \frac{a_i}{a_i^*} \leq \frac{\sigma}{\sigma-1}\}} \left[a_i^{1-\sigma} - (a_i^*)^{1-\sigma} \right] \beta_i^\sigma di + \frac{L}{\sigma-1} \int_{i \in \{\frac{a_i}{a_i^*} > \frac{\sigma}{\sigma-1}\}} \left[\left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} - 1 \right] (a_i^*)^{1-\sigma} \beta_i^\sigma di}_{\text{negative consumer surplus}} \\
&+ \underbrace{\frac{L^*}{\sigma^*-1} \int_{i \in \{\frac{a_i}{a_i^*} < \frac{\sigma^*-1}{\sigma^*}\}} \left(\frac{\sigma^*-1}{\sigma^*} \right)^{\sigma^*} a_i^{1-\sigma^*} (\beta_i^*)^{\sigma^*} di + \int_{i \in \{\frac{\sigma^*-1}{\sigma^*} \leq \frac{a_i}{a_i^*} < 1\}} L^* \left(\frac{a_i^*}{\beta_i^*} \right)^{-\sigma^*} (a_i^* - a_i) di}_{\text{positive tax revenue}}.
\end{aligned} \tag{35}$$

The sign of $(\mathcal{W}^{NE} - \mathcal{W}^{FreeTrade})$ depends on two factors: the relative size of the labor endowment L/L^* and the distribution of comparative advantage, or technology, $G(a_i/a_i^*)$. Simple algebra leads us to the following proposition:

Proposition 3 *All else constant, Home's welfare under Nash trade policies relative to free trade $(\mathcal{W}^{NE} - \mathcal{W}^{FreeTrade})$ is higher if Home has a smaller relative labor endowment size L/L^* , and if the technology distribution $G(a_i/a_i^*)$ is more left-skewed compared to $F(a_i/a_i^*)$; in other words, if $F(a_i/a_i^*)$ first-order stochastically dominates $G(a_i/a_i^*)$.*

Proposition 3 suggests that Home may even be strictly better off under noncooperative trade policies compared to free trade, i.e., $\mathcal{W}^{NE} - \mathcal{W}^{FreeTrade} > 0$. An extreme example illustrating this outcome occurs when $G(a_i/a_i^*)$ is sufficiently left-skewed such that only the first region remains non-empty. In this scenario, Home can collect strictly positive tax revenues while maintaining its consumer surplus at the same level as under free trade. Naturally, this advantage for Home comes at the expense of a welfare loss for Foreign.

Home may also benefit from having a smaller relative size L/L^* , due to welfare gains from relatively larger positive tax revenue in its competitive sectors. This result is somewhat counter-intuitive, as the literature generally suggests that larger countries tend to gain more from trade policies or “win” in trade wars.⁵ The underlying mechanism is as follows. With quasilinear preferences, wages are fixed, eliminating the traditional general-equilibrium considerations associated with wage adjustments. Instead, in this context, the optimal tax policy is driven by demand conditions and relative market size. In sectors where Home has a stronger comparative advantage, it produces, exports, and imposes taxes. As shown in (35), the larger the foreign country, the more tax revenue Home can collect, leading to higher domestic welfare.⁶ It is important to note that the finding that a small economy can prevail in a

⁵See, for example, Kennan and Riezman (1988), Syropoulos (2002), and Ossa (2011, 2014).

⁶Interestingly, Bagwell and Staiger (2012) demonstrate in the Cournot delocation model that Nash import and export tariffs are generally both positive. Similarly, based on the Melitz and Ottaviano model with quasilinear preferences, Bagwell and Lee (2020) also find that a country can raise welfare by collecting additional tariff revenues. Our theoretical prediction and empirical application confirm the possibility of a comparable effect within the Ricardian framework of Dornbusch et al. (1977).

trade war is mainly driven by the presence of Nash competition rather than by the assumption of quasilinear preferences. As Costinot et al. (2015) show, even under CES preferences, a country's optimal trade policy may consist of zero import tariffs and positive export taxes, consistent with our Proposition 1. However, their results always correspond to the unilateral optimal case in which the other country remains passive. Only when both countries act strategically in a Nash setting does the possibility arise that a small country may be better off under noncooperative trade policies than under free trade.

A related question is whether a country that converges to a small open economy can still benefit from tariff wars, given that many studies argue the optimal policy for a small open economy is free trade. In fact, this issue does not present a challenge to our framework for two reasons. On the one hand, we adopt the Ricardian model of Dornbusch et al. (1977), in which technology is the fundamental driver of international trade. In this setting, even a very small country exerts some influence on world prices. A small open economy can still constrain a larger country's ability to manipulate world prices, even if it does not produce at all. On the other hand, from a technical perspective, quasilinear preferences require that the labor endowment be sufficiently large to ensure that the numeraire good is always produced. Thus, even a small open economy must retain a relative labor size consistent with the quasilinear specification.⁷

Recall that the traditional general-equilibrium wage effect is absent under quasilinear preferences (Ossa, 2011; Costinot et al., 2015; Lashkaripour and Lugovskyy, 2023), and therefore the classic terms-of-trade (ToT) channel does not operate in the present Ricardian framework (Bagwell and Staiger, 1999, 2004; Bagwell and Lee, 2020). Since wages are fixed by the productivity of the numeraire good, a country cannot influence another country's wage, and thus its production costs, through general-equilibrium adjustments in foreign labor markets. Nevertheless, trade taxes can still enhance Home's welfare on a good-by-good basis by raising export prices, with the scope for manipulating world prices arising from superior productivity or comparative advantage. To illustrate, consider a good i produced and exported by Home. Its world price, $p_i = a_i / (1 + t_i)$, increases as the export tax rises (i.e., as t_i becomes more negative). In this case, Home collects substantial tax revenues from the foreign country while incurring only a modest loss in consumer surplus. Overall, Home can derive significant welfare gains from optimal trade taxes in sectors where it possesses a comparative advantage.

⁷See Section 2.1 for a detailed discussion of the quasilinear assumption and related trade policy literature employing this framework.

6 Application

In this section, we apply our theoretical findings to two empirical scenarios. The first considers two major players—China and the United States—while the second expands the analysis to three players: China, the United States, and the European Union (EU). For each scenario, we quantitatively assess welfare, tax revenue, and consumer surplus across different trade situations—namely, autarky, free trade, Nash Equilibrium, and unilateral optimal trade policy. Our objective is to analyze how comparative advantage and trade scenarios drive quantitative changes in welfare. Furthermore, we aim to validate Proposition 3 and highlight several intriguing insights from our findings.

Our quantitative analysis focuses on the agricultural sector for several compelling reasons. First, from a theoretical perspective, the agricultural market structure aligns well with the assumptions of our quasilinear model. Second, from a policy perspective, agricultural security and trade policy are highly debated topics of global interest, as demonstrated by the lengthy and significant Doha Round negotiations within the WTO. Lastly, from a measurement standpoint, comparative advantage in agriculture tends to be stable over time, driven primarily by natural conditions and the cumulative scientific expertise of agronomists.

6.1 Data

Our data primarily come from the Global Agro-Ecological Zones (GAEZ) version 4 project, developed by the Food and Agriculture Organization (FAO). To compute domestic welfare as shown in Section 5, we use the following parameters: (1) the unit factor requirements in each area for each of the 49 crops,⁸ denoted by a_i and a_i^* ; (2) exogenous preference parameters, $\beta_i = \beta_i^* = 1$, under the assumption that all crops enter utility symmetrically in both countries; (3) the elasticity of substitution, $\sigma = \sigma^* = 2.9$, assuming a constant elasticity of substitution across all non-numeraire goods, consistent with the value for the elasticity of substitution as used in Costinot et al. (2015); (4) the sizes of the regions, L and L^* , determined by the hectare devoted to the crops considered and vary according to specific scenarios. With these parameters defined, we calculate welfare, tax revenue, and consumer surplus for each trade scenario.

The single factor of production in our model is agricultural land. Consequently, the size of a country is measured by its amount of agricultural land, data for which is available from the GAEZ project. The unit factor requirements for each crop also come from the GAEZ project

⁸In the original GAEZ v4 database, we consolidate the crop listings into 49 categories by grouping sub-varieties under their main varieties. For example, dryland rice and wetland rice are combined into a single category, “combined rice,” using the maximum predicted GAEZ yield as the representative yield for this category. Other crops requiring similar consolidation include sorghum, cocoa, coffee, maize, millet, barley, wheat, yam, rye, rice, and phaseolus beans and grams.

(Costinot et al., 2015; Costinot and Donaldson, 2016). Using data on local conditions such as soil, elevation, topography, and climate, the GAEZ agronomic model computes the yield that parcels of land worldwide could achieve for each of the 49 crops we consider, assuming “high” levels of inputs and “rain-fed” water supply. We interpret a_i and a_i^* as the average hectare per kilogram of output across lands in Home and Foreign, respectively.⁹

6.2 Two-Country Scenario: China and the US

In the first scenario, we analyze the welfare implications of different trade settings between China and the US. China is designated as the home country and the US as the foreign country. We set $L = 529,038,600$ and $L^* = 412,857,600$ to reflect the hectares dedicated to the crops under consideration. Table 1 presents the welfare, tax revenue, and consumer surplus for China and the US across four trade situations: autarky, free trade, Nash Equilibrium, and unilateral optimal trade policy. For comparability, we normalize each region’s welfare to 100% under autarky when reporting all numerical results.

Table 1: China and the US

Region	Trade Situation	Welfare	Tax Revenue	Consumer Surplus
China	Autarky	100.00%	0.00%	100.00%
	Free trade	130.71%	0.00%	130.71%
	Nash Equilibrium	101.01%	0.99%	100.02%
	Unilateral optimal	131.70%	0.99%	130.71%
US	Autarky	100.00%	0.00%	100.00%
	Free trade	101.44%	0.00%	101.44%
	Nash Equilibrium	123.25%	23.18%	100.07%
	Unilateral optimal	124.63%	23.18%	101.44%

Notes: This table presents welfare, tax revenue, and consumer surplus for two regions—China and the US—under four trade situations: autarky, free trade, Nash Equilibrium, and unilateral optimal trade policy. Welfare in each region is normalized to 100% under autarky for consistent numerical comparison.

As shown in Table 1, both welfare and consumer surplus are lowest under autarky. Moving from autarky to free trade raises welfare in both regions. As expected, each region attains its highest welfare when it is able to set trade policy unilaterally while the other remains passive. These quantitative findings are consistent with the theoretical results in Section 5. Notably, China’s welfare under Nash Equilibrium falls below that under free trade or unilateral optimal policy. By contrast, the US experiences a substantial welfare gain under Nash Equilibrium relative to free trade, as documented in Table 1. This welfare improvement arises primarily from significant tax revenues generated in its competitive sectors, which more than offset the decline in consumer surplus.

⁹We conduct robustness checks using the average hectare per ton of output across lands. The welfare results are nearly identical, differing only in decimal points as a percentage. Alternative tables are available upon request.

The China–US case illustrates the role of relative country size and technological distribution in shaping trade policy outcomes. Here, country size is measured by agricultural land area, rendering the US the smaller country relative to China. Transitioning from free trade to a noncooperative Nash Equilibrium alters the distribution of gains: the smaller country, the US, suffers a decline in consumer surplus due to higher prices in sectors where it is less competitive, but simultaneously expands production and exports in sectors where it enjoys a comparative advantage. Given strong demand from China, the US leverages its superior agricultural productivity to collect substantial tax revenues, which more than compensate for its consumer surplus loss. Consequently, the US’ welfare rises while China’s falls, reinforcing our proposition that, under certain conditions, smaller countries can prevail in a trade war.

6.3 Three-Country Scenario: China, the US, and the EU

In the second scenario, we investigate the welfare implications of alternative trade arrangements in a three-country setting: China, the US, and the EU. Due to the non-smooth production possibility frontier in the Ricardian model of [Dornbusch et al. \(1977\)](#), standard optimization techniques such as Lagrangian methods or gradient-based approaches cannot be applied. Instead, one must assume quasilinear preferences, as in this paper, or adopt the primal approach suggested by [Costinot et al. \(2015\)](#). Extending the policy implications from a two-country to a three-country environment is also challenging, since the Ricardian model of [Dornbusch et al. \(1977\)](#) characterizes deterministic productivity and comparative advantage, rather than the stochastic technology distributions employed in the Ricardian model of [Eaton and Kortum \(2002\)](#).¹⁰

Accordingly, we study the three-country case in a quantitative manner. The agricultural land endowments devoted to the crops under consideration are 529,038,600 hectares for China, 412,857,600 hectares for the US, and 170,956,490 hectares for the EU. Table 2 reports welfare, tax revenue, and consumer surplus for all three regions under four trade regimes: autarky, free trade, Nash Equilibrium, and unilateral optimal trade policy. In this setting, China and the US are relatively larger in size but technologically less efficient compared with the EU. Consistent with Proposition 3, a smaller but more productive economy can fare better in a trade war. Indeed, the results show that the EU experiences a substantial welfare gain under Nash Equilibrium relative to free trade, driven primarily by large tariff revenues. However, the EU’s gains come at the expense of the larger economies, as both China and the US incur considerable welfare losses. To conclude, our prediction that smaller economies with supe-

¹⁰[Collins \(1985\)](#) incorporates three countries into the framework of [Dornbusch et al. \(1977\)](#). However, that study imposes a strong simplifying assumption: the unit labor requirements in the intermediate-efficient country are fixed as a constant fraction of those in the least-efficient country. Such an ad hoc restriction renders the model unrealistic and unsuitable for serious quantitative policy evaluation.

Table 2: China, the US, and EU

Region	Trade Situation	Welfare	Tax Revenue	Consumer Surplus
China	Autarky	100.00%	0.00%	100.00%
	Free trade	236.17%	0.00%	236.17%
	Nash Equilibrium	138.18%	1.40%	136.78%
	Unilateral optimal	237.57%	1.40%	236.17%
US	Autarky	100.00%	0.00%	100.00%
	Free trade	183.29%	0.00%	183.29%
	Nash Equilibrium	111.32%	3.77%	107.55%
	Unilateral optimal	187.06%	3.77%	183.29%
EU	Autarky	100.00%	0.00%	100.00%
	Free trade	106.19%	0.00%	106.19%
	Nash Equilibrium	241.75%	137.97%	103.78%
	Unilateral optimal	244.16%	137.97%	106.19%

Notes: This table presents welfare, tax revenue, and consumer surplus for three regions—China, the US, and the EU—under four trade situations: autarky, free trade, Nash Equilibrium, and unilateral optimal trade policy. Welfare in each region is normalized to 100% under autarky for consistent numerical comparison.

rior technology can benefit from tariff wars holds not only in the two-country case but also extends to the three-country scenario in a quantitative setting.

7 Conclusion

In this paper, we examine the optimal trade policy within a Ricardian model framework where both countries act strategically. Our findings reveal that, in a Nash Equilibrium, a country’s optimal import tariffs are zero, while optimal export taxes increase with its comparative advantage. This outcome aligns structurally with the optimal trade policy model of [Costinot et al. \(2015\)](#), where the Foreign country is passive; however, the welfare gains from trade policy are lower in the strategic context. We further validate our theoretical predictions through empirical simulations in the agricultural sector, finding that the numerical results support the model’s predictions. A particularly intriguing insight is that, under certain conditions, a smaller country may achieve greater welfare under a Nash Equilibrium of optimal trade policies compared to a free trade scenario. This finding emerges robustly across simulations, whether in the bilateral policy interactions between the United States and China or in the three-country setting that additionally includes the European Union.

Several limitations in this study point to future research directions. First, we employ a numeraire good to fix the wage, which precludes consideration of the Lerner Symmetry Theorem (cf. [Costinot and Werning, 2019](#)). Second, extending the model to incorporate multiple sectors with sector-specific elasticities of substitution could further enrich the conclusions, as shown in [Ossa \(2014\)](#) and [Beshkar and Lashkaripour \(2017\)](#). Finally, incorporating dynamic decision-making into this Nash policy game could offer new insights into the protracted WTO

negotiations. We leave these avenues open for future research.

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A Unilateral Optimal Trade Policy with Trade Costs

We extend the model to include exogenous iceberg trade costs, which are asymmetric. Specifically, if one unit of good i arrives at Home, $\delta \geq 1$ units must be shipped from Foreign; similarly, if one unit of good i arrives at Foreign, $\delta^* \geq 1$ units must be shipped from Home. With these trade costs, two distinct world prices emerge: p_i , the price at Home, and p_i^* the price in Foreign. The relationship between p_i and p_i^* can be expressed as:

$$\phi(m_i) \equiv \begin{cases} \delta, & \text{if } m_i \geq 0, \\ 1/\delta^*, & \text{if } m_i < 0. \end{cases} \quad (36)$$

Thus, the trade balance condition implies that Foreign's net imports are given by:

$$m_i^* = -m_i \phi(m_i) = \begin{cases} -\delta m_i, & \text{if } m_i \geq 0, \\ -\frac{1}{\delta^*} m_i, & \text{if } m_i < 0. \end{cases} \quad (37)$$

Accordingly, Equations (20)-(22) in Section 3 are modified as follows:

$$p_i^*(m_i, t_i, t_i^*) \equiv \min\{u_i^{*f}(-(m_i \phi(m_i))/L^*)/(1+t_i^*), a_i^*/(1+t_i^*)\}, \quad (38)$$

$$q_i^*(m_i, t_i, t_i^*) \equiv \max\{0, L^*(a_i^*/\beta_i^*)^{-\sigma^*} + m_i \phi(m_i)\}, \quad (39)$$

$$c_i^*(m_i, t_i, t_i^*) \equiv \max\{-m_i \phi(m_i), L^*(a_i^*/\beta_i^*)^{-\sigma^*}\}. \quad (40)$$

Equations (20)-(22) indicate that the threshold of net imports, above which Foreign is required to produce good i , now adjusts to $\delta^* M_i^I = -\delta^* L^* \left(\frac{a_i^*}{\beta_i^*}\right)^{-\sigma^*}$ due to the inclusion of iceberg trade costs. Additionally, as discussed in [Dornbusch et al. \(1977\)](#) and [Costinot et al. \(2015\)](#), these trade costs result in the emergence of a range of nontraded goods, identified by a new cut-off at $m_i = 0$. Together with the original cut-off $M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$, below which Home must produce good i , we now have three cut-offs defining the boundaries of Home's net imports. Consequently, the range of Home's net imports for good i can be divided into four distinct regions: Region I with $m_i \leq \delta^* M_i^I$, Region II with $\delta^* M_i^I \leq m_i \leq 0$, Region III with $0 \leq m_i \leq M_i^{II}$, and Region IV with $m_i \geq M_i^{II}$. Each region represents a different production and trade scenario based on the relative size of Home's net imports and the trade cost-adjusted thresholds.

The introduction of iceberg trade costs does not alter the qualitative findings presented in Sections 3 and 4. Accordingly, we can extend the previous analysis to generalize Propositions 1 and 2. To proceed, let us examine the domestic government's welfare optimization problem on a region-by-region basis.

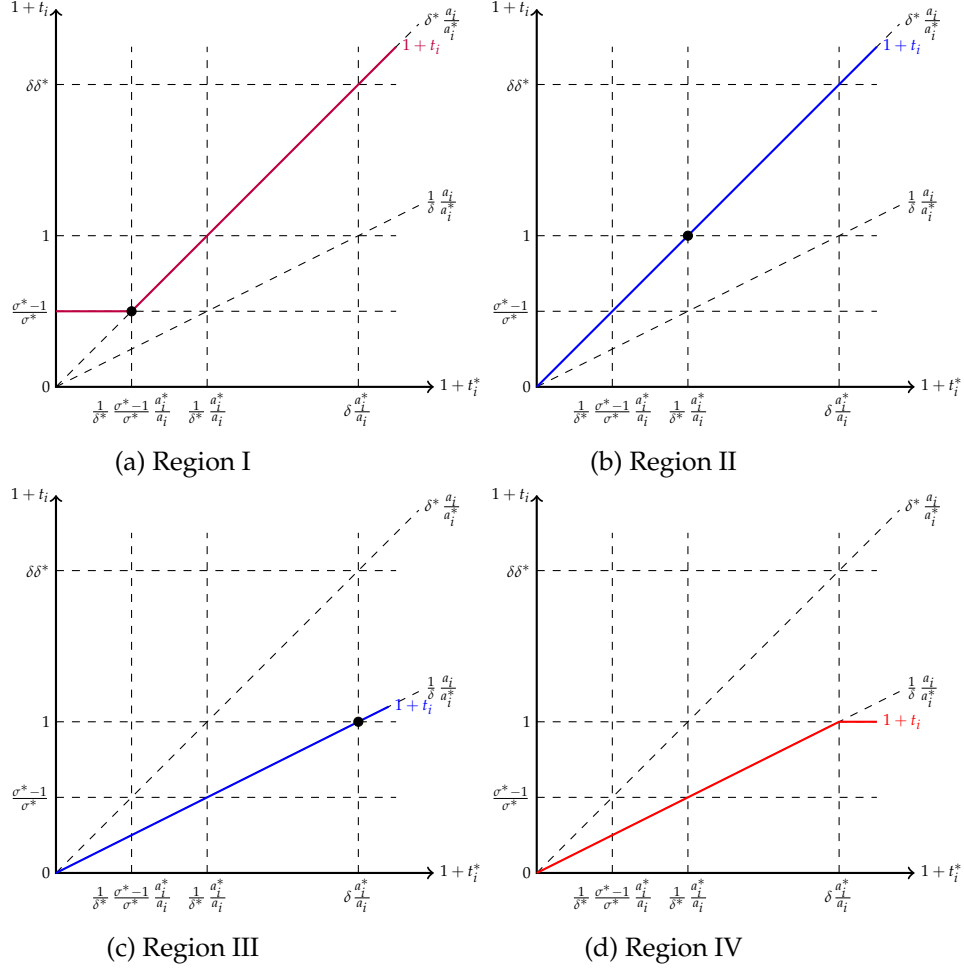


Figure A.1: Best Response with Trade Costs in Regions I, II, III, and IV

Region I: $m_i \leq \delta^* M_i^I$. In this region, only Home produces good i . If $1 + t_i^* \leq \frac{1}{\delta^*} \frac{\sigma^* - 1}{\sigma^*} \frac{a_i^*}{a_i} = \frac{1}{\delta^*} B_i^I$, then optimal net imports are $(\delta^*)^{1-\sigma^*} m_i^I = -L^* \left[\frac{a_i}{\beta_i^*} \frac{\sigma^*}{\sigma^* - 1} (1 + t_i^*) \right]^{-\sigma^*} (\delta^*)^{1-\sigma^*}$, with $p_i = \frac{a_i}{1+t_i} = \frac{\sigma^*}{\sigma^* - 1} a_i$ and $1 + t_i = \frac{\sigma^* - 1}{\sigma^*}$. Thus, Home's optimized welfare is $\mathcal{W}_i^{Io} = \frac{L}{\sigma - 1} a_i^{1-\sigma} \beta_i^\sigma + \frac{L^* a_i}{\sigma^* - 1} \left[\frac{a_i}{\beta_i^*} \frac{\sigma^*}{\sigma^* - 1} (1 + t_i^*) \right]^{-\sigma^*} (\delta^*)^{1-\sigma^*}$. In contrast, if $1 + t_i^* \geq \frac{1}{\delta^*} \frac{\sigma^* - 1}{\sigma^*} \frac{a_i^*}{a_i}$, then optimal net imports are $\delta^* M_i^I$, with $p_i = \frac{a_i}{1+t_i} = \frac{1}{\delta^*} \frac{a_i^*}{1+t_i^*}$ and $1 + t_i = \frac{a_i^*}{a_i} (1 + t_i^*) \delta^* \geq \frac{\sigma^* - 1}{\sigma^*}$. Thus, Home's optimized welfare is $\mathcal{W}_i^{Io} = \frac{L}{\sigma - 1} a_i^{1-\sigma} \beta_i^\sigma + (a_i - \frac{1}{\delta^*} \frac{a_i^*}{1+t_i^*}) \left[-L^* \left(\frac{a_i^*}{\beta_i^*} \right)^{-\sigma^*} \delta^* \right]$.

Region II: $\delta^* M_i^I \leq m_i \leq 0$. In this region, both Home and Foreign can produce good i , but only Home exports, or no trade occurs. If $1 + t_i^* \leq \frac{1}{\delta^*} \frac{a_i^*}{a_i} = \frac{1}{\delta^*} B_i^{II}$, then optimal net imports are $\delta^* M_i^I$, with $p_i = \frac{a_i}{1+t_i} = \frac{1}{\delta^*} \frac{a_i^*}{1+t_i^*}$ and $1 + t_i = \frac{a_i^*}{a_i} (1 + t_i^*) \delta^*$. Thus, Home's optimized welfare is $\mathcal{W}_i^{IIo} = \frac{L}{\sigma - 1} a_i^{1-\sigma} \beta_i^\sigma + (a_i - \frac{1}{\delta^*} \frac{a_i^*}{1+t_i^*}) \left[-L^* \left(\frac{a_i^*}{\beta_i^*} \right)^{-\sigma^*} \delta^* \right]$. In contrast, if $1 + t_i^* \geq \frac{1}{\delta^*} \frac{a_i^*}{a_i}$, then optimal

net imports are zero, with $p_i = \frac{a_i}{1+t_i} = \frac{1}{\delta^*} \frac{a_i^*}{1+t_i^*}$ and $1+t_i = \frac{a_i}{a_i^*}(1+t_i^*)\delta^* \geq 1$. Thus, Home's optimized welfare is $\mathcal{W}_i^{IIo} = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma$.

Region III: $0 \leq m_i \leq M_i^{II}$. In this region, both Home and Foreign can produce good i , but only Home imports, or no trade occurs. If $1+t_i^* \leq \delta \frac{a_i^*}{a_i} = \delta B_i^{II}$, then optimal net imports are zero, with $p_i = \frac{a_i}{1+t_i} = \delta \frac{a_i^*}{1+t_i^*}$ and $1+t_i = \frac{1}{\delta} \frac{a_i}{a_i^*}(1+t_i^*) \leq 1$. Thus, Home's optimized welfare is $\mathcal{W}_i^{IIIo} = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma$. In contrast, if $1+t_i^* \geq \delta \frac{a_i^*}{a_i}$, then optimal net imports are M_i^{II} , with $p_i = \frac{a_i}{1+t_i} = \delta \frac{a_i^*}{1+t_i^*}$ and $1+t_i = \frac{1}{\delta} \frac{a_i}{a_i^*}(1+t_i^*) \geq 1$. Thus, Home's optimized welfare is $\mathcal{W}_i^{IIIo} = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + (a_i - \delta \frac{a_i^*}{1+t_i^*}) \left[L \left(\frac{a_i}{\beta_i} \right)^{-\sigma} \right]$.

Region IV: $m_i \geq M_i^{II}$. In this region, only Foreign produces good i . If $1+t_i^* \geq \delta \frac{a_i^*}{a_i} = \delta B_i^{II}$, then optimal net imports are $\delta^{-\sigma} L \left[\frac{a_i^*}{\beta_i} \frac{1}{1+t_i^*} \right]^{-\sigma} = \delta^{-\sigma} m_i^{III}$, with $p_i = \delta p_i^* = \delta \frac{a_i^*}{1+t_i^*}$ and $1+t_i = 1$. Thus, Home's optimized welfare is $\mathcal{W}_i^{IVo} = \frac{L}{\sigma-1} [p_i(1+t_i)]^{1-\sigma} \beta_i^\sigma = \frac{L}{\sigma-1} \left[\delta \frac{a_i^*}{1+t_i^*} \right]^{1-\sigma} \beta_i^\sigma$. In contrast, if $0 < 1+t_i^* \leq \delta \frac{a_i^*}{a_i}$, then optimal net imports are M_i^{II} , with $p_i = \delta p_i^* = \delta \frac{a_i^*}{1+t_i^*} = \frac{a_i}{1+t_i}$ and $1+t_i = \frac{1}{\delta} \frac{a_i}{a_i^*}(1+t_i^*) \leq 1$. Thus, Home's optimized welfare is $\mathcal{W}_i^{IVo} = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + (a_i - \delta \frac{a_i^*}{1+t_i^*}) \left[L \left(\frac{a_i}{\beta_i} \right)^{-\sigma} \right]$.

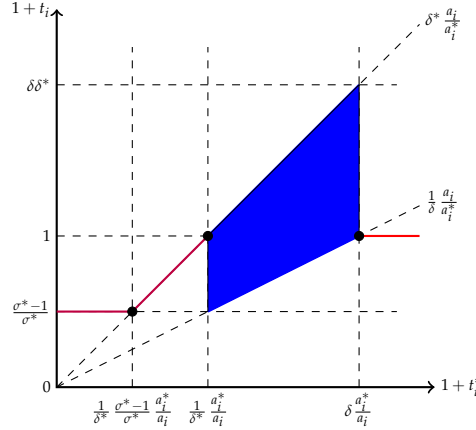


Figure A.2: Home's Unilateral Optimal Trade Policy with Trade Costs

Home's optimal responses across the four regions are illustrated in Figure A.1. Notably, there are three critical cut-offs for $(1+t_i^*)$: $\frac{1}{\delta^*} \frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} = \frac{1}{\delta^*} B_i^I$, $\frac{1}{\delta^*} \frac{a_i^*}{a_i} = \frac{1}{\delta^*} B_i^{II}$, and $\delta \frac{a_i^*}{a_i} = \delta B_i^{II}$. Based on these cut-offs, Home's optimal trade policy $(1+t_i)$ can be categorized into seven scenarios, determined by foreign trade taxes (t_i^*) , technology parameters (a_i, a_i^*) , and asymmetric iceberg trade costs (δ, δ^*) . By comparing the optimized welfare across the four regions within each scenario, we can summarize Home's optimal trade policy with trade costs as outlined in the following proposition, presented in Figure A.2.

Proposition 4 *If the two strategic countries set their optimal trade policies noncooperatively, given the foreign trade taxes (t_i^*), technology parameters (a_i, a_i^*), and asymmetric iceberg trade costs (δ, δ^*), Home's unilateral optimal trade policy is characterized as follows. The producer, optimal trade taxes (t_i), prevailing world prices at Home (p_i), Home's net imports (m_i), and Home's optimized welfare ($\mathcal{W}_i$) satisfy:*

(a) *only Home produces, $1 + t_i = \frac{\sigma^* - 1}{\sigma^*}$, $p_i = \frac{\sigma^*}{\sigma^* - 1} a_i$, $m_i = (\delta^*)^{1 - \sigma^*} m_i^I$, and $\mathcal{W}_i = \mathcal{W}_i^{Io}$, if $1 + t_i^* \in (0, \frac{1}{\delta^*} B_i^I)$;*

(b) *only Home produces, $1 + t_i = \frac{\sigma^* - 1}{\sigma^*} = \frac{a_i}{a_i^*} (1 + t_i^*) \delta^*$, $p_i = \frac{\sigma^*}{\sigma^* - 1} a_i = \frac{1}{\delta^*} \frac{a_i^*}{1 + t_i^*} = \frac{a_i}{1 + t_i}$, $m_i = \delta^* m_i^I = \delta^* M_i^I$, and $\mathcal{W}_i = \mathcal{W}_i^{Io} = \mathcal{W}_i^{IIo}$, if $1 + t_i^* = \frac{1}{\delta^*} B_i^I$;*

(c) *only Home produces, $1 + t_i = \frac{a_i}{a_i^*} (1 + t_i^*) \delta^*$, $p_i = \frac{a_i}{1 + t_i} = \frac{1}{\delta^*} \frac{a_i^*}{1 + t_i^*}$, $m_i = \delta^* M_i^I$, and $\mathcal{W}_i = \mathcal{W}_i^{IIo}$, if $1 + t_i^* \in (\frac{1}{\delta^*} B_i^I, \frac{1}{\delta^*} B_i^{II})$;*

(d) *case (A) only Home produces, $1 + t_i = 1$, $p_i = a_i$, $m_i = \delta^* M_i^I$, and $\mathcal{W}_i = \mathcal{W}_i^{Io} = \mathcal{W}_i^{IIIo}$; case (B) both produce, $1 + t_i = \frac{a_i}{p_i} \in [\frac{1}{\delta \delta^*}, 1]$, $p_i = \frac{a_i}{1 + t_i} \in [a_i, a_i \delta \delta^*]$, $m_i \in [\delta^* M_i^I, 0]$, and $\mathcal{W}_i = \mathcal{W}_i^{IIIo}$, if $1 + t_i^* = \frac{1}{\delta^*} B_i^{II}$;*

(e) *both produce, $1 + t_i = \frac{a_i}{p_i} \in [\frac{1}{\delta} \frac{a_i^*}{a_i^*} (1 + t_i^*), \frac{a_i}{a_i^*} (1 + t_i^*) \delta^*]$, $p_i = \frac{a_i}{1 + t_i} \in [\frac{1}{\delta^*} \frac{a_i^*}{1 + t_i^*}, \delta \frac{a_i^*}{1 + t_i^*}]$, $m_i = 0$, and $\mathcal{W}_i = \mathcal{W}_i^{IIIo}$, if $1 + t_i^* \in (\frac{1}{\delta^*} B_i^{II}, \delta B_i^{II})$;*

(f) *case (A) both produce, $1 + t_i = \frac{a_i}{p_i} \in [1, \delta \delta^*]$, $p_i = \frac{a_i}{1 + t_i} \in [\frac{a_i}{\delta \delta^*}, a_i]$, $m_i = 0$, and $\mathcal{W}_i = \mathcal{W}_i^{IIIo}$; case (B) only Foreign produces, $1 + t_i = 1$, $p_i = a_i$, $m_i = M_i^{II} = \delta^{-\sigma} m_i^{III}$, and $\mathcal{W}_i = \mathcal{W}_i^{IIIo} = \mathcal{W}_i^{IVo}$, if $1 + t_i^* = \delta B_i^{II}$;*

(g) *only Foreign produces, $1 + t_i = 1$, $p_i = \delta p_i^* = \delta \frac{a_i^*}{1 + t_i^*}$, $m_i = \delta^{-\sigma} m_i^{III}$, and $\mathcal{W}_i = \mathcal{W}_i^{IVo}$, if $1 + t_i^* > \delta B_i^{II}$, where $B_i^I \equiv \frac{\sigma^* - 1}{\sigma^*} \frac{a_i^*}{a_i}$, $B_i^{II} \equiv \frac{a_i^*}{a_i}$; $M_i^I \equiv -L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}$, $M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$; $m_i^I \equiv -L^*[\frac{a_i}{\beta_i^*} \frac{\sigma^*}{\sigma^* - 1} (1 + t_i^*)]^{-\sigma^*}$, $m_i^{III} \equiv L[\frac{a_i^*}{\beta_i^*} \frac{1}{1 + t_i^*}]^{-\sigma}$; $\mathcal{W}_i^{Io} = \frac{L}{\sigma - 1} a_i^{1 - \sigma} \beta_i^\sigma + \frac{L^* a_i}{\sigma^* - 1} [\frac{a_i}{\beta_i^*} \frac{\sigma^*}{\sigma^* - 1} (1 + t_i^*)]^{-\sigma^*} (\delta^*)^{1 - \sigma^*}$, $\mathcal{W}_i^{IIo} = \frac{L}{\sigma - 1} a_i^{1 - \sigma} \beta_i^\sigma + (a_i - \frac{1}{\delta^*} \frac{a_i^*}{1 + t_i^*}) [-L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*} \delta^*]$, $\mathcal{W}_i^{IIIo} = \frac{L}{\sigma - 1} a_i^{1 - \sigma} \beta_i^\sigma$, $\mathcal{W}_i^{IVo} \equiv \frac{L}{\sigma - 1} [\frac{a_i^*}{1 + t_i^*}]^{1 - \sigma} \beta_i^\sigma$.*