A Dynamic Quantitative Analysis of Labor Market Implications of Taiwan's WTO Accession: Skill, Capital, and the China Shock / Online Appendix

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A Proof of Propositions

A.1 Proof of Proposition 1

Applying the definition of dot algebra ($\dot{y}_{t+1} \equiv y_{t+1}/y_t$) to (10) yields (22). Applying the dot algebra to (13) and using (14) yields:

$$\begin{aligned}
\dot{P}^{nj}_{t+1} &= \left[\frac{\sum_o \left(\kappa^{nj,oj}_{t+1} x^{oj}_{t+1}\right)^{-\theta^j} \left(A^{oj}_{t+1}\right)^{\gamma^{oj}\theta^j}}{\sum_o \left(\kappa^{nj,oj}_{t} x^{oj}_{t}\right)^{-\theta^j} \left(A^{oj}_{t}\right)^{\gamma^{oj}\theta^j}}\right]^{-\frac{1}{\theta^j}} \\
&= \left[\sum_o \frac{\left(\kappa^{nj,oj}_{t+1} x^{oj}_{t+1}\right)^{-\theta^j} \left(A^{oj}_{t+1}\right)^{\gamma^{oj}\theta^j}}{\sum_{\text{o}} \left(\kappa^{nj,\text{o}j}_{t} x^{\text{o}j}_{t}\right)^{-\theta^j} \left(A^{\text{o}j}_{t}\right)^{\gamma^{oj}\theta^j}}\right]^{-\frac{1}{\theta^j}} \\
&= \left[\sum_o \left(\dot{\kappa}^{nj,oj}_{t+1} \dot{x}^{oj}_{t+1}\right)^{-\theta^j} \left(\dot{A}^{oj}_{t+1}\right)^{\gamma^{oj}\theta^j} \pi^{nj,oj}_t\right]^{-\frac{1}{\theta^j}},
\end{aligned}$$

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which is (23). Applying the dot algebra to (14) gives:

$$\begin{aligned}\dot{\pi}_{t+1}^{nj,oj} &= \left(\dot{\kappa}_{t+1}^{nj,oj}\dot{x}_{t+1}^{oj}\right)^{-\theta^j}\left(\dot{A}_{t+1}^{oj}\right)^{\gamma^{oj}\theta^j}\frac{\sum_{\mathrm{o}}\left(\kappa_t^{nj,\mathrm{o}j}x_t^{\mathrm{o}j}\right)^{-\theta^j}\left(A_t^{\mathrm{o}j}\right)^{\gamma^{\mathrm{o}j}\theta^j}}{\sum_{\mathrm{o}}\left(\kappa_{t+1}^{nj,\mathrm{o}j}x_{t+1}^{\mathrm{o}j}\right)^{-\theta^j}\left(A_{t+1}^{\mathrm{o}j}\right)^{\gamma^{\mathrm{o}j}\theta^j}}\\ &= \left(\dot{\kappa}_{t+1}^{nj,oj}\dot{x}_{t+1}^{oj}\right)^{-\theta^j}\left(\dot{A}_{t+1}^{oj}\right)^{\gamma^{oj}\theta^j}\left[\frac{\sum_{\mathrm{o}}\left(\kappa_{t+1}^{nj,\mathrm{o}j}x_{t+1}^{\mathrm{o}j}\right)^{-\theta^j}\left(A_{t+1}^{\mathrm{o}j}\right)^{\gamma^{\mathrm{o}j}\theta^j}}{\sum_{\mathrm{o}}\left(\kappa_t^{nj,\mathrm{o}j}x_t^{\mathrm{o}j}\right)^{-\theta^j}\left(A_t^{\mathrm{o}j}\right)^{\gamma^{\mathrm{o}j}\theta^j}}\right]^{-1},\end{aligned}$$

which leads to (24) by using (23). The change in the price index $\dot{P}_{t+1}^n$ in (28) follows from its definition.

For (26), we simply use (16) and replace $w_{t+1}^{njs}L_{t+1}^{njs}$ with $\dot{w}_{t+1}^{njs}\dot{L}_{t+1}^{njs}w_t^{njs}L_t^{njs}$. For (27), we use (17) and (6), and replace $r_{t+1}^nK_{t+1}^n$ with $\dot{r}_{t+1}^n\dot{K}_{t+1}^nr_t^nK_t^n$. Equation (25) simply follows from the definition of X_{t+1}^{nj}.

We use (12), (20), (21) and the definition of $\gamma^{L,njh}$ to obtain:

$$\begin{aligned}\left(\dot{\chi}_{t+1}^{njh}\right)^{1-\xi^{njh}} =& \frac{\zeta^{nj3}\left(w_{t+1}^{nj3}\right)^{1-\xi^{njh}}+\zeta^{njK}\left(r_{t+1}^n\right)^{1-\xi^{njh}}}{\zeta^{nj3}\left(w_t^{nj3}\right)^{1-\xi^{njh}}+\zeta^{njK}\left(r_t^n\right)^{1-\xi^{njh}}}\\ =&\frac{\gamma_t^{L,nj3}}{\gamma_t^{L,njh}}\left(\dot{w}_{t+1}^{nj3}\right)^{1-\xi^{njh}}+\frac{\gamma_t^{K,nj}}{\gamma_t^{L,njh}}\left(\dot{r}_{t+1}^n\right)^{1-\xi^{njh}}.\end{aligned}$$

Then by (11), (18), (19) and the definition of $\gamma^{L,njh}$, we have:

$$\left(\dot{\chi}_{t+1}^{nj}\right)^{1-\xi^{nj}} =\frac{\gamma_t^{L,nj1}}{\gamma^{nj}}\left(\dot{w}_{t+1}^{nj1}\right)^{1-\xi^{nj}}+\frac{\gamma_t^{L,nj2}}{\gamma^{nj}}\left(\dot{w}_{t+1}^{nj2}\right)^{1-\xi^{nj}}+\frac{\gamma_t^{L,njh}}{\gamma^{nj}}\left(\dot{\chi}_{t+1}^{njh}\right)^{1-\xi^{nj}}.$$

Finally, $\left\{\dot{\gamma}_{t+1}^{L,njs}\right\}_{s=1}^3$, $\dot{\gamma}_{t+1}^{L,njh}$, and $\dot{\gamma}_{t+1}^{K,nj}$ follow directly from (18)–(21) by applying the dot algebra.

A.2 Proof of Proposition 2

Given the definition of $u_t^{njs}\equiv e^{V_t^{njs}}$, applying the dot algebra to (3) yields:

$$\dot{\mu}_0^{njs,nki} =e^{\frac{\beta\delta\left(V_1^{nki}-V_0^{nki}\right)}{\nu}}\frac{\sum_{\mathrm{K}=0}^J\sum_{\mathrm{I}\geq s}^3 e^{\frac{\beta\delta V_0^{n\mathrm{KI}}-\rho^{njs,n\mathrm{KI}}}{\nu}}}{\sum_{\mathrm{K}=0}^J\sum_{\mathrm{I}\geq s}^3 e^{\frac{\beta\delta V_1^{n\mathrm{KI}}-\rho^{njs,n\mathrm{KI}}}{\nu}}}$$

$$
\begin{aligned}
&= \frac{e^{\frac{\beta\delta\left(V_1^{nki}-V_0^{nki}\right)}{\nu}}}{\frac{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3} e^{\frac{\beta\delta V_1^{n\text{KI}}-\rho^{njs,n\text{KI}}}{\nu}} \frac{e^{\frac{\beta\delta V_0^{n\text{KI}}-\rho^{njs,n\text{KI}}}{\nu}}}{e^{\frac{\beta\delta V_0^{n\text{KI}}-\rho^{njs,n\text{KI}}}{\nu}}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3} e^{\frac{\beta\delta V_0^{n\text{KI}}-\rho^{njs,n\text{KI}}}{\nu}}}} \\
&= \frac{e^{\frac{\beta\delta\left(V_1^{nki}-V_0^{nki}\right)}{\nu}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3} e^{\frac{\beta\delta\left(V_1^{n\text{KI}}-V_0^{n\text{KI}}\right)}{\nu}} \frac{e^{\frac{\beta\delta V_0^{n\text{KI}}-\rho^{njs,n\text{KI}}}{\nu}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3} e^{\frac{\beta\delta V_0^{n\text{KI}}-\rho^{njs,n\text{KI}}}{\nu}}}} \\
&= \frac{\left(\dot{u}_1^{nki}\right)^{\frac{\beta\delta}{\nu}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3} \left(\dot{u}_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} \mu_{-1}^{njs,n\text{KI}}},
\end{aligned}
$$

which is (29) at $t=-1$. The proof applies to general t. The derivations above also imply that:

$$
\frac{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3} e^{\frac{\beta\delta V_{t+1}^{n\text{KI}}-\rho^{njs,n\text{KI}}}{\nu}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3} e^{\frac{\beta\delta V_{t+2}^{n\text{KI}}-\rho^{njs,n\text{KI}}}{\nu}}} = \left[\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3} \left(\dot{u}_{t+2}^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} \mu_t^{njs,n\text{KI}}\right]^{-1}.
$$

Using (2) and the above equation, we have:

$$
\begin{aligned}
V_1^{njs} - V_0^{njs} &= \ln \dot{\omega}_1^{njs} + \nu \ln \frac{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3} e^{\frac{\beta\delta V_2^{n\text{KI}}-\rho^{njs,n\text{KI}}}{\nu}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3} e^{\frac{\beta\delta V_1^{n\text{KI}}-\rho^{njs,n\text{KI}}}{\nu}}} \\
&= \ln \dot{\omega}_1^{njs} + \nu \ln \sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3} \left(\dot{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} \mu_0^{njs,n\text{KI}},
\end{aligned}
$$

which implies (30) for $t=0$. The proof applies to general t.

We can solve for $\{K_t^n\}_{t=1}^{\infty}$, given the temporary allocations $\left\{\dot{r}_t^n, \dot{P}_t^n\right\}_{t=1}^{\infty}$, and the initial values of K_0^n and φ_0^n. By (9) and the definition of $u_t^{nK} \equiv e^{V_t^{nK}}$, we obtain:

$$
\dot{u}_{t+1}^{nK} = \dot{C}_{t+1}^{nK} \left(\dot{u}_{t+2}^{nK}\right)^{\beta}.
$$

Equations (7) and (8) further imply:

$$
C_t^{nK} = \frac{1-\beta}{\beta} K_{t+1}^n.
$$

Equation (31) then follows by combining the equations above.

Equations (32)–(34) simply restate the laws of motion for labor in (4) and (5), and for capital in (8).

A.3 Proof of Proposition 3

Equations (41) and (43) are obtained by simply applying the definition of $\widehat{y}_{t+1} \equiv \dot{y}'_{t+1}/\dot{y}_{t+1}$ to (22) and (24), respectively. To derive (42), note that we can write the counterfactual version of (23) as:

$$\begin{aligned}
\dot{P}'^{nj}_{t+1} &= \left[\sum_o \left(\dot{\kappa}'^{nj,oj}_{t+1}\dot{x}'^{oj}_{t+1}\right)^{-\theta^j} \left(\dot{A}'^{oj}_{t+1}\right)^{\gamma^{oj}\theta^j} \pi'^{nj,oj}_t\right]^{-\frac{1}{\theta^j}} \\
&= \left[\sum_o \left(\widehat{\kappa}^{nj,oj}_{t+1}\widehat{x}^{oj}_{t+1}\right)^{-\theta^j} \left(\widehat{A}^{oj}_{t+1}\right)^{\gamma^{oj}\theta^j} \pi'^{nj,oj}_t \left(\dot{\kappa}^{nj,oj}_{t+1}\dot{x}^{oj}_{t+1}\right)^{-\theta^j} \left(\dot{A}^{oj}_{t+1}\right)^{\gamma^{oj}\theta^j}\right]^{-\frac{1}{\theta^j}} \\
&= \left[\sum_o \left(\widehat{\kappa}^{nj,oj}_{t+1}\widehat{x}^{oj}_{t+1}\right)^{-\theta^j} \left(\widehat{A}^{oj}_{t+1}\right)^{\gamma^{oj}\theta^j} \pi'^{nj,oj}_t \left(\dot{P}^{nj}_{t+1}\right)^{-\theta^j} \dot{\pi}^{nj,oj}_{t+1}\right]^{-\frac{1}{\theta^j}},
\end{aligned}$$

where (24) is used in the last step. Equation (42) then follows by rearranging the terms. The counterfactual change in the price index $\widehat{P}^n_{t+1}$ in (47) follows from its definition.

For (45), we simply write the counterfactual version of (26) and replace $\dot{w}'^{njs}_{t+1}\dot{L}'^{njs}_{t+1}w'^{njs}_t L'^{njs}_t$ with its equivalent $\widehat{w}^{nks}_{t+1}\widehat{L}^{nks}_{t+1}w'^{nks}_t L'^{nks}_t \dot{w}^{nks}_{t+1}\dot{L}^{nks}_{t+1}$. For (46), we use the counterfactual version of (27) and replace $\dot{r}'^n_{t+1}\dot{K}'^n_{t+1}r'^n_t K'^n_t$ with $\widehat{r}^n_{t+1}\widehat{K}^n_{t+1}r'^n_t K'^n_t \dot{r}^n_{t+1}\dot{K}^n_{t+1}$. Equation (44) simply follows from the definition of X'^{nj}_{t+1}.

Using the expression of $\dot{\chi}^{njh}_{t+1}$ in Proposition 1, we have:

$$\begin{aligned}
\left(\widehat{\chi}^{njh}_{t+1}\right)^{1-\xi^{njh}} &= \frac{\gamma^{L,njh}_t}{\gamma^{L,njh\prime}_t} \frac{\gamma^{L,nj3\prime}_t \left(\dot{w}^{nj3\prime}_{t+1}\right)^{1-\xi^{njh}} + \gamma^{K,nj\prime}_t \left(\dot{r}^{n\prime}_{t+1}\right)^{1-\xi^{njh}}}{\gamma^{L,nj3}_t \left(\dot{w}^{nj3}_{t+1}\right)^{1-\xi^{njh}} + \gamma^{K,nj}_t \left(\dot{r}^{n}_{t+1}\right)^{1-\xi^{njh}}} \\
&= \frac{1}{\gamma^{L,njh\prime}_t} \frac{\frac{\gamma^{L,nj3\prime}_t \left(\dot{w}^{nj3\prime}_{t+1}\right)^{1-\xi^{njh}}}{\dot{\gamma}^{L,njh}_{t+1}\left(\dot{w}^{nj3}_{t+1}\right)^{1-\xi^{njh}}} \dot{\gamma}^{L,njh}_{t+1}\left(\dot{w}^{nj3}_{t+1}\right)^{1-\xi^{njh}} + \frac{\gamma^{K,nj\prime}_t \left(\dot{r}^{n\prime}_{t+1}\right)^{1-\xi^{njh}}}{\dot{\gamma}^{L,njh}_{t+1}\left(\dot{r}^{n}_{t+1}\right)^{1-\xi^{njh}}} \dot{\gamma}^{L,njh}_{t+1}\left(\dot{r}^{n}_{t+1}\right)^{1-\xi^{njh}}}{\frac{\gamma^{L,nj3}_t}{\gamma^{L,njh}_t}\left(\dot{w}^{nj3}_{t+1}\right)^{1-\xi^{njh}} + \frac{\gamma^{K,nj}_t}{\gamma^{L,njh}_t}\left(\dot{r}^{n}_{t+1}\right)^{1-\xi^{njh}}} \\
&= \frac{1}{\gamma^{L,njh\prime}_t} \frac{\gamma^{L,nj3\prime}_t \left(\dot{w}^{nj3\prime}_{t+1}\right)^{1-\xi^{njh}}}{\dot{\gamma}^{L,njh}_{t+1}\left(\dot{w}^{nj3}_{t+1}\right)^{1-\xi^{njh}}} \dot{\gamma}^{L,nj3}_{t+1} + \frac{1}{\gamma^{L,njh\prime}_t} \frac{\gamma^{K,nj\prime}_t \left(\dot{r}^{n\prime}_{t+1}\right)^{1-\xi^{njh}}}{\dot{\gamma}^{L,njh}_{t+1}\left(\dot{r}^{n}_{t+1}\right)^{1-\xi^{njh}}} \dot{\gamma}^{K,nj}_{t+1} \\
&= \frac{1}{\gamma^{L,njh\prime}_t \dot{\gamma}^{L,njh}_{t+1}} \left[\gamma^{L,nj3\prime}_t \dot{\gamma}^{L,nj3}_{t+1}\left(\widehat{w}^{nj3}_{t+1}\right)^{1-\xi^{njh}} + \gamma^{K,nj\prime}_t \dot{\gamma}^{K,nj}_{t+1}\left(\widehat{r}^{n}_{t+1}\right)^{1-\xi^{njh}}\right].
\end{aligned}$$

Similarly, based on the expression of $\dot{\chi}^{nj}_{t+1}$ in Proposition 1, we have:

$$\left(\widehat{\chi}^{nj}_{t+1}\right)^{1-\xi^{nj}} = \frac{\frac{\gamma'^{L,nj1}_t}{\gamma^{nj}}\left(\dot{w}'^{nj1}_{t+1}\right)^{1-\xi^{nj}} + \frac{\gamma'^{L,nj2}_t}{\gamma^{nj}}\left(\dot{w}'^{nj2}_{t+1}\right)^{1-\xi^{nj}} + \frac{\gamma'^{L,njh}_t}{\gamma^{nj}}\left(\dot{\chi}'^{njh}_{t+1}\right)^{1-\xi^{nj}}}{\frac{\gamma^{L,nj1}_t}{\gamma^{nj}}\left(\dot{w}^{nj1}_{t+1}\right)^{1-\xi^{nj}} + \frac{\gamma^{L,nj2}_t}{\gamma^{nj}}\left(\dot{w}^{nj2}_{t+1}\right)^{1-\xi^{nj}} + \frac{\gamma^{L,njh}_t}{\gamma^{nj}}\left(\dot{\chi}^{njh}_{t+1}\right)^{1-\xi^{nj}}}$$

$$
\begin{aligned}
=&\frac{\frac{\gamma_t^{\prime L,nj1}}{\gamma^{nj}}\left(\dot{w}_{t+1}^{nj1}\right)^{1-\xi^{nj}}\left(\widehat{w}_{t+1}^{nj1}\right)^{1-\xi^{nj}}+\frac{\gamma_t^{\prime L,nj2}}{\gamma^{nj}}\left(\dot{w}_{t+1}^{nj2}\right)^{1-\xi^{nj}}\left(\widehat{w}_{t+1}^{nj2}\right)^{1-\xi^{nj}}+\frac{\gamma_t^{\prime L,njh}}{\gamma^{nj}}\left(\dot{\chi}_{t+1}^{njh}\right)^{1-\xi^{nj}}\left(\widehat{\chi}_{t+1}^{njh}\right)^{1-\xi^{nj}}}{\frac{\gamma_t^{L,nj1}}{\gamma^{nj}}\left(\dot{w}_{t+1}^{nj1}\right)^{1-\xi^{nj}}+\frac{\gamma_t^{L,nj2}}{\gamma^{nj}}\left(\dot{w}_{t+1}^{nj2}\right)^{1-\xi^{nj}}+\frac{\gamma_t^{L,njh}}{\gamma^{nj}}\left(\dot{\chi}_{t+1}^{njh}\right)^{1-\xi^{nj}}}\\
=&\frac{\gamma_t^{\prime L,nj1}}{\gamma^{nj}}\dot{\gamma}_{t+1}^{L,nj1}\left(\widehat{w}_{t+1}^{nj1}\right)^{1-\xi^{nj}}+\frac{\gamma_t^{\prime L,nj2}}{\gamma^{nj}}\dot{\gamma}_{t+1}^{L,nj2}\left(\widehat{w}_{t+1}^{nj2}\right)^{1-\xi^{nj}}+\frac{\gamma_t^{\prime L,njh}}{\gamma^{nj}}\dot{\gamma}_{t+1}^{L,njh}\left(\widehat{\chi}_{t+1}^{njh}\right)^{1-\xi^{nj}}.
\end{aligned}
$$

Finally, $\left\{\dot{\gamma}_{t+1}^{\prime L,njs}\right\}_{s=1}^{3}$, $\dot{\gamma}_{t+1}^{\prime L,njh}$ and $\dot{\gamma}_{t+1}^{\prime K,nj}$ follow from the expressions of $\left\{\dot{\gamma}_{t+1}^{L,njs}\right\}_{s=1}^{3}$, $\dot{\gamma}_{t+1}^{L,njh}$ and $\dot{\gamma}_{t+1}^{K,nj}$ in Proposition 1 by applying the hat algebra.

To obtain (35), apply the hat algebra to (29) to obtain:

$$
\begin{aligned}
\widehat{\mu}_{t+1}^{njs,nki}=&\left(\widehat{u}_{t+2}^{nki}\right)^{\frac{\beta\delta}{\nu}}\frac{\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\left(\dot{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}\mu_t^{njs,n\mathrm{KI}}}{\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\left(\dot{u}_{t+2}^{\prime n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}\mu_t^{\prime njs,n\mathrm{KI}}}\\
=&\frac{\left(\widehat{u}_{t+2}^{nki}\right)^{\frac{\beta\delta}{\nu}}}{\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\frac{\left(\dot{u}_{t+2}^{\prime n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}\mu_t^{\prime njs,n\mathrm{KI}}}{\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\left(\dot{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}\mu_t^{njs,n\mathrm{KI}}}}\\
=&\frac{\left(\widehat{u}_{t+2}^{nki}\right)^{\frac{\beta\delta}{\nu}}}{\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\frac{\left(\dot{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}\left(\dot{u}_{t+2}^{\prime n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}\mu_t^{\prime njs,n\mathrm{KI}}\left(\dot{u}_{t+2}^{n\mathrm{KI}}\right)^{-\frac{\beta\delta}{\nu}}}{\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\left(\dot{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}\mu_t^{njs,n\mathrm{KI}}}}.
\end{aligned}
$$

Applying (29) to the denominator above yields:

$$
\widehat{\mu}_{t+1}^{njs,nki}=\frac{\left(\widehat{u}_{t+2}^{nki}\right)^{\frac{\beta\delta}{\nu}}}{\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\dot{\mu}_{t+1}^{njs,n\mathrm{KI}}\mu_t^{\prime njs,n\mathrm{KI}}\left(\widehat{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}},
$$

which is equivalent to (35). To obtain (36), apply the hat algebra to (30) to obtain:

$$
\begin{aligned}
\widehat{u}_{t+1}^{njs}=&\widehat{\omega}_{t+1}^{njs}\left[\frac{\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\mu_t^{\prime njs,n\mathrm{KI}}\left(\dot{u}_{t+2}^{\prime n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\mu_t^{njs,n\mathrm{KI}}\left(\dot{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}}\right]^{\nu}\\
=&\widehat{\omega}_{t+1}^{njs}\left[\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\frac{\mu_t^{njs,n\mathrm{KI}}\left(\dot{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\mu_t^{njs,n\mathrm{KI}}\left(\dot{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}}\frac{\mu_t^{\prime njs,n\mathrm{KI}}}{\mu_t^{njs,n\mathrm{KI}}}\frac{\left(\dot{u}_{t+2}^{\prime n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\left(\dot{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}}\right]^{\nu}.
\end{aligned}
$$

Applying (29) to the above equation yields:

$$
\widehat{u}_{t+1}^{njs}=\widehat{\omega}_{t+1}^{njs}\left[\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\mu_{t+1}^{njs,n\mathrm{KI}}\frac{\mu_t^{\prime njs,n\mathrm{KI}}}{\mu_t^{njs,n\mathrm{KI}}}\frac{\left(\dot{u}_{t+2}^{\prime n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\left(\dot{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}}\right]^{\nu}
$$

$$
\begin{aligned}
=&\widehat{\omega}_{t+1}^{njs}\left[\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\frac{\mu_{t+1}^{njs,n\text{KI}}}{\mu_{t}^{njs,n\text{KI}}}\mu_{t}^{\prime njs,n\text{KI}}\frac{\left(\dot{u}_{t+2}^{\prime n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\left(\dot{u}_{t+2}^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}\right]^{\nu}\\
=&\widehat{\omega}_{t+1}^{njs}\left[\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\dot{\mu}_{t+1}^{njs,n\text{KI}}\mu_{t}^{\prime njs,n\text{KI}}\left(\widehat{u}_{t+2}^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}\right]^{\nu},
\end{aligned}
$$

which corresponds to (36).

The welfare dynamics for capital owners in (37) follows by applying the hat algebra to (31). Equations (38)–(40) are simply the counterfactual versions of the laws of motion for labor in (4)–(5), and for capital in (8). Since the counterfactual allocations at $t=0$ are the same as in the baseline economy, we can recover $\dot{\varphi}_{t+1}^{\prime n}$ from $\widehat{\varphi}_{t+1}^{n}$ implied by the solutions of (41)–(47), given φ_0^n, K_0^n, and $\dot{\varphi}_{t+1}^{n}$.

The above derivations handle the change of endogenous variables between t and $t+1$ in general. Special treatment is required for $\widehat{u}_1^{njs}$ and $\mu_1^{\prime njs,nki}$, because the shocks to fundamentals start at $t=1$ and the decisions made at $t=0$ by agents are taken as given. First, by (2), we have:

$$
\begin{aligned}
u_1^{\prime njs} &= \omega_1^{\prime njs}\left(\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\left(u_2^{\prime n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}\right)^{\nu},\\
u_0^{njs} &= \omega_0^{njs}\left(\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\left(u_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}\right)^{\nu}.
\end{aligned}
$$

Because the shocks to fundamentals start at $t=1$, it follows that $\dot{u}_1^{\prime njs}\equiv\frac{u_1^{\prime njs}}{u_0^{\prime njs}}=\frac{u_1^{\prime njs}}{u_0^{njs}}$ and $\dot{\omega}_1^{\prime njs}\equiv\frac{\omega_1^{\prime njs}}{\omega_0^{\prime njs}}=\frac{\omega_1^{\prime njs}}{\omega_0^{njs}}$. Combining these observations and defining $\phi_1^{n\text{KI}}\equiv\left(\frac{u_1^{n\text{KI}}}{u_1^{\prime n\text{KI}}}\right)^{\frac{\beta\delta}{\nu}}$, we have:

$$
\begin{aligned}
\dot{u}_1^{\prime njs} &= \dot{\omega}_1^{\prime njs}\left(\frac{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\left(u_2^{\prime n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\left(u_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}}\right)^{\nu}\\
&= \dot{\omega}_1^{\prime njs}\left(\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\frac{\left(u_2^{\prime n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\phi_1^{n\text{KI}}\left(u_1^{\prime n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}}\right)^{\nu}.
\end{aligned}
$$

Next, given (3) and the definition of $\phi_1^{n\text{KI}}$, we have:

$$
\mu_0^{njs,nki} = \frac{\left(u_1^{nki}\right)^{\frac{\beta\delta}{\nu}}e^{-\frac{\rho^{njs,nki}}{\nu}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\left(u_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}}
$$

$$= \frac{\phi_1^{nki} \left(u_1'^{nki}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,nki}}{\nu}}}{\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \phi_1^{n\text{KI}} \left(u_1'^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}}.$$

Combining $\mu_0^{njs,nki}$ and $\dot{u}_1'^{njs}$ yields:

$$\begin{aligned}
\dot{u}_1'^{njs} &= \dot{\omega}_1'^{njs} \left(\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \frac{\left(u_2'^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}}{\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \phi_1^{n\text{KI}} \left(u_1'^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}} \right)^{\nu} \\
&= \dot{\omega}_1'^{njs} \left(\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \frac{\phi_1^{n\text{KI}} \left(u_1'^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}}{\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \phi_1^{n\text{KI}} \left(u_1'^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}} \frac{\left(u_2'^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\left(u_1'^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}} \frac{1}{\phi_1^{n\text{KI}}} \right)^{\nu} \\
&= \dot{\omega}_1'^{njs} \left(\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \frac{\mu_0^{njs,n\text{KI}}}{\phi_1^{n\text{KI}}} \left(\dot{u}_2'^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} \right)^{\nu}.
\end{aligned}$$

Note that (30) at $t = 0$ gives us:

$$\dot{u}_1^{njs} = \dot{\omega}_1^{njs} \left[\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \mu_0^{njs,n\text{KI}} \left(\dot{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} \right]^{\nu}.$$

Using the definition of $\mu_0^{njs,n\text{KI}}$, we therefore have:

$$\begin{aligned}
\widehat{u}_1^{njs} &= \widehat{\omega}_1^{njs} \left(\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \frac{\frac{\mu_0^{njs,n\text{KI}}}{\phi_1^{n\text{KI}}} \left(\dot{u}_2'^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \mu_0^{njs,n\text{KI}} \left(\dot{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}} \right)^{\nu} \\
&= \widehat{\omega}_1^{njs} \left(\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \frac{\left(\widehat{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\phi_1^{n\text{KI}}} \frac{\mu_0^{njs,n\text{KI}} \left(\dot{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \mu_0^{njs,n\text{KI}} \left(\dot{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}} \right)^{\nu} \\
&= \widehat{\omega}_1^{njs} \left(\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \frac{\left(\widehat{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\phi_1^{n\text{KI}}} \frac{\frac{\left(u_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}}{\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \left(u_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}} \left(\dot{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \frac{\left(u_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}}{\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \left(u_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}} \left(\dot{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}} \right)^{\nu} \\
&= \widehat{\omega}_1^{njs} \left(\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \frac{\left(\widehat{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\phi_1^{n\text{KI}}} \frac{\left(u_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}}{\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \left(u_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}} \right)^{\nu}, \\
&= \widehat{\omega}_1^{njs} \left(\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \frac{\left(\widehat{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\phi_1^{n\text{KI}}} \mu_1^{njs,n\text{KI}} \right)^{\nu}, \\
&= \widehat{\omega}_1^{njs} \left(\sum_{\text{K}=0}^{J} \sum_{\text{I}\geq s}^{3} \mu_1^{njs,n\text{KI}} \left(\widehat{u}_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} \left(\widehat{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} \right)^{\nu},
\end{aligned}$$

which is the expression used in (B.2). Note that we have used the fact that

$$\frac{1}{\phi_1^{n\text{KI}}} = \left(\frac{u_1'^{n\text{KI}}}{u_1^{n\text{KI}}}\right)^{\frac{\beta\delta}{\nu}} = \left(\frac{u_1'^{n\text{KI}}/u_0'^{n\text{KI}}}{u_1^{n\text{KI}}/u_0^{n\text{KI}}}\right)^{\frac{\beta\delta}{\nu}} = \left(\widehat{u}_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}.$$

Next, we examine the sector-skill transition probability. Given (3), we have:

$$\begin{aligned}
\frac{\mu_1'^{njs,nki}}{\mu_1^{njs,nki}} &= \frac{\dfrac{\left(u_2'^{nki}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,nki}}{\nu}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\left(u_2'^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}}}{\dfrac{\left(u_2^{nki}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,nki}}{\nu}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\left(u_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}} e^{-\frac{\rho^{njs,n\text{KI}}}{\nu}}}} \\
&= \frac{\dfrac{\left(u_2'^{nki}\right)^{\frac{\beta\delta}{\nu}}}{\left(u_2^{nki}\right)^{\frac{\beta\delta}{\nu}}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\mu_1^{njs,n\text{KI}}\dfrac{\left(u_2'^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\left(u_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}} \\
&= \frac{\left(\widehat{u}_2^{nki}\right)^{\frac{\beta\delta}{\nu}}\dfrac{\left(u_1'^{nki}\right)^{\frac{\beta\delta}{\nu}}}{\left(u_1^{nki}\right)^{\frac{\beta\delta}{\nu}}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\mu_1^{njs,n\text{KI}}\left(\widehat{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}\dfrac{\left(u_1'^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}{\left(u_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}}} \\
\Rightarrow \mu_1'^{njs,nki} &= \frac{\mu_1^{njs,nki}\left(\widehat{u}_1^{nki}\right)^{\frac{\beta\delta}{\nu}}\left(\widehat{u}_2^{nki}\right)^{\frac{\beta\delta}{\nu}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\mu_1^{njs,n\text{KI}}\left(\widehat{u}_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}\left(\widehat{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}},
\end{aligned}$$

where we have again used the fact that $\left(\frac{u_1'^{n\text{KI}}}{u_1^{n\text{KI}}}\right)^{\frac{\beta\delta}{\nu}} = \left(\frac{u_1'^{n\text{KI}}/u_0'^{n\text{KI}}}{u_1^{n\text{KI}}/u_0^{n\text{KI}}}\right)^{\frac{\beta\delta}{\nu}} = \left(\widehat{u}_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}$. The above result corresponds to the expression used in (B.1).

B Implementation Algorithm of the Simulations

We first construct a baseline economy during 1995 – 2050 based on observed data. For counterfactual simulation, we use the allocations in the baseline economy $\dot{y}_t$ and Proposition 3 to recover the counterfactual allocations $\dot{y}'_t$ as $\dot{y}'_t = \widehat{y}_t \dot{y}_t$. Throughout this section we refer to 1995 as the initial year and denote by $t = 0$, and to 1996 as the "shock year" wherein the unexpected shock kicks in.

B.1 Constructing Baseline Economy

The construction of baseline economy come in three parts. We first use the data of value-added, tradeflow, labor allocation and capital in the initial year to compute the model-consistent counterparts of value-added and tradeflow using a similar approach to Proposition 1. We refer to this constructed allocation as the **base year allocation**. Then we combine the base year allocations with observed capital stock in 1995, the level of real rental rate in 1995 implied by the observed change in capital stock from 1995 to 1996, and the observed labor allocations and bilateral tariffs during 1995 – 2007 to construct changes in temporary allocations following Proposition 1 and capital dynamic (34). The allocations after 2008 are then computed using the allocations in 2007 following Propositions 1 and 2. The whole computed series for 1995 – 2050 are referred to as the **baseline economy**.

B.1.1 Computing Base Year Allocation

Our data in 1995 include country level capital $K^n_{0,data}$, labor allocations $L^{njs}_{0,data}$, country-sector labor value-added $VAL^{nj}_{0,data} \equiv w^{nj}_{0,data} L^{nj}_{0,data}$, country-level capital value-added $VAK^n_{0,data} \equiv r^n_{0,data} K^n_{0,data}$, cost shares of input factors $\gamma^{L,njs}_{0,data}$, $\gamma^{K,nj}_{0,data}$, country-sector value-added share γ^{nj}, and bilateral tradeflow $\pi^{nj,oj}_{0,data}$. The cost share of input factors are obtained as follows. First we compute capital share in value-added for 1995 at country-sector level using the value-added and gross output retrieved from TiVA ICIO, which implies the labor share to value-added at country-sector level. By multiplying the labor shares to wage-bill share of workers at country-sector-skill in 1995 obtained from WIOD 2013 we recover the country-sector-skill share to value-added. Dividing it by γ^{nj} yields $\gamma^{L,njs}_{0,data}$. The cost share of capital is similarly obtained by dividing the capital share to value-added with γ^{nj}. These information also allows us to compute value-added of labor at country-sector-skill level $VAL^{njs}_{0,data}$, and value-added of capital at country-sector level $VAK^{nj}_{0,data}$.

Our objective is to compute model-consistent $\Big\{VAL^{njs}_0 \equiv w^{njs}_0 L^{njs}_0, VAK^n_0 \equiv r^n_0 K^n_0, \gamma^{L,njs}_{1995}, \gamma^{K,nj}_{1995}, \pi^{nj,oj}_0, \ldots$ given the same labor allocations, capital stock and fundamental parameters such as technology A and tariffs τ. To speed-up computation, we normalize the observed value-added such that

$$\sum_n \sum_j \sum_s VAL^{njs}_{0,data} + \sum_n VAK^n_{0,data} = 1.$$

The temporary allocation of our model is exactly a static Eaton-Kortum (2002) model. Suppose that the observed data also follows the same data generating process, we can apply a hat-algebra-like approach that is isomorphic to Proposition 1 to recover the model-consistent base year allocations. Let $x_{0,data}$ denote the observed allocation in 1995, and $\overline{x}_{0,data} \equiv$

$x_0/x_{0,data}$ as the relative difference between the base year allocation and data observation. It is readily verified that

$$
\begin{aligned}
\overline{x}_0^{nj} &\equiv \left(\overline{\chi}_0^{nj}\right)^{\gamma^{nj}} \prod_{k=1}^{J} \left(\overline{P}_0^{nk}\right)^{\gamma^{nj,nk}} \\
\overline{P}_0^{nj} &= \left[\sum_{o} \pi_{0,data}^{nj,oj} \left(\overline{\kappa}_0^{nj,oj}\overline{x}_0^{oj}\right)^{-\theta^j} \left(\overline{A}_t^{0j}\right)^{\gamma^{ij}\theta^j}\right]^{-\frac{1}{\theta^j}} \\
\pi_0^{nj,oj} &= \left(\frac{\overline{\kappa}_0^{nj,oj}\overline{x}_0^{oj}}{\overline{P}_0^{nj}}\right)^{-\theta^j} \left(\overline{A}_0^{oj}\right)^{\gamma^{ij}\theta^j} \pi_{0,data}^{nj,oj} \\
X_0^{nj} &= \sum_{k=1}^{J} \gamma^{nk,nj} \sum_{o=1}^{N} \frac{\pi_0^{ok,nk} X_0^{ok}}{1+\tau_0^{ok,nk}} \\
&\quad + \alpha^{nj} \left(\sum_{k=1}^{J}\sum_{s=1}^{3} \overline{w}_0^{nks}\overline{L}_0^{nks}\left[w_{0,data}^{nks}L_{0,data}^{nks}\right] + \overline{r}_0^{n}\overline{K}_0^{n}\left[r_{0,data}^{n}K_{0,data}^{n}\right]\right) \\
&\quad + \alpha^{nj} \left(\sum_{k=1}^{J}\sum_{o=1}^{N} \tau_0^{nk,ok} \frac{\pi_0^{nk,ok} X_0^{nk}}{1+\tau_0^{nk,ok}} + D_t^n\right) \\
D_t^n &\equiv \iota_t^n \sum_{k=1}^{J}\sum_{o=1}^{N} X_t^{ok} \\
\overline{w}_0^{nks}\overline{L}_0^{nks}\left[w_{0,data}^{nks}L_{0,data}^{nks}\right] &= \gamma_0^{L,njs} \sum_{o=1}^{N} \frac{\pi_0^{oj,nj}X_0^{oj}}{1+\tau_0^{oj,nj}} \equiv VAL_0^{njs} \\
\overline{r}_0^{n}\overline{K}_0^{n}\left[r_{0,data}^{n}K_{0,data}^{n}\right] &= \sum_{j=1}^{J} \gamma_0^{K,nj} \sum_{o=1}^{N} \frac{\pi_0^{oj,nj}X_0^{oj}}{1+\tau_0^{oj,nj}} \equiv VAK_0^{n} \\
\left[\overline{\chi}_0^{njh}\right]^{1-\xi^{njh}} &= \frac{\gamma_{0,data}^{L,nj3}}{\gamma_{0,data}^{L,njh}} \left[\overline{w}_0^{nj3}\right]^{1-\xi^{njh}} + \frac{\gamma_{0,data}^{K,nj}}{\gamma_{0,data}^{L,njh}} \left[\overline{r}_0^{n}\right]^{1-\xi^{njh}} \\
\left[\overline{\chi}_0^{nj}\right]^{1-\xi^{nj}} &= \frac{\gamma_{0,data}^{L,nj1}}{\gamma^{nj}} \left(\overline{w}_0^{nj1}\right)^{1-\xi^{nj}} + \frac{\gamma_{0,data}^{L,nj2}}{\gamma^{nj}} \left(\overline{w}_0^{nj2}\right)^{1-\xi^{nj}} + \frac{\gamma_{0,data}^{L,njh}}{\gamma^{nj}} \left(\overline{\chi}_0^{njh}\right)^{1-\xi^{nj}} \\
\gamma_0^{L,nj1} &= \left(\frac{\overline{w}_0^{nj1}}{\overline{\chi}_0^{nj}}\right)^{1-\xi^{nj}} \gamma_{0,data}^{L,nj1} \\
\gamma_0^{L,nj2} &= \left(\frac{\overline{w}_0^{nj2}}{\overline{\chi}_0^{nj}}\right)^{1-\xi^{nj}} \gamma_{0,data}^{L,nj2} \\
\gamma_0^{L,nj3} &= \left(\frac{\overline{w}_0^{njh}}{\overline{\chi}_0^{nj}}\right)^{1-\xi^{nj}} \left(\frac{\overline{w}_0^{nj3}}{\overline{\chi}_0^{njh}}\right)^{1-\xi^{njh}} \gamma_{0,data}^{L,nj3}
\end{aligned}
$$

$$\gamma_0^{L,njh} = \gamma^{nj} - \gamma_0^{L,nj1} - \gamma_0^{L,nj2}$$
$$\gamma_0^{K,nj} = \gamma^{nj} - \gamma_0^{L,nj1} - \gamma_0^{L,nj2} - \gamma_0^{L,nj3}$$

where we have $\overline{\kappa}_0^{nj,ij} = \overline{A}_0^{ij} = \overline{L}_0^{nks} = \overline{K}_0^{n} = 1$.

The algorithm is as follows:

1. Guess a vector of $\left\{\overline{w}_0^{njs}(0), \overline{r}_0^n(0)\right\}$ such that $\sum_n \sum_j \sum_s VAL_0^{njs} + \sum_n VAK_0^n = 1$. Then solve for $\left\{\overline{\chi}_0^{nj}(0), \gamma_0^{L,njs}(0), \gamma_0^{K,nj}(0)\right\}$ with the initial guesses and $\left\{\gamma_{0,data}^{L,njs}, \gamma_{0,data}^{K,nj}\right\}$.

2. Guess a vector of $\overline{P}_0^{nj}$ to obtain $\overline{x}_0^{nj}$ along using $\overline{\chi}_0^{nj}(0)$, then iterate with $\overline{P}_0^{nj}$ until convergence. Denote the converged outcome by $\overline{P}_0^{nj}(0)$ and $\overline{x}_0^{nj}(0)$.

3. Solve for $\pi_0^{nj,oj}(0)$ using $\left\{\overline{P}_0^{nj}(0), \overline{x}_0^{nj}(0), \pi_{0,data}^{nj,oj}\right\}$.

4. Use $\left\{\overline{w}_0^{njs}(0), \overline{r}_0^n(0), \gamma_0^{L,njs}(0), \gamma_0^{K,nj}(0), \pi_0^{nj,oj}(0), VAK_{0,data}^n, VAL_{0,data}^{njs}\right\}$ to solve for $X_0^{nj}(0)$ from the goods market clearing conditions.

5. Use $\left\{\gamma_0^{L,njs}(0), \gamma_0^{K,nj}(0), \pi_0^{nj,oj}(0), X_0^{nj}(0)\right\}$ to obtain value-added of labor and capital for the base year allocation $\left\{VAL_0^{njs}, VAK_0^n\right\}$, then use $\left\{VAL_0^{njs}, VAK_0^n, VAL_{0,data}^{njs}, VAK_{0,data}^n\right\}$ to recover $\left\{\overline{w}_0^{njs}(1), \overline{r}_0^n(1)\right\}$ from the factor market clearing conditions.

6. Check whether $\left\{\overline{w}_0^{njs}(1), \overline{r}_0^n(1)\right\} \approx \left\{\overline{w}_0^{njs}(0), \overline{r}_0^n(0)\right\}$. If yes, stop iteration and return the outcome. If not, form a new initial guess for factor prices based on $\left\{\overline{w}_0^{njs}(1), \overline{r}_0^n(1)\right\}$ following Alvarez and Lucas (2007) and repeat steps 2 to 6 until convergence.

B.1.2 Computing Allocations for Data Years (1995 – 2007)

We use the base year allocations $\left\{VAL_0^{njs}, VAK_0^n, \gamma_0^{L,njs}, \gamma_0^{K,nj}, \pi_0^{nj,oj}, X_0^{nj}\right\}$ as a starting point, and combine with the actual labor allocation series $\left\{L_{t,data}^{njs}\right\}_{t=0}^{12}$ and capital allocations $\left\{K_{0,data}^n, K_{1,data}^n\right\}$ hence the implied φ_0^n from (34), $\left\{r_{t,data}^n \equiv VAK_{t,data}^n / K_{t,data}^n\right\}_{t=0}^{12}$ and implied $\left\{P_{t,data}^n\right\}_{t=0}^{12}$ from $\{\varphi_t^n\}_{t=1}^{12}$, actual bilateral trade shares $\left\{\pi_{t,data}^{nj,oj}\right\}_{t=0}^{12}$, actual changes in factor cost shares $\left\{\dot{\gamma}_{t,data}^{L,njs}, \dot{\gamma}_{t,data}^{K,nj}\right\}$, and actual changes in tariff rates $\left\{\tau_{t,data}^{nj,oj}\right\}_{t=0}^{12}$ to compute the series $\left\{\dot{w}_t^{njs}, \dot{r}_t^n, \dot{P}_t^n, \phi_t^n, K_t^n, VAL_t^{njs}, VAK_t^{nj}, \gamma_t^{L,njs}\gamma_t^{K,nj}, \pi_t^{nj,oj}, X_t^{nj}\right\}_{t=1}^{12}$ for 1996 – 2007 using equations (41) - (47) in Proposition 3 and (40). We define $\widehat{y}_{t+1} \equiv \dot{y}_{t+1}/\dot{y}_{t+1,data}$, where $\dot{y}_{t+1}$ is the model-consistent allocations implied by the data allocations $\dot{y}_{t+1,data}$. The algorithm starts by using allocations at period $t = 0$ to compute the allocations for $t = 1$, and then using the computed allocations for $t = 1$ as the starting point to compute the allocations for $t = 2$ and so on. For each $t \in \{0, ..., 11\}$ we compute the allocations for $t + 1$ as follows:

1. Use observed $\left\{L^{njs}_{t,data}\right\}^{12}_{t=1}$ to compute $\dot{L}^{njs}_{t+1}$ hence $\left\{\mu^{njs,nki}_{t,data}\right\}^{11}_{t=1}$.

2. Guess a vector of changes in factor prices $\left\{\dot{w}^{njs}_{t+1}(0), \dot{r}^{n}_{t+1}(0)\right\}$ such that

$$VA^{world}_{t+1}(0) = \sum_n \sum_j \sum_s w^{njs}_{t+1}(0)\, L^{nj,s}_{t+1} + \sum_n r^n_{t+1}(0)\, K^n_{t+1} = 1.$$

   Then solve for $\left\{\dot{\chi}^{nj}_{t+1}(0), \gamma^{L,njs}_{t+1}(0), \gamma^{K,nj}_{t+1}(0)\right\}$ with the initial guesses and $\left\{\gamma^{L,njs}_t, \gamma^{K,nj}_t\right\}$.

3. Guess a vector of $\widehat{P}^{nj}_{t+1}$ to obtain $\widehat{x}^{nj}_{t+1}$ using $\dot{\chi}^{nj}_{t+1}(0)$ from (41), then iterate (42) along with $\pi^{nj,oj}_t$ and $\dot{\pi}^{nj,oj}_{t+1,data}$ until $\widehat{P}^{nj}_{t+1}$ converges. Denote the converged outcome by $\widehat{P}^{nj}_{t+1}(0)$ and $\widehat{x}^{nj}_{t+1}(0)$.

4. Solve for $\pi^{nj,oj}_{t+1}(0)$ using $\left\{\widehat{P}^{nj}_{t+1}(0), \widehat{x}^{nj}_{t+1}(0), \pi^{nj,oj}_t, \dot{\pi}^{nj,oj}_{t+1,data}\right\}$ from (43).

5. Use $\left\{\dot{w}^{njs}_{t+1}(0), \dot{r}^n_{t+1}(0), \pi^{nj,oj}_{t+1}(0), VAL^{njs}_t, VAK^{nj}_t, \dot{L}^{njs}_{t+1}(0), \dot{K}^n_{t+1}(0)\right\}$ to solve for $X^{nj}_{t+1}(0)$ from (44), where $VAL^{njs}_{t+1} \equiv \dot{w}^{njs}_{t+1}\dot{L}^{njs}_{t+1}VAL^{njs}_t$ and $VAK^n_{t+1} \equiv \dot{r}^n_{t+1}\dot{K}^n_{t+1}VAK^n_t$.

6. Use $\left\{\gamma^{L,njs}_{t+1}(0), \gamma^{K,nj}_{t+1}(0), \pi^{nj,oj}_{t+1}(0), X^{nj}_{t+1}(0)\right\}$ to obtain $\left\{VAL^{njs}_{t+1}(0), VAK^n_{t+1}(0)\right\}$ as the RHS of (45) and (46). Then combine with $\left\{VAK^n_t, VAL^{njs}_t, \dot{L}^{njs}_{t+1}, \dot{K}^n_{t+1}\right\}$ to recover $\left\{\dot{w}^{njs}_{t+1}(1), \dot{r}^n_{t+1}(1)\right\}$ using (45) and (46).

7. Check whether $\left\{\dot{w}^{njs}_{t+1}(1), \dot{r}^n_{t+1}(1)\right\} \approx \left\{\dot{w}^{njs}_{t+1}(0), \dot{r}^n_{t+1}(0)\right\}$. If not, form a new initial guess for factor prices based on $\left\{\dot{w}^{njs}_{t+1}(0), \dot{r}^n_{t+1}(0)\right\}$ following Alvarez and Lucas (2007) and repeat steps 2 to 7 until convergence. If yes, enter the next step.

8. Use $\dot{P}^{nj}_{t+1,data}$ and $\widehat{P}^{nj}_{t+1}$ to compute $\dot{P}^n_{t+1}$. Then use $\dot{r}^n_{t+1}$ and $\dot{w}^{njs}_{t+1}$ and $\dot{P}^n_{t+1}$ to compute $\dot{\varphi}^n_{t+1}$ and $\dot{\omega}^{njs}_{t+1}$. Recover $\varphi^n_{t+1} = \dot{\varphi}^n_{t+1}\varphi^n_t$.

9. Use K^n_t and φ^n_t to compute K^n_{t+1} hence $\dot{K}^n_{t+1}$ from (34).

B.1.3 Computing Allocations after 2007

For the allocations beyond the data period we assume that the economy fundamentals such as bilateral tariffs are fixed at the levels in 2007. In other words, $\dot{x}_{t+1} = 1$ for these economy fundamentals. Then we use the computed allocations in 2007 ($t = 12$) to compute the time series during 2008 − 2050 ($t \in \{13, ..., 55\}$). The algorithm comes in two layers. The outer layer computes the labor transition, while the inner layer computes the temporary allocations given the labor transition using exactly the same algorithm that computes the allocations for the data years. The algorithm is as follows:

1. Guess a vector of $\left\{\dot{u}_t^{njs}(0)\right\}_{t=13}^{55}$ such that $\dot{u}_{55}^{njs}(0) = 1$, i.e., the economy no longer adjusts since 2051. Then we use the initial guess to compute $\left\{\mu_t^{njs,nki}(0)\right\}_{t=12}^{54}$ and $\left\{L_t^{njs}(0)\right\}_{t=13}^{55}$ by (29), (32) and (33) along with $\left\{\mu_{11}^{njs,nki}, L_{12}^{njs}\right\}$ observed in the data.

2. Given $\left\{\mu_t^{njs,nki}(0)\right\}_{t=12}^{54}$ and $\left\{L_t^{njs}(0)\right\}_{t=13}^{55}$ we compute the temporary allocations for $t = \{13, ..., 55\}$ with the following algorithm to obtain the series of real wages $\left\{\omega_t^{njs}(0)\right\}_{t=13}^{54}$: Use $t = 12$ as the starting point, we compute the allocations for $t+1 \in \{13, ..., 55\}$ as follows:

   (a) Guess a vector of changes in factor prices $\left\{\dot{w}_{t+1}^{njs}(0), \dot{r}_{t+1}^{n}(0)\right\}$ such that

   $$VA_{t+1}^{world}(0) = \sum_n \sum_j \sum_s w_{t+1}^{njs}(0) L_{t+1}^{nj,s} + \sum_n r_{t+1}^{n}(0) K_{t+1}^{n} = 1.$$

   Then solve for $\left\{\dot{\chi}_{t+1}^{nj}(0), \gamma_{t+1}^{L,njs}(0), \gamma_{t+1}^{K,nj}(0)\right\}$ with the initial guesses and $\left\{\gamma_t^{L,njs}, \gamma_t^{K,nj}\right\}$.

   (b) Guess a vector of $\dot{P}_{t+1}^{nj}$ to obtain $\dot{x}_{t+1}^{nj}$ using $\dot{\chi}_{t+1}^{nj}(0)$ from (22), then iterate (23) along with $\pi_t^{nj,oj}$ until $\dot{P}_{t+1}^{nj}$ converges. Denote the converged outcome by $\dot{P}_{t+1}^{nj}(0)$ and $\dot{x}_{t+1}^{nj}(0)$.

   (c) Solve for $\pi_{t+1}^{nj,oj}(0)$ using $\left\{\dot{P}_{t+1}^{nj}(0), \dot{x}_{t+1}^{nj}(0), \pi_t^{nj,oj}\right\}$ from (24).

   (d) Use $\left\{\dot{w}_{t+1}^{njs}(0), \dot{r}_{t+1}^{n}(0), \pi_{t+1}^{nj,oj}(0), VAL_t^{njs}, VAK_t^{nj}, \dot{L}_{t+1}^{njs}(0), \dot{K}_{t+1}^{n}(0)\right\}$ to solve for $X_{t+1}^{nj}(0)$ from (25).

   (e) Use $\left\{\gamma_{t+1}^{L,njs}(0), \gamma_{t+1}^{K,nj}(0), \pi_{t+1}^{nj,oj}(0), X_{t+1}^{nj}(0)\right\}$ to obtain $\left\{VAL_{t+1}^{njs}(0), VAK_{t+1}^{n}(0)\right\}$ as the RHS of (26) and (27). Then combine with $\left\{VAK_t^{n}, VAL_t^{njs}, \dot{L}_{t+1}^{njs}, \dot{K}_{t+1}^{n}\right\}$ to recover $\left\{\dot{w}_{t+1}^{njs}(1), \dot{r}_{t+1}^{n}(1)\right\}$ using (26) and (27).

   (f) Check whether $\left\{\dot{w}_{t+1}^{njs}(1), \dot{r}_{t+1}^{n}(1)\right\} \approx \left\{\dot{w}_{t+1}^{njs}(0), \dot{r}_{t+1}^{n}(0)\right\}$. If not, form a new initial guess for factor prices based on $\left\{\dot{w}_{t+1}^{njs}(0), \dot{r}_{t+1}^{n}(0)\right\}$ following Alvarez and Lucas (2007) and repeat steps 2 to 7 until convergence. If yes, enter the next step.

   (g) Use $\dot{P}_{t+1}^{nj}$ to compute $\dot{P}_{t+1}^{n}$. Then use $\dot{r}_{t+1}^{n}$ and $\dot{w}_{t+1}^{njs}$ and $\dot{P}_{t+1}^{n}$ to compute $\dot{\varphi}_{t+1}^{n}$ and $\dot{\omega}_{t+1}^{njs}$. Recover $\varphi_{t+1}^{n} = \dot{\varphi}_{t+1}^{n}\varphi_t^{n}$.

   (h) Use K_t^{n} and φ_t^{n} to compute K_{t+1}^{n} hence $\dot{K}_{t+1}^{n}$ from (34).

3. Use $\left\{\mu_t^{njs,nki}(0)\right\}_{t=12}^{53}$, $\left\{\omega_t^{njs}(0)\right\}_{t=13}^{54}$ and $\left\{\dot{u}_t^{njs}(0)\right\}_{t=14}^{55}$ to compute $\left\{\dot{u}_t^{njs}(1)\right\}_{t=13}^{54}$ following (30). Check if $\left\{\dot{u}_t^{njs}(1)\right\}_{t=13}^{54} \approx \left\{\dot{u}_t^{njs}(0)\right\}_{t=13}^{54}$. Note that $\dot{u}_{55}^{njs}(1) = \dot{u}_{55}^{njs}(0) = 1$ by construction. If $\dot{u}$ converges, return the temporary allocations, $\left\{\mu_t^{njs,nki}\right\}_{t=12}^{54}$, $\left\{L_t^{njs}\right\}_{t=13}^{55}$, and $\left\{\dot{u}_t^{njs}\right\}_{t=13}^{55}$ as the dynamic equilibrium path. If not, use $\left\{\dot{u}_t^{njs}(1)\right\}_{t=13}^{55}$ to construct a new initial guess and repeat steps 1 to 3 until convergence.

B.2 Computing Counterfactual Economy

Consider a counterfactual change in economy fundamentals that kicks-in in the "shock year" 1996, the resulting counterfactual changes in allocations $\dot{x}'_t$ during 1995 – 2050 can be recovered using the constructed baseline changes $\dot{x}_t$ and hat algebra $\widehat{x}_t$ following Proposition 3. Since the shock is unexpected, the allocations before 1995 are identical between the counterfactual and baseline economy. Following the proof for Proposition 3, it implies that $\widehat{\mu}_1^{njs,nki}$ and $\widehat{u}_1^{njs}$ satisfy

$$\mu_1'^{njs,nki} = \frac{\mu_1^{njs,nki}\left(\widehat{u}_1^{nki}\right)^{\frac{\beta\delta}{\nu}}\left(\widehat{u}_2^{nki}\right)^{\frac{\beta\delta}{\nu}}}{\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\mu_1^{njs,n\text{KI}}\left(\widehat{u}_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}\left(\widehat{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}} \tag{B.1}$$

$$\widehat{u}_1^{njs} = \widehat{\omega}_1^{njs}\left(\sum_{\text{K}=0}^{J}\sum_{\text{I}\geq s}^{3}\mu_1^{njs,n\text{KI}}\left(\widehat{u}_1^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}\left(\widehat{u}_2^{n\text{KI}}\right)^{\frac{\beta\delta}{\nu}}\right)^{\nu}. \tag{B.2}$$

The algorithm is similar to that used to compute the baseline allocations after the data years. Idea wise, we first guess $\widehat{u}$ and compute counterfactual labor dynamics $\{\mu', L'\}$ using their baseline counterparts. Then we use $\{L, K\}$ to construct $\left\{\widehat{L}, \widehat{K}\right\}$, and then combine with the baseline economy to compute the counterfactual temporary allocations from period to period. Then we take the implied $\widehat{\omega}$ to check the convergence of $\widehat{u}$. The algorithm is detailed as follows.

1. Guess $\left\{\widehat{u}_t^{njs}(0)\right\}_{t=1}^{55}$ such that $\widehat{u}_{55}^{njs}(0) = 1$. Then compute $\left\{\mu_t'^{njs,nki}(0)\right\}_{t=1}^{54}$ and $\left\{L_t'^{njs}(0)\right\}_{t=1}^{55}$ with $\left\{\mu_t^{njs,nki}\right\}_{t=1}^{54}$ from (35), (B.1), (38) and (39). Note that $L_1'^{njs}(0) = L_1^{njs}$ since μ_0 and L_0 are the same.

2. Compute temporary equilibrium for the counterfactual economy given $\left\{L_t'^{njs}(0)\right\}_{t=1}^{55}$, $\varphi_0^n = \varphi_0'^n$, $K_0^n = K_0'^n$, $\gamma_0^{L,njs} = \gamma_0'^{L,njs}$, $\gamma_0^{K,nj} = \gamma_0'^{K,nj}$, $\pi_0^{nj,oj} = \pi_0'^{nj,oj}$, $VAL_0^{njs} = VAL_0'^{njs}$ and $VAK_0^{nj} = VAK_0'^{nj}$ for each $t \in \{1, ..., 55\}$:

(a) Guess a vector of changes in factor prices $\left\{\dot{w}_{t+1}^{\prime njs}(0), \dot{r}_{t+1}^{\prime n}(0)\right\}$ such that

$$VA_{t+1}^{\prime world}(0) = \sum_{n}\sum_{j}\sum_{s} w_{t+1}^{\prime njs}(0) L_{t+1}^{\prime nj,s} + \sum_{n} r_{t+1}^{\prime n}(0) K_{t+1}^{\prime n} = 1.$$

Then solve for $\left\{\widehat{w}_{t+1}^{njs}(0), \widehat{r}_{t+1}^{n}(0), \widehat{\chi}_{t+1}^{nj}(0), \gamma_{t+1}^{\prime L,njs}(0), \gamma_{t+1}^{\prime K,nj}(0)\right\}$ with the initial guesses and $\left\{\dot{w}_{t+1}^{njs}, \dot{r}_{t+1}^{n}, \dot{\gamma}_{t+1}^{L,njs}, \dot{\gamma}_{t+1}^{K,nj}, \gamma_{t}^{\prime L,njs}, \gamma_{t}^{\prime K,nj}\right\}$.

(b) Guess a vector of $\widehat{P}_{t+1}^{nj}$ to obtain $\widehat{x}_{t+1}^{nj}$ using $\widehat{\chi}_{t+1}^{nj}(0)$ from (41), then iterate (42) along with $\left\{\pi_{t}^{\prime nj,oj}, \dot{\pi}_{t+1}^{nj,oj}\right\}$ until $\widehat{P}_{t+1}^{nj}$ converges. Denote the converged outcome by $\widehat{P}_{t+1}^{nj}(0)$ and $\widehat{x}_{t+1}^{nj}(0)$.

(c) Solve for $\pi_{t+1}^{\prime nj,oj}(0)$ using $\left\{\widehat{P}_{t+1}^{nj}(0), \widehat{x}_{t+1}^{nj}(0), \pi_{t}^{\prime nj,oj}, \dot{\pi}_{t+1}^{nj,oj}\right\}$ from (43).

(d) Use $\left\{\dot{w}_{t+1}^{\prime njs}(0), \dot{r}_{t+1}^{\prime n}(0), \pi_{t+1}^{\prime nj,oj}(0), VAL_{t}^{\prime njs}, VAK_{t}^{\prime nj}, \dot{L}_{t+1}^{\prime njs}(0), \dot{K}_{t+1}^{\prime n}(0)\right\}$ to solve for $X_{t+1}^{\prime nj}(0)$ from (44).

(e) Use $\left\{\gamma_{t+1}^{\prime L,njs}(0), \gamma_{t+1}^{\prime K,nj}(0), \pi_{t+1}^{\prime nj,oj}(0), X_{t+1}^{\prime nj}(0)\right\}$ to obtain $\left\{VAL_{t+1}^{\prime njs}(0), VAK_{t+1}^{\prime n}(0)\right\}$ as the RHS of (26) and (27). Then combine with $\left\{VAK_{t}^{\prime n}, VAL_{t}^{\prime njs}, \dot{L}_{t+1}^{\prime njs}(0), \dot{K}_{t+1}^{\prime n}(0)\right\}$ to recover $\left\{\dot{w}_{t+1}^{njs}(1), \dot{r}_{t+1}^{n}(1)\right\}$ using (26) and (27).

(f) Check whether $\left\{\dot{w}_{t+1}^{\prime njs}(1), \dot{r}_{t+1}^{\prime n}(1)\right\} \approx \left\{\dot{w}_{t+1}^{\prime njs}(0), \dot{r}_{t+1}^{\prime n}(0)\right\}$. If not, form a new initial guess for factor prices based on $\left\{\dot{w}_{t+1}^{\prime njs}(1), \dot{r}_{t+1}^{\prime n}(1)\right\}$ following Alvarez and Lucas (2007) and repeat steps (a) to (g) until convergence. If yes, enter the next step.

(g) Use $\dot{P}_{t+1}^{n}$ and $\widehat{P}_{t+1}^{nj}$ to recover $\dot{P}_{t+1}^{\prime n}$. Then use $\dot{r}_{t+1}^{\prime n}$ and $\dot{w}_{t+1}^{\prime njs}$ and $\dot{P}_{t+1}^{\prime n}$ to compute $\dot{\varphi}_{t+1}^{\prime n}$ and $\dot{\omega}_{t+1}^{\prime njs}$. Recover $\varphi_{t+1}^{\prime n} = \dot{\varphi}_{t+1}^{\prime n}\varphi_{t}^{\prime n}$.

(h) Use $K_{t}^{\prime n}$ and $\varphi_{t}^{\prime n}$ to compute $K_{t+1}^{\prime n}$ hence $\dot{K}_{t+1}^{\prime n}$ from (40).

3. Construct $\left\{\widehat{\omega}_{t}^{njs}\right\}_{t=1}^{55}$ using the outcome from step 2 and the baseline allocations. Then compute $\left\{\widehat{u}_{t}^{njs}(1)\right\}_{t=1}^{54}$ using $\left\{\widehat{\omega}_{t}^{njs}\right\}_{t=1}^{54}$, $\left\{\dot{\mu}_{t}^{njs,nki}\right\}_{t=1}^{54}$, $\left\{\mu_{t}^{\prime njs,nki}(0)\right\}_{t=1}^{53}$ and $\left\{\widehat{u}_{t}^{njs}(0)\right\}_{t=1}^{55}$ following (36). Note that $\widehat{u}_{55}^{njs}(0) = \widehat{u}_{55}^{njs}(1) = 1$ by construction. Check if $\left\{\widehat{u}_{t}^{njs}(0)\right\}_{t=1}^{55} \approx \left\{\widehat{u}_{t}^{njs}(1)\right\}_{t=1}^{55}$. If yes return the results as the counterfactual equilibrium path. If not, construct a new initial guess based on $\left\{\widehat{u}_{t}^{njs}(1)\right\}_{t=1}^{55}$ then repeat steps 1 to 3.

B.3 Welfare Computation

The long-run welfare change for workers in country n with sector-skill combination js in 1995 is given by the compensation variation. Following Caliendo, Dvorkin and Parro (2019) the welfare change is given by:

$$\widehat{W}^{njs} = \sum_{t=1995}^{\infty} (\beta\delta)^{t-1995} \ln \left(\frac{\widehat{\omega}_t^{njs}}{\left(\widehat{\mu}_t^{njs,njs}\right)^{\nu}} \right).$$

Note that the allocations in 1995 are identical for both the baseline and the counterfactual economies, the welfare change formula can be equivalently stated as:

$$\widehat{W}^{njs} = (1-\beta\delta) \sum_{t=1995}^{\infty} (\beta\delta)^{t-1995} \ln \left(\frac{\omega_t^{\prime njs}/\omega_t^{njs}}{\left(\mu_t^{\prime njs,njs}/\mu_t^{njs,njs}\right)^{\nu}} \right).$$

As we simulate the model up to year 2050, we assume that the allocations are fixed at the 2050 levels ever after so that the resulting welfare formula is given by:

$$\begin{aligned}
\widehat{W}^{njs} &= (1-\beta\delta) \left[\sum_{t=1995}^{2049} (\beta\delta)^{t-1995} \ln \frac{(\omega_t/\omega_t')}{(\mu_t/\mu_t')^{v}} + (\beta\delta)^{2050-1995} \ln \frac{(\omega_{2050}/\omega_{2050}')}{(\mu_{2050}/\mu_{2050}')^{v}} \right. \\
&\qquad \left. + \sum_{t=2051}^{\infty} (\beta\delta)^{t-1995} \ln \frac{(\omega_{2050}/\omega_{2050}')}{(\mu_{2050}/\mu_{2050}')^{v}} \right] \\
&= (1-\beta\delta) \left[\sum_{t=1995}^{2049} (\beta\delta)^{t-1995} \ln \frac{(\omega_t/\omega_t')}{(\mu_t/\mu_t')^{v}} + \frac{(\beta\delta)^{55}}{1-\beta\delta} \ln \frac{(\omega_{2050}/\omega_{2050}')}{(\mu_{2050}/\mu_{2050}')^{v}} \right].
\end{aligned}$$

Our implementation inspects the welfare changes at both broad sector and skill levels up to 2020. For this purpose, the welfare changes are defined as the sum of components during 1995–2020. Then we aggregate the njs-level welfare changes up to the relevant levels using labor value-added shares in 1995 as weight.

The welfare formula for the capital owner is similarly derived. Using (7)–(9), we obtain the expected lifetime welfare as:

$$\begin{aligned}
V_t^{nK} &= \ln (1-\beta) \left[\varphi_t^n + \left(1-\delta^K\right) \right] K_t^n + \beta V_{t+1}^{nK} \\
&= \ln (1-\beta) \frac{K_{t+1}^n}{\beta} + \beta V_{t+1}^{nK} \\
&= \ln \left(\frac{1-\beta}{\beta} \right) + \ln K_{t+1}^n + \beta \left[\ln \left(\frac{1-\beta}{\beta} \right) + \ln K_{t+2}^n + \beta V_{t+2}^{nK} \right] \\
&= (1+\beta) \ln \left(\frac{1-\beta}{\beta} \right) + \ln K_{t+1}^n + \beta \ln K_{t+2}^n + \beta^2 V_{t+2}^{nK} \\
&= \sum_{s=t}^{\infty} \beta^{s-t} \ln \left(\frac{1-\beta}{\beta} \right) + \sum_{s=t}^{\infty} \beta^{s-t} \ln K_{s+1}^n.
\end{aligned}$$

Let Δ^{nK} be the compensation variation in 1995, defined by:

$$V_t'^{nK} = V_t^{nK} + \sum_{s=t}^{\infty} \beta^{s-t} \ln \Delta^{nK}.$$

It follows that:

$$\begin{aligned}
\ln \Delta^{nK} &= (1-\beta)\left(V_t'^{nK} - V_t^{nK}\right) \\
&= (1-\beta) \sum_{s=t}^{\infty} \beta^{s-t} \ln \frac{K_{s+1}'^{n}}{K_{s+1}^{n}} \\
&= \sum_{s=t}^{\infty} \beta^{s-t} \ln \frac{K_{s+1}'^{n}}{K_{s+1}^{n}} - \sum_{s=t}^{\infty} \beta^{s-t+1} \ln \frac{K_{s+1}'^{n}}{K_{s+1}^{n}} \\
&= \ln \frac{K_{t+1}'^{n}}{K_{t+1}^{n}} + \beta \ln \frac{K_{t+2}'^{n}}{K_{t+2}^{n}} + \beta^2 \ln \frac{K_{t+3}'^{n}}{K_{t+3}^{n}} + \beta^3 \ln \frac{K_{t+4}'^{n}}{K_{t+4}^{n}} + ... \\
&\qquad - \beta \ln \frac{K_{t+1}'^{n}}{K_{t+1}^{n}} - \beta^2 \ln \frac{K_{t+2}'^{n}}{K_{t+2}^{n}} - \beta^3 \ln \frac{K_{t+3}'^{n}}{K_{t+3}^{n}} + ... \\
&= \ln \frac{K_{t+1}'^{n}}{K_{t+1}^{n}} + \beta \ln \widehat{K}_{t+2}^{n} + \beta^2 \ln \widehat{K}_{t+3}^{n} + ... \\
&= \ln \frac{K_{t+1}'^{n}}{K_{t+1}^{n}} + \sum_{s=t+1}^{\infty} \beta^{s-t} \ln \widehat{K}_{s+1}^{n}.
\end{aligned}$$

Because the shock occurs unexpectedly in the year ($t = 1996$) following the initial year, the start-of-year capital stock K_{1996} remains the same, given K_{1995} and φ_{1995}. It follows that:

$$\begin{aligned}
\widehat{W}^{nK} \equiv \ln \Delta^{nK} &= \sum_{s=1996}^{\infty} \beta^{s-1995} \ln \widehat{K}_{s+1}^{n} \\
&= \sum_{s=1996}^{2049} \beta^{s-1995} \ln \widehat{K}_{s+1}^{n} + \beta^{2050-1995} \widehat{K}_{2051}^{n} + \sum_{s=2051}^{\infty} \beta^{s-1995} \widehat{K}_{2051}^{n} \\
&= \sum_{s=1996}^{2049} \beta^{s-1995} \ln \widehat{K}_{s+1}^{n} + \frac{\beta^{55}}{1-\beta} \widehat{K}_{2051}^{n}.
\end{aligned}$$

C Model Extension: Time-varying Sector-Skill Transition Costs

We have assumed the sector-skill transition costs to be time-invariant in the benchmark. This appendix shows that the dynamic hat algebra can be generalized to allow for time-varying sector-skill transition costs. This alternative framework can be used to accommodate changes to the sector-skill transition costs in a counterfactual such as that analyzed in Section 6.2 of the paper. In general, it can also be used to study the effects of supply-side shocks such as education reforms that change the costs of skill upgrading.

Let an individual's objective function be given by:

$$v_t^{njs} = \ln C_t^{njs} + \max_{\{k,i\}_{k=0,i=1}^{J,3}} \left\{ \beta\delta V_{t+1}^{nki} - \rho_t^{njs,nki} + \nu\epsilon_t^{ki} \right\}.$$

The objective function is similar to (1) in the benchmark except that the transition cost $\rho_t^{njs,nki}$ is now time-varying. The assumption that ϵ is drawn *i.i.d.* from the Type-I extreme value distribution implies that the value function and transition probability are respectively given by:

$$\begin{aligned} V_t^{njs} &= \ln C_t^{njs} + \nu \ln \sum_{\mathrm{K}=0}^{J} \sum_{\mathrm{I}\geq s}^{3} e^{\frac{\beta\delta V_{t+1}^{n\mathrm{KI}} - \rho_t^{njs,n\mathrm{KI}}}{\nu}}, \\ \mu_t^{njs,nki} &= \frac{e^{\frac{\beta\delta V_{t+1}^{nki} - \rho_t^{njs,nki}}{\nu}}}{\sum_{\mathrm{K}=0}^{J} \sum_{\mathrm{I}\geq s}^{3} e^{\frac{\beta\delta V_{t+1}^{n\mathrm{KI}} - \rho_t^{njs,n\mathrm{KI}}}{\nu}}}. \end{aligned}$$

The laws of motion for the labor pool in each sector-skill combination are unaffected. As the conditions on the production side remain the same, Proposition 1 is also unaffected.

Both $\dot{\mu}_{t+1}^{njs,nki}$ and $\dot{u}_{t+1}^{njs}$ are derived using exactly the same technique as for (29) and (30) in the benchmark. Let $\varrho_t^{njs,nki} \equiv e^{\rho_t^{njs,nki}}$. It can be readily checked that both the changes across time periods in utility and in transition probability are identical to the benchmark model up to the inclusion of the change in $\varrho_t^{njs,nki}$:

$$\begin{aligned} \dot{u}_{t+1}^{njs} &= \dot{\omega}_{t+1}^{njs} \left[\sum_{\mathrm{K}=0}^{J} \sum_{\mathrm{I}\geq s}^{3} \mu_t^{njs,n\mathrm{KI}} \left(\dot{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}} \left(\dot{\varrho}_{t+1}^{njs,n\mathrm{KI}}\right)^{-\frac{1}{\nu}} \right]^{\nu}, \\ \dot{\mu}_{t+1}^{njs,nki} &= \frac{\left(\dot{u}_{t+2}^{nki}\right)^{\frac{\beta\delta}{\nu}} \left(\dot{\varrho}_{t+1}^{njs,nki}\right)^{-\frac{1}{\nu}}}{\sum_{\mathrm{K}=0}^{J} \sum_{\mathrm{I}\geq s}^{3} \mu_t^{njs,n\mathrm{KI}} \left(\dot{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}} \left(\dot{\varrho}_{t+1}^{njs,n\mathrm{KI}}\right)^{-\frac{1}{\nu}}}. \end{aligned}$$

The dynamic hat algebra can also be derived with the same technique as for (35) and (36). For $t > 1$, we have:

$$\widehat{\mu}_{t+1}^{njs,nki} \equiv \frac{\dot{\mu}_{t+1}^{\prime njs,nki}}{\dot{\mu}_{t+1}^{njs,nki}}$$

$$=\frac{\left(\widehat{u}_{t+2}^{nki}\right)^{\frac{\beta\delta}{\nu}}\left(\widehat{\varrho}_{t+1}^{njs,nki}\right)^{-\frac{1}{\nu}}}{\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\mu_t^{\prime njs,n\mathrm{KI}}\dot{\mu}_{t+1}^{njs,n\mathrm{KI}}\left(\widehat{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}\left(\widehat{\varrho}_{t+1}^{njs,n\mathrm{KI}}\right)^{-\frac{1}{\nu}}},$$

and

$$\begin{aligned}\widehat{u}_{t+1}^{njs}\equiv&\frac{\dot{u}_{t+1}^{\prime njs}}{\dot{u}_{t+1}^{njs}},\\=&\widehat{\omega}_{t+1}^{njs}\left[\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\mu_t^{\prime njs,n\mathrm{KI}}\dot{\mu}_{t+1}^{njs,n\mathrm{KI}}\left(\widehat{u}_{t+2}^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}\left(\widehat{\varrho}_{t+1}^{njs,n\mathrm{KI}}\right)^{-\frac{1}{\nu}}\right]^{\nu}.\end{aligned}$$

Since the path of counterfactual fundamentals is observed only at $t=1$ while the decisions are made in $t=0$, we need to derive $\mu_1^{\prime njs,nki}$ and $\widehat{u}_1^{njs}$ differently from the general case of $t>1$. Recall that the allocations at $t=0$ are such that $\widehat{u}_0^{njs}=1$, $\mu_0^{\prime njs,nki}=\mu_0^{njs,nki}$ and $L_1^{\prime njs}=L_1^{njs}$. Also note that $\varrho_0^{\prime njs,nki}=\varrho_0^{njs,nki}$ since we assume that the shocks to the fundamentals occur unexpectedly at $t=1$. Following the approach for Proposition 3 as detailed in Appendix A.3 yields:

$$\begin{aligned}\widehat{u}_1^{njs}=&\widehat{\omega}_1^{njs}\left[\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\upsilon_0^{njs,n\mathrm{KI}}\left(\widehat{u}_2^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}\left(\widehat{\varrho}_1^{njs,n\mathrm{KI}}\right)^{-\frac{1}{\nu}}\right]^{\nu},\\ \mu_1^{\prime njs,nki}=&\frac{\upsilon_0^{njs,nki}\left(\widehat{u}_2^{nki}\right)^{\frac{\beta\delta}{\nu}}\left(\widehat{\varrho}_1^{njs,nki}\right)^{-\frac{1}{\nu}}}{\sum_{\mathrm{K}=0}^{J}\sum_{\mathrm{I}\geq s}^{3}\upsilon_0^{njs,n\mathrm{KI}}\left(\widehat{u}_2^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}\left(\widehat{\varrho}_1^{njs,n\mathrm{KI}}\right)^{-\frac{1}{\nu}}},\end{aligned}$$

where $\upsilon_0^{njs,n\mathrm{KI}}\equiv\mu_1^{njs,n\mathrm{KI}}\left(\widehat{u}_1^{n\mathrm{KI}}\right)^{\frac{\beta\delta}{\nu}}$.

D Model Extension: Skill-Job Disparity

In the paper, we have assumed that jobs and skills are paired perfectly, such that a worker with a given skill level always does a job that requires exactly the skill level. Therefore, a high-skilled worker is always assigned to an occupation that requires a high skill level. In this appendix, we consider setups where workers might not end up with jobs that match their skill levels. For example, a middle-skilled worker may choose to upgrade his skill level but end up doing a job that requires a low skill level in the next period. The following analysis aims to generalize the model by allowing workers to undertake jobs with lower skill requirement than the worker's current skill level.

When a worker of a given skill level can choose a job of lower skill requirement, the number of jobs that requires the given skill level to perform does not necessarily equals the number of workers of that skill level. We therefore need to distinguish between skill requirement of jobs and skill attainment of workers. Let a_t denotes the skill attainment of an individual at time t, and s_t denotes the skill requirement of the individual's job at time t. We assume that $a_t \geq s_t$ holds for each individual so that workers can "match downwards" to jobs in terms of skill level. The Bellman equation becomes:

$$v_t^{njs;a_t \geq s} = \ln C_t^{njs;a_t \geq s} + \max_{\{k,i\}_{k=0,i=1}^{J,3}} \left\{ \beta\delta V_{t+1}^{nki;a_{t+1}=\max\{a_t,i\}} - \rho^{njs,nki;a_t \geq s} + \nu\epsilon_t^{ki} \right\},$$

such that:

$$\begin{aligned} \rho^{njs,nki;a_t \geq s} &= \rho_{job}^{njs,nki;a_t \geq s} + \rho^{a_t,i} \\ \rho^{a_t,i} &= 0 \text{ if } a_t \geq i. \end{aligned}$$

The parameter $\rho_{job}^{njs,nki;a_t \geq s}$ governs the cost to choose a job in sector k that requires skill level i given the worker's skill attainment as a_t. The parameter $\rho^{a_t,i}$ is the cost of *skill cultivation*, which occurs when the worker chooses a job that requires a *higher* skill level than his current skill attainment. In other words, if a worker with a skill attainment a_t chooses a job with a skill requirement $i > a_t$, then he bears an additional cost to upgrade his skill attainment to i in the next period. Note that the setup reduces to the benchmark case when $\rho_{job}^{njs,nki;a_t \geq s} = \infty$ for $s \neq a_t$ and $i < s$.

Following similar analysis as in the main text, the probability of switching between sector-job combinations and the lifetime utility are, respectively:

$$\mu_t^{njs,nki;a_t \geq s} = \frac{e^{\frac{\beta\delta V_{t+1}^{nki;a_{t+1}=\max\{a_t,i\}} - \rho^{njs,nki;a_t \geq s}}{\nu}}}{\sum_{\text{K}=0}^{J} \sum_{\text{I}=1}^{3} e^{\frac{\beta\delta V_{t+1}^{n\text{KI};a_{t+1}=\max\{a_t,\text{I}\}} - \rho^{njs,n\text{KI};a_t \geq s}}{\nu}}}, \tag{D.3}$$

$$V_t^{njs;a_t \geq s} = \ln C_t^{njs;a_t \geq s} + \nu \ln \sum_{\text{K}=0}^{J} \sum_{\text{I}=1}^{3} e^{\frac{\beta\delta V_{t+1}^{n\text{KI};a_{t+1}=\max\{a_t,\text{I}\}} - \rho^{njs,n\text{KI};a_t \geq s}}{\nu}}. \tag{D.4}$$

Equations (D.3) and (D.4) are pretty much the same as those in the benchmark model, except that workers can do jobs with lower skill requirements than their skill attainment.

Because skill attainment is weakly greater than the skill requirement of jobs, the number of workers choosing a *sector-job combination* js is no longer identical to the workers with skill attainment a in sector j. The laws of motion (4) and (5) need to be replaced by equations (D.5)–(D.8) below. In particular, the laws of motion for the numbers of workers choosing a sector-job combination js, with skill attainment a_{t+1} in the next period are given by:

$$L_{t+1}^{njs;a_{t+1}>s} = \delta \sum_{k=0}^{J} \sum_{i=1}^{i<a_t} \mu_t^{nki,njs;a_t>i}|_{s<a_t} L_t^{nki;a_t>i} + \delta \sum_{k=0}^{J} \mu_t^{nki,njs;a_t=i}|_{s<a_t} L_t^{nki;a_t=i} \tag{D.5}$$

$$L_{t+1}^{njs;a_{t+1}=s} = \delta \sum_{k=0}^{J} \sum_{i=1}^{i<a_t} \mu_t^{nki,njs;a_t>i}|_{s\geq a_t} L_t^{nki;a_t>i} + \delta \sum_{k=0}^{J} \mu_t^{nki,njs;a_t=i}|_{s\geq a_t} L_t^{nki;a_t=i} \tag{D.6}$$

$$L_{t+1}^{n01;a_{t+1}=1} = \delta \sum_{k=0}^{J} \mu_t^{nk1,n01;a_t=1} L_t^{nk1;a_t=1} + (1-\delta) L^n \tag{D.7}$$

$$L_{t+1}^{njs} = \sum_{a_{t+1}>s}^{3} L_{t+1}^{njs;a_{t+1}>s} + L_{t+1}^{njs;a_{t+1}=s}. \tag{D.8}$$

Equation (D.5) governs the motion of workers with skill attainment a_t that eventually choose a sector-job combination js with a lower skill requirement than his skill attainment. There are two possibilities: the worker with a skill attainment a_t currently taking a job with a skill requirement $i < a_t$, and the worker with a skill attainment a_t currently taking a job with exactly the same skill requirement $i = a_t$. These workers choose the sector-job combination js where s is still lower than their own skill attainment a_t with a probability $\mu_t^{nki,njs;a_t>i}|_{s<a_t}$, and $\mu_t^{nki,njs;a_t=i}|_{s<a_t}$, respectively. There are $L_t^{nki;a_t>i}$ and $L_t^{nki;a_t=i}$ of these types of workers, respectively.

Equation (D.6) refers to the case where the worker chooses to take a sector-job combination js in the next period, with the skill requirement (weakly) greater than his current skill attainment. In this case, this worker experiences a skill upgrading and his skill attainment improves from a_t to s in the next period. The probability for this worker to do so is $\mu_t^{nki,njs;a_t>i}|_{s\geq a_t}$ when the worker's skill attainment is strictly higher than the skill requirement of his current job, and $\mu_t^{nki,njs;a_t=i}|_{s\geq a_t}$ when his skill attainment is exactly the same as the skill requirement of his current job.

Equations (D.5) and (D.6) characterize the labor supply to sector-job combination js in the *next* period conditional on the worker's *current* skill attainment. For labor market clearing, we require equation (D.8), where the LHS denotes the total labor demand of sector-job combination js in the *next* period, and the RHS corresponds to the total labor supply to sector-job combination js in the *next* period with agents of skill attainment a_{t+1} in the *next* period.

Equation (D.7) plays the same role as (5) in the benchmark model. The second term follows from our setting that the population growth rate is zero and all newborns begin with skill attainment 1 and start with sector-job combination 01. The first term represents the mass of agents with skill attainment 1 in the current period who decide to take sector-job combination 01 for the next period. Since skill attainment is nondecreasing, we do not need to worry about other possibilities of a_t in this case. Therefore, (D.7) and (5) are equivalent.

We can also characterise the laws of motion in terms of worker skill attainment a_t for each sector. Let $M_t^{nj;a_t=s}$ be the number of workers currently working in sector j with a skill attainment $a_t = s$. From (D.5)–(D.8), it follows that:

$$\begin{aligned} M_{t+1}^{nj;a_{t+1}=s} &= \delta \left[\sum_{k=0}^{J} \mu_t^{nki,njs;a_t=i}|_{s>a_t} L_t^{nki;a_t=i} + \sum_{k=0}^{J} \sum_{i=1}^{i<a_t} \mu_t^{nki,njs;a_t>i}|_{s>a_t} L_t^{nki;a_t>i} \right] \\ &+ \delta \left[\sum_{k=0}^{J} \sum_{i=1}^{i<a_t} \mu_t^{nki,njs;a_t>i}|_{s=a_t} L_t^{nki;a_t>i} + \sum_{k=0}^{J} \sum_{i=1}^{i<a_t} \mu_t^{nki,njh;a_t>i}|_{h<s=a_t} L_t^{nki;a_t>i} \right] \\ &+ \delta \left[\sum_{k=0}^{J} \mu_t^{nks,njs;a_t=s}|_{s=a_t} L_t^{nks;a_t=s} + \sum_{k=0}^{J} \mu_t^{nks,njh;a_t=s}|_{h<s=a_t} L_t^{nks;a_t=s} \right]. \end{aligned} \quad \text{(D.9)}$$

The first line represents the agents that experience skill upgrading from $a_t < s$ to $a_{t+1} = s$ by taking a job of skill requirement s in sector j, who currently work in sector k with a skill requirement $i \leq a_t$. The second line and third lines denote agents who already have the skill attainment $a_t = s$ in the current period and do not experience skill upgrading. In particular, the second line consists of agents who take jobs with skill requirements strictly less than s currently, but choose to take jobs with skill requirements either equal or less than s for the next period. The third line sums up agents who currently have jobs with exactly skill requirement s, but choose jobs that require either equal or less than s in the next period.

Equations (10)–(17) in the paper continue to hold in this alternative setup. Therefore, the same temporary equilibrium conditions (22)–(28) continue to hold, except that L_t^{njs} is now governed by (D.8) and that the wage now depends on skill requirement instead of skill attainment. For the sequential equilibrium, equations (29) and (30) remain to hold but with an additional superscript $a_t \geq s$. The skill attainment a_t does not affect the derivation, as can be verified following the proof in Appendix A.2. The laws of motion are replaced by (D.5)–(D.7). Because the techniques of derivation are unaffected by the superscript a_t, it is obvious that the dynamic hat algebra (35)–(47) continues to hold here except for the laws of motion (38) and (39). The laws of motion are replaced by the counterfactual versions of (D.5)–(D.7).

The above discussion shows that we can solve for the entire path of equilibrium once initial points are given. This gives us the demand for jobs of different skill requirement in different sectors and economies. These information allows us to obtain the skill attainment in each economy-sector-job combination by plugging the equilibrium outcome into (D.9). The probability for an agent with a given skill attainment to "under match" with a job that requires a lower skill level can also be obtained in the same manner as for (D.9).