# General Purpose Technology, Growth, and Dynamic Gains from Trade

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August 2, 2024

**Abstract**

This paper develops a dynamic general-equilibrium model of trade, innovation, and growth rooted in the evolution of general purpose technology that is applied to all of the differentiated products. While innovation in general purpose technology can occur only in the North, differentiated products are produced both in the North and the South. We find that trade liberalization induces more R&D on general purpose technology and hence faster long-run growth, where the North-South trade and innovation structure plays a key role in this process. The model is generalized and calibrated for multi-country quantitative analyses, in which we find large total gains from trade with the dynamic component far dominating the conventional static and an income-gains components. The share of dynamic gains in total gains is higher when the initial trade costs are higher. This is because trade liberalization lowers the North-South wage gap, which induces more innovation, and the drop in the wage gap is larger when trade costs are higher.

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[†]We are grateful for the valuable comments and suggestions from George Alessandria, Costas Arkolakis, Eric Bond, Been-Lon Chen, Jonathan Eaton, Chang-Tai Hsieh, Samuel Kortum, Erzo Luttmer, Paul Segerstrom, Michael Sposi, and Michael Waugh, as well as conference participants at the Midwest Trade Meetings, the Society for Advanced Economic Theory Conference, SMU Trade Workshop, and the Taiwan Economic Research Conference. Hsu would like to acknowledge the financial support provided by the Sing Lun Fellowship of Singapore Management University. We also acknowledge grant support from Taiwan's Ministry of Science and Technology (MOST 104-2811-H-001-007) to facilitate the coauthor work at Academia Sinica.

# 1 Introduction

The past decade has seen substantial progress in evaluating dynamic gains from trade.[1] This is likely due to two reasons. First, the tools are ready, as there has been much progress in both firm dynamics and the trade literature on heterogeneous firms since the early 2000s. Second, it is motivated by the findings in Atkeson and Burstein (2010) and Arkolakis, Costinot and Rodríguez-Clare (2012) (henceforth, ACR) that welfare gains from trade are typically small in static frameworks. In other words, it is a response to the natural and important questions of how large the welfare gains from trade are and whether dynamics matter. While several aspects of dynamics and growth have been studied in this regard, one important piece is missing. That is, as much as science is hierarchically structured – applied sciences build on basic sciences, technology is also hierarchically structured – production technology of various products build on general purpose technology (henceforth GPT).

A GPT is defined by the following three features (Bresnahan and Trajtenberg, 1995; Jovanovic and Rousseau, 2005): (1) **pervasiveness** – the GPT is applied to most sectors and products; (2) **improvement** – the GPT should get better over time and hence keep lowering the costs of its users; (3) **innovation spawning** – the GPT should make it easier to invent and produce new products or processes. Historical examples of GPT include internal combustion engines, electricity, and, most recently, information technology (semiconductors in particular). Jovanovic and Rousseau (2005) provides empirical evidence for these features in the cases of electrification and information technology.[2] In addition, they show that both cases were accompanied by a rise in creative destruction and turbulence as measured by the entry and exit of firms, as well as by mergers and takeovers. Taken together, these GPT features and the empirical evidence suggest that the GPT evolution is essential for long-run economic growth. But would trade play a role in the economic growth consequences of GPT evolution? This paper develops a dynamic trade model in which the GPT evolution is the driver for growth. We ask how trade affects growth, how large are the welfare gains from trade, and how much do dynamics matter.

We model the evolution of the GPT technology using the endogenous growth framework *a là* Aghion and Howitt (1992) (henceforth, AH) in which the sustained growth of the economy is driven by R&D with creative destruction. In particular, the advancement of the GPT is a "productivity ladder", which captures the **improvement** feature of the

[1] See, for example,Sampson (2015), Buera and Oberfield (2020), Perla, Tonetti and Waugh (2021), Somale (2021), Cai, Li and Santacreu (2022), among many others cited in our literature review.

[2] For other earlier theoretical and empirical work on GPT, see Helpman and Trajtenberg (1994), Aghion, Howitt and Violante (2002), and Crafts (2004).

GPT. We extend this productivity-ladder model to a general-equilibrium trade setting of Bernard et al. (2003) (henceforth, BEJK). Each entrepreneur in our economy enters a market for a specific product and develops an idea (a blueprint) for how this product may be produced. The productivities of their blueprints are ex post heterogeneous, and entrepreneurs compete globally in a Bertrand fashion as in BEJK. All differentiated products require the use of the GPT to produce (the **pervasiveness** feature), and hence whenever the GPT advances, the ideas (or blueprints) for producing differentiated products need to be redrawn. Moreover, we assume that better GPT allows more ideas to be drawn (the **innovation spawning** feature). As there is only one winner in the Bertrand competition among entrepreneurs for the same product, the idea redraws when the GPT advances imply that the incumbents may be replaced by other entrepreneurs, causing creative destruction for the differentiated products as documented by Jovanovic and Rousseau (2005). We also assume that GPT innovation is conducted only in a subset of countries (North countries), but the GPT is used by all differentiated-product firms in all countries.[3]

Our analysis proceeds in three steps. First, we lay out the model with only two countries (the North and the South). This simple structure emphasizes the essential forces at work in the model. Second, we extend the model to allow for an arbitrary number of countries and a flexible structure of bilateral trade costs to prepare for the quantitative analysis. We study the properties related to welfare gains from trade in the quantitative model. Finally, we calibrate the model and conduct a set of counter-factual analyses to gauge the size of the total gains from trade and how much the dynamics matter.

In the two-country model, we show that the link between GPT innovation and trade costs is captured by a multiplier that is the ratio of aggregate world revenue of differentiated products to the North's wages. Trade liberalization induces this multiplier to increase, hence incentivizing GPT innovation and increasing the economic growth rate. The intuition is that aggregate world revenue reflects the marginal benefit of GPT innovation, whereas the labor costs in the North reflect the marginal cost. A more integrated economy reduces the wage gap between the North and the South and increases the global market size of differentiated products in terms of the North's labor. Thus, R&D effort increases, as does the growth rate of the economy. This mechanism of trade and growth, with globalization characterized by falling tariffs and improving transport technology contributing to the narrowing of income and wage gaps between developed and developing countries, is consistent with recent research (see, e.g., Caliendo et al. 2015).

[3] Innovation on the GPT is generally more difficult than other innovations and requires an environment with high institutional quality. Empirical studies have shown that the spatial distribution of innovation (let alone the GPT innovation) is highly uneven, with the North countries having a dominant share (e.g., see Egger and Loumeau 2018).

Next, we extend the model to allow for any number of countries and a flexible structure of bilateral trade costs for quantitative purposes. We first show that the long-run welfare change in response to a change in trade cost can be decomposed into a growth-rate effect (GR), an income-gains effect (IG), and the ACR statistic. The GR effect is a direct dynamic effect of a trade shock, as it reflects how growth amplifies the gains from trade. The IG comes from payments the North receives from the differentiated-product firms worldwide for using the GPT. ACR gains come from the usual static sources as in the ACR setting. However, as the general equilibrium effects (on wages and goods prices) are intertwined among the three channels, the GR effect alone does not capture dynamic gains from trade. To isolate the dynamic gains, our quantitative analysis will also compare the benchmark model with a purely static model by removing the endogenous growth mechanism in a counterfactual economy.

The generalized model is calibrated to the world economy during 2000-2014. We estimate bilateral trade costs using a standard gravity-equation estimation approach. We back out the country-specific technology stock for differentiated products using the exporter fixed effects and the model structure, following Fieler (2011) and Ravikumar, Santacreu and Sposi (2019). The parameters for the GPT's arrival rate function (which we will call innovation parameters throughout the paper) are chosen such that the annual real income growth rate and the employment share of R&D researchers generated by the model match the data counterparts.

We quantitatively analyze two scenarios of trade liberalization: (i) the first studies trade liberalization from autarky to the benchmark trade costs, and (ii) the second examines the effects of a moderate 25% reduction in trade costs from observed ones. For the first scenario, the population-weighted average of total gains is $30.6\%$, with the North gaining slightly more than the South. The gain from the growth-rate component is $27.4\%$, and for most countries this component makes the bulk of the total gains. Thus, the weighted average of the share of the growth-rate component is $89.5\%$ with this share being slightly lower for northern countries because they have higher ACR gains and income gains on average.

For the *true dynamic gains*, we shut down the growth driver from GPT innovation and compute the static gains. The dynamic gains are, therefore, the difference between the total gains and these static gains. It turns out that the size of dynamic gains and their share in total gains are rather similar to the growth-rate component in the simple decomposition, indicating that the general equilibrium effects via GPT innovation are small. This is intuitive because even though GPT innovation is critical to the overall welfare gains from trade, it accounts for only a small share of the labor force and, therefore, does not induce large effects via wages and trade.

In the second scenario, a $25\%$ reduction in estimated trade costs, the weighted average of the total gains and growth-rate gains are $3.7\%$ and $0.11\%$, respectively, much smaller than in the first scenario. The total gains are smaller because the change in trade costs are much less than the first scenario. However, the decomposition exhibits an interesting pattern in that the share of growth-rate component (and the dynamic gains) is drastically reduced, on weighted-average, from $89.5\%$ to $3.0\%$. The intuition is that a more integrated economy reduces the wage gap between the North countries and South ones, and this tends to increase the global market size of differentiated products in terms of the North's labor, hence increasing the R&D multiplier. Because the wage gap are at their largest levels under autarky and because the first scenario features a large drop in trade costs, the reduction in wage gap in the first scenario is much larger than that in the second scenario. This leads to a novel virtuous cycle between trade and innovation that amplifies dynamic gains from trade, thereby explaining why the growth-rate effect is much larger in the first scenario. In addition, by Shepherd's Lemma, the ACR statistic reflects essentially a direct effect of trade costs on the prices of imported goods. Under lower trade costs, consumed quantities of imported goods tend to be larger, and this amplifies the direct price effect and leads to a much larger ACR effect in the second scenario.

The dynamic and total gains from trade in our calibrated model seem large compared with the literature. To see the source of such large gains, we conduct a series of sensitivity analyses. We examine the roles of intermediate inputs, trade elasticity, the size of the productivity ladder, and the GPT's bargaining power. In each exercise, we change one parameter at a time, recalibrate the model, and compute the dynamic and total gains. Although the dynamic and total gains are sensitive to each of these factors except for the size of the productivity ladder, whenever there are large changes, they are mainly due to the changes in the innovation parameters. In other words, the large dynamic gains are the reflection of the importance of the growth channel as the arrival rate of the GPT is high and sensitive to R&D effort. Even with large-scale sensitivity exercises, total gains remain sizable throughout (all above 10%), whereas the shares of dynamic gains continue to be most dominant (all above two-thirds).

Related Literature

This paper is closely related to the strand the literature that studies trade, innovation, and growth by building on the BEJK trade structure. Somale (2021) builds a model with directed research effort and shows that trade shocks may change the allocation of R&D resources across sectors and hence comparative advantages. Buera and Oberfield (2020) develop a model of international technology diffusion, which drives endogenous growth and is affected by international trade. Cai, Li and Santacreu (2022) encompass both directed research effort as in Somale (2021) and technology diffusion as in Eaton and Kortum (1999) in a model with input-output linkages. They emphasize the role of cross-country-sector linkages in both trade and technology diffusion in terms of shaping comparative advantages and gains from trade. Hsieh, Klenow and Nath (2023) embed Klette and Kortum (2004) creative destruction mechanism into the BEJK trade structure to study the global competition in innovation and growth and examine the implications on gains from trade. Our work differs from all of these studies in its focus on how trade affects the GPT innovation that drives long-run growth in a North-South structure.

Our study is also related to a series of recent studies on dynamic gains from trade.[4] Sampson (2015) develops an endogenous growth model in which the productivity distribution of firms constantly moves to the right over time as entrants learn from the incumbent firms that survived selection previously. Perla, Tonetti and Waugh (2021) quantify a model of trade and growth in which all firms learn from each other (not just the entrants learning from the incumbents). Impullitti and Licandro (2017) construct a two-country model with process innovation. Alessandria, Choi and Ruhl (2021) evaluate welfare gains from tariff reductions in a model of exporting firms' life cycles that features a tradeoff between new firm creation and export capacity expansion. Bloom et al. (2020) examine gains from trade in the presence of frictions that impede factor mobility, which spurs innovation, increases economic growth rate in the medium run, and results in large gains from trade. Ravikumar, Santacreu and Sposi (2019) investigate a trade model with capital accumulation and fit the model to a sample of 44 countries. Our focus and model mechanism differ from all of these studies.

The rest of the paper is organized as follows. Section 2 lays out a two-country North-South model of trade, innovation, and growth; Section 3 analyzes how trade affects growth. Section 4 extends the model to a quantitative one and studies the welfare properties. Section 5 quantifies the model, Section 6 conducts the welfare analysis of gains from trade, and Section 7 concludes.

[4]For earlier studies on dynamic gains from trade, see Baldwin (1992), who shows that the gains in output in the steady state from trade liberalization in a model with capital accumulation can be several times larger than the static gains. The earlier work byGrossman and Helpman (1991) and Rivera-Batiz and Romer (1991) are also related because they are concerned with the implications of trade on the allocation of labor resources as in our paper, even though they are not concerned with gains from trade per se.

# 2 Model

We first present a North-South two-country model to clarify the mechanism and intuitions of the model, and will extend the model to allow an arbitrary number of countries and flexible trade-cost structure in Section 4 for quantitative analysis.

## 2.1 Environment and Preference

There are two countries, $i = 1, 2$, each with population $N_i$, which is constant over time. Without loss of generality, we shall label country $1$ and $2$ as the North and the South, respectively. Time is continuous, with calendar time indexed by $t$. At time $0$, there is a given version of the General Purpose Technology (GPT) indexed by $\nu = 0$; subsequent $\nu$-th innovations lead to the $\nu$-th generations of the GPT. It will be clear that between any two consecutive GPT innovations, the economy stays static.

The life time utility of each individual in country $i$ is:[5]

$$U_i = \int_0^\infty Q_{it} e^{-\rho t} dt,$$

where $\rho$ is the time preference rate, and instantaneous utility $Q_{it}$ is given by the standard CES aggregator:

$$Q_{it} = \left( \int_0^1 (q_{it}(\omega))^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma$ is the elasticity of substitution among differentiated goods which are indexed by $\omega$.

## 2.2 Innovation

GPT R&D and the subsequent innovation only occur in the North. Each good $\omega$ requires a blueprint of the production process and production workers to implement such a process to produce. Each production process requires the use of GPT, which is embodied in a "chip" (or an operation system, or engine, etc.). Here, a new generation of GPT has two effects. First, the production process of every good has to be revamped to accommodate this new chip, and hence new ideas about how to produce have to be drawn again. Second, because chips become better over time and allow more flexibility in how to produce the final good $\omega$, the number of ideas that entrepreneurs can draw grows by a constant

[5] Note that linear flow utility is assumed here. As will be explained in Section 4.2, adding curvature to the flow utility function will not affect the dynamic component of the welfare.

factor $\gamma > 1$, which resembles the size of the quality ladder as in AH and Segerstrom, Anant and Dinopoulos (1990). The GPT monopoly can make each chip good-specific at zero cost.

In the North, there is a unit continuum of GPT innovating firms, each of which engages in hiring R&D labor (researchers), and innovation occurs in a Poisson arrival fashion. So, this market is perfectly competitive, and innovating firms take the wages for R&D labor as given. As these firms are ex-ante identical, the GPT arrival rate for an individual firm is the GPT arrival for the whole continuum. Once the GPT innovation occurs, without loss of generality, one firm is randomly selected as the new GPT monopoly.[6] We will formally describe the GPT innovation process in Section 2.4.

Let $\tilde{t}_{i,0}$ denote the number of ideas per unit of the $M_{i,0}$ entrepreneurial labor at time zero. When $\nu$-th generation of GPT is invented, at time $\nu_+$ each unit of the $M_{i,\nu}$ entrepreneurial labor for each good $\omega$ works in that instant to come up $\gamma^\nu \tilde{t}_{i,0}$ ideas, and the productivity of each idea is drawn from a Fréchet distribution, $F_i^{\text{draw}}(z) = e^{-z^{-\theta}}$. Firms engage in Bertrand competition as in BEJK, and the best idea prevails. Then, at $\nu_{++}$ the GPT monopoly makes the chip $\omega$-specific to each $\omega$ and engages in a Nash bargaining with each entrepreneur who owns the best production process to produce $\omega$. For each $\omega$, the bargaining is one-time over the expected profits earned by the $\omega$-firm for the entire duration of the GPT (until it is replaced by the next GPT innovation), and the bargaining power of the GPT firm is $\beta \in (0,1)$. For simplicity, the marginal cost of producing each additional chip is assumed to be zero. In expected terms, this bargaining outcome can also be interpreted as the GPT monopoly selling the right at every instant to use $\omega$-chip for each $\omega$ at the price of $\beta$ times the profit of the $\omega$-firm, and this is indeed a royalty payment.

Entrepreneurs in both countries need to purchase the $\omega$-chip from GPT innovators. The evolution of the total number of idea draws in each country $i$ is $\widetilde{T}_{i,\nu} = M_{i,\nu}\gamma^\nu \tilde{t}_{i,0}$. Let $Z_1$ and $Z_2$ denote the top two productivities (as random variables). To utilize the limiting joint distribution of the top two productivities in BEJK, assume that $\tilde{t}_{i,0} = t_{i,0}K$, where $K$ is an arbitrarily large number. Defining $T_{i,\nu} \equiv \widetilde{T}_{i,\nu}/K$, we have

$$T_{i,\nu} = M_{i,\nu}\gamma^\nu t_{i,0}. \tag{1}$$

While details are relegated to Appendix A, the limiting joint distribution of the scaled

[6] The model can be straightforwardly modified to the AH setting with $R$ fixed and $\varphi$ function being constant returns to scale so that there are numerous R&D firms to compete for the lone monopolist in the future.

order statistics $K^{-1/\theta}Z_1$ and $K^{-1/\theta}Z_2$ is given by[7]

$$\lim_{K\to\infty} \Pr\left[K^{-1/\theta}Z_1 \le z_1, K^{-1/\theta}Z_2 \le z_2\right] = \left[1 + T_{i,\nu}\left(z_2^{-\theta} - z_1^{-\theta}\right)\right] e^{-T_{i,\nu}z_2^{-\theta}}.$$

Thus, with proper rescaling, the c.d.f. of the maximum productivity of these draws and that of the top two productivities are given by,

$$F_{i,\nu}(z) = e^{-T_{i,\nu}z^{-\theta}}, \quad z \ge 0, \tag{2}$$

$$F_{i,\nu}(z_1, z_2) = [1 + T_{i,\nu}(z_2^{-\theta} - z_1^{-\theta})]e^{-T_{i,\nu}z_2^{-\theta}}. \tag{3}$$

The BEJK trade model starts with (2) and (3) with given $T_{i,\nu}$. Here, $T_{i,\nu}$ is endogenous and evolves with $M_{i,\nu}$ and GPT innovations.[8] Cross-country difference in $T_{i,\nu}$ is also affected by $t_{i,0}$. From this point onward, we refer to $\gamma^\nu t_{i,0}$ as country $i$'s *knowledge stock* and $T_{i,\nu}$ as its *technology stock*, which depends on how many entrepreneurs $M_{i,\nu}$ tap on the knowledge stock. Due to historical reasons, the North has a larger initial knowledge stock so that $t_{1,0} > t_{2,0}$. We now turn to the trade environment in BEJK.

## 2.3 Production and Trade

We briefly explain how the cost and price distributions are determined in BEJK and then collect several useful results from BEJK. For each good $\omega$, the unit cost of supplying to consumers in location $n$ by the $k$-th most efficient producers located in $i$ is given by,

$$C_{kni}(\omega) = \left(\frac{w_i}{Z_{ki}(\omega)}\right)\tau_{ni},$$

where $\tau_{ni} = 1$ if $n = i$, $\tau_{ni} = \tau$ if $n \neq i$, and the $Z_{1i}(\omega)$ and $Z_{2i}(\omega)$ are random variables whose joint distribution is given by (3). The producer serving market $n$ has unit cost $C_{1n}(\omega) = \min_i\{C_{1ni}(\omega)\}$, and the second lowest cost to supply to $n$ is $C_{2n}(\omega) = \min\{C_{2i^*n}(\omega), \min_{i\neq i^*}\{C_{1in}(\omega)\}\}$, where $i^*$ is the country in which the lowest cost supplier to $n$ is located. Bertrand competition implies that the producer with $C_{1n}(\omega)$ charges $C_{2n}(\omega)$, resulting in a markup of $C_{2n}(\omega)/C_{1n}(\omega)$. Under the CES specification of the utility function, this markup cannot exceed the monopoly markup $\sigma/(\sigma-1)$ if $\sigma > 1$. Bertrand

[7] We thank Erzo Luttmer for his comments which help clarify the setup here.

[8] Note that whereas GPT innovation enhances productivities of all goods, it is not a typical sense of process innovation, which is often considered a firm-level effort.

limit pricing yields:

$$P_n(\omega) = \begin{cases} \min\{C_{2n}(\omega), \frac{\sigma}{\sigma-1}C_{1n}(\omega)\} & \text{if } \sigma > 1 \\ C_{2n}(\omega) & \text{if } \sigma \leq 1 \end{cases}.$$

Using (2) and (3), the joint distribution function of $C_{1n}$ and $C_{2n}$ is $H_n(c_1, c_2) = 1 - e^{-\Phi_n c_1^\theta} - \Phi_n c_1^\theta e^{-\Phi_n c_2^\theta}$, where $\Phi_n = \sum_{i=1}^2 T_i(w_i \tau_{ni})^{-\theta}$, which distills the parameters of productivity distributions, wages, and the trade cost into one single term governing the cost and price distributions. The following properties either recap or extend the BEJK's analytical results that are useful for our analysis.

**BEJK Result 1** The probability that country $i$ provides a good at the lowest cost in country $n$ is:

$$\pi_{ni} = \frac{T_i(w_i\tau_{ni})^{-\theta}}{\sum_{k=1}^2 T_k(w_k\tau_{nk})^{-\theta}} = \frac{T_i(w_i\tau_{ni})^{-\theta}}{\Phi_n}.$$

Since there is a continuum of goods, $\pi_{ni}$ is also the fraction of goods that the consumers at $n$ purchases from $i$.

**BEJK Result 2** In a country $n$, the probability of buying a good with price lower than $p$ is independent from where the good is purchased from. Thus, $\pi_{ni}$ is also the share of expenditure of consumers in $n$ on the goods from $i$:

$$X_{ni} = \pi_{ni} Y_n,$$

where $X_{ni}$ is the total sales from country $i$ to $n$, and $Y_n$ denote the total (nominal) income in $n$. Let $X_n$ denote the total sales of producers in $n$, and note that the above equation differs from the formula in BEJK, $X_{ni} = \pi_{ni} X_n$, because in our model total revenue from firms producing differentiated products is not equal to total income due to the international royalty payments.

**BEJK Result 3** Under $\theta + 1 > \sigma$, the price index in country $n$ is:

$$P_n = \eta \Phi_n^{-\frac{1}{\theta}}, \tag{4}$$

where $\eta \equiv \left\{ \left[ 1 + \frac{(\sigma-1)}{1+\theta-\sigma} \left( \frac{\sigma}{\sigma-1} \right)^{-\theta} \right] \Gamma \left( \frac{1+2\theta-\sigma}{\theta} \right) \right\}^{\frac{1}{1-\sigma}}$ with $\Gamma$ denoting the gamma function.

**BEJK Result 4** A fraction $\theta/(1+\theta)$ of revenue goes to variable cost.

## 2.4 Labor Markets

There are three occupations in this economy: entrepreneurs $M_i$ and production workers $L_i$ who are present in both countries and research workers $R$ who are present only in the North.[9] Labor is homogenous; thus, the labor source constraints are

$$R + M_1 + L_1 = N_1, \qquad M_2 + L_2 = N_2.$$

The labor markets in both countries are perfectly competitive. By BEJK Result 4 and the fact that part of the revenue is paid to the GPT firm, entrepreneurs' expected payoffs $v_{i,\nu}$ and production workers' wages $w_{i,\nu}$ are given by

$$v_{i,\nu} = \frac{1-\beta}{1+\theta}\frac{X_{i,\nu}}{M_{i,\nu}}, \qquad w_{i,\nu} = \frac{\theta}{1+\theta}\frac{X_{i,\nu}}{L_{i,\nu}}.$$

In equilibrium, $w_{i,\nu} = v_{i,\nu}$, which implies that $M_{i,\nu} = \frac{1-\beta}{\theta}L_{i,\nu}$. In the South, this implies that

$$L_{2,\nu} = \frac{\theta}{1-\beta+\theta}N_2, \qquad M_{2,\nu} = \frac{1-\beta}{1-\beta+\theta}N_2. \tag{5}$$

In the North, we have $N_1 - R_\nu = M_{1,\nu} + L_{1,\nu} = \frac{1-\beta+\theta}{\theta}L_{1,\nu}$, or alternatively,

$$L_{1,\nu} = \frac{\theta}{1-\beta+\theta}(N_1 - R_\nu) \tag{6}$$

$$M_{1,\nu} = \frac{1-\beta}{1-\beta+\theta}(N_1 - R_\nu) \tag{7}$$

We now shift our attention to research workers. As the aggregate profit earned by all differentiated-product firms is $\frac{X_{1,\nu}+X_{2,\nu}}{1+\theta}$ at any point in time between $\nu$-th and $\nu+1$-th innovations, the value of an innovation is, in a similar fashion to AH, specified by

$$\begin{aligned} V_{\nu+1} &= \beta\int_0^\infty \left[\int_0^t \frac{(X_{1,\nu+1}+X_{2,\nu+1})\,e^{-r\tau}}{1+\theta}d\tau\right]\lambda(R_{\nu+1})\,e^{-\lambda(R_{\nu+1})t}dt \\ &= \frac{\beta\,(X_{1,\nu+1}+X_{2,\nu+1})}{(1+\theta)\,[r+\lambda(R_{\nu+1})]}. \end{aligned} \tag{8}$$

where $r$ is the real interest rate, and the Poisson arrival rate $\lambda$ is an increasing function of R&D labor $R_\nu$ hired for innovating a new GPT. Here, the value of innovation is the sum of the GPT profits over time properly discounted by both the real interest rate and the rate

[9] This can be justified by viewing that there is no suitable environment for GPT innovation in the South, and hence all individuals can only choose between being an entrepreneur or a production worker; natively-born research type in the South can only migrate to the North to engage in GPT innovation.

of creative destruction, $\lambda(R_{\nu+1})$.

Labor being homogeneous and the labor market being perfectly competitive imply that the wages for researchers are the same as $w_{1,\nu}$, the North's wages for production workers. Taking $V_{\nu+1}$, the wages, and the price indices as given,[10] a GPT innovator solves:

$$\max_{R_\nu} \lambda(R_\nu)\frac{V_{\nu+1}}{P_{1,\nu+1}} - \frac{w_{1,\nu}}{P_{1,\nu}}R_\nu.$$

This formulation involves the North's price indices at different generations, as what matters is the real profits. Whenever an innovation arrives, the innovator becomes the GPT monopoly. With (8), the first-order condition is:

$$MB \equiv \lambda'(R_\nu)\frac{V_{\nu+1}}{P_{1,\nu+1}} = \frac{\beta(X_{1,\nu+1}+X_{2,\nu+1})}{(1+\theta)P_{1,\nu+1}}\frac{\lambda'(R_\nu)}{r+\lambda(R_{\nu+1})} = \frac{w_{1,\nu}}{P_{1,\nu}} \equiv MC, \tag{9}$$

where the left-hand side is the marginal benefit of $R_\nu$ and the right-hand side the marginal cost. We restrict $\lambda'' < 0$ so that the second-order condition $\lambda''(R_\nu)V_{\nu+1}/P_{1,\nu+1} < 0$ holds. The first-order condition (10) can be further rewritten as

$$\frac{\beta}{1+\theta}\frac{(X_{1,\nu+1}+X_{2,\nu+1})/P_{1,\nu+1}}{w_{1,\nu}/P_{1,\nu}} = \frac{r+\lambda(R_{\nu+1})}{\lambda'(R_\nu)}. \tag{10}$$

The right-hand side of (10) is the AH component capturing the effects of arrival, interest, and create-destruction rates. Along a balance growth path where $R_\nu = R_{\nu+1} = R$, this term strictly increases with $R$. Therefore, the left-hand side of the equation can be viewed as an *R&D multiplier* – the larger the R&D multiplier, the more GPT R&D activities in equilibrium. The numerator term $\beta(X_{1,\nu+1}+X_{2,\nu+1})/(1+\theta)P_{1,\nu+1}$ reflects the real royalty payments received from differentiated products firms globally, which depends on real global sales of these products, whereas the denominator term is the marginal cost of R&D captured by the North's real wage. As we will explain in Section 3.2, the R&D multiplier is the key to linking trade and growth.

Using (5) and (6), the labor market clearing conditions are

$$\frac{\theta}{1+\theta}X_{1,\nu} = w_{1,\nu}L_1 = \frac{\theta}{1-\beta+\theta}(N_1-R_\nu)w_{1,\nu}, \tag{11}$$

$$\frac{\theta}{1+\theta}X_{2,\nu} = w_{2,\nu}L_2 = \frac{\theta}{1-\beta+\theta}N_2 w_{2,\nu}. \tag{12}$$

[10] As there is a continuum of GPT innovators, each innovator is an atomless agent, who takes both the expected GPT monopoly revenue $\beta(X_{1\nu+1}+X_{2\nu+1})/(1+\theta)$ and the expected rate of creative destruction of his own prey $\lambda(R_{\nu+1})$ as given.

Define $x_\nu \equiv X_{1,\nu}/X_{2,\nu}$, $\chi \equiv N_1/N_2$, and $w_\nu = w_{1,\nu}/w_{2,\nu}$. Taking the ratio of the above two equations entails

$$x_\nu = \chi \left(1 - \frac{R_\nu}{N_1}\right) w_\nu. \tag{13}$$

### 2.5 Goods Markets and Equilibrium Definition

National income is given by

$$\begin{aligned} Y_{1,\nu} &= (1-\beta)\,\Pi_{1,\nu} + w_{1,\nu}L_1 + \frac{\beta}{1+\theta}\left(X_{1,\nu} + X_{2,\nu}\right), \\ Y_{2,\nu} &= (1-\beta)\,\Pi_{2,\nu} + w_{2,\nu}L_2. \end{aligned}$$

That is, there is an international redistribution of royalty payments from South to North whenever $\beta > 0$. Market clearing condition for final goods is

$$Y_{1,\nu} = X_{1,\nu} + \frac{\beta}{1+\theta}X_{2,\nu} = X_{11,\nu} + X_{12,\nu}, \tag{14}$$

$$Y_{2,\nu} = X_{2,\nu} - \frac{\beta}{1+\theta}X_{2,\nu} = X_{21,\nu} + X_{22,\nu}. \tag{15}$$

Using $X_{i,\nu} = \sum_{n=1,2} X_{ni,\nu}$ to combine with the above, we get

$$X_{12,\nu} = X_{21,\nu} + \frac{\beta}{1+\theta}X_{2,\nu}. \tag{16}$$

The lack of balanced trade in final goods is because country 1's royalty income from country 2 makes up its excessive net import. Using BEJK Results 1 and 2 in conjunction with (14) and (15) and with a few algebraic manipulations, (17) gives the ratio of total revenues proportional to the ratio of entrepreneurship in skill units:

$$\frac{X_{1,\nu}}{X_{2,\nu}} = \left[\frac{\Phi_{1,\nu}}{\Phi_{2,\nu}}\frac{1+\theta-\beta}{1+\theta} + \frac{\beta\tau^\theta}{1+\theta}\right]\frac{M_{1,\nu}}{M_{2,\nu}}w_\nu^{-\theta}, \tag{17}$$

where $\Phi_{n,\nu} = \sum_{i=1,2} T_{i,\nu}\left(w_{i,\nu}\tau_{ni}\right)^{-\theta}$. With (1) and letting $t_0 = t_{1,0}/t_{2,0}$ and $m_\nu = M_{1,\nu}/M_{2,\nu}$, the above becomes

$$x_\nu = m_\nu w_\nu^{-\theta}\left[\frac{m_\nu t_0 w_\nu^{-\theta} + \tau^{-\theta}}{m_\nu t_0\left(w_\nu\tau\right)^{-\theta} + 1}\frac{1+\theta-\beta}{1+\theta} + \frac{\beta\tau^\theta}{1+\theta}\right].$$

Combining (5), (7), and (13) entails

$$m_\nu = \chi\left(1 - \frac{R_\nu}{N_1}\right) = \frac{x_\nu}{w_\nu}. \tag{18}$$

Combing the above two equations entail

$$w_\nu^{1+\theta} = \frac{m_\nu t_0 w_\nu^{-\theta} + \tau^{-\theta}}{m_\nu t_0 \left(w_\nu \tau\right)^{-\theta} + 1} \frac{1+\theta-\beta}{1+\theta} + \frac{\beta\tau^\theta}{1+\theta}. \tag{19}$$

We choose the North's labor as the numeraire for the general equilibrium in each generation. That is, $w_{1,\nu} = 1$ for all $\nu$, and $w_\nu = 1/w_{2\nu}$. A *dynamic world equilibrium* is a sequence of a tuple $\left\{R_\nu, w_\nu, \{L_{i,\nu}, M_{i,\nu}, T_{i,\nu}, P_{i,\nu}\}_{i=1,2}\right\}_{\nu=0}^{\infty}$ that satisfies, at any generation $\nu$, the two occupational-choice conditions and two labor resource constraints (5), (6), and (7), the evolution of technology stock and price index (1) and (4) in both countries, the trade condition (19), and the first-order condition for innovators' problems (10).

A simple algorithm is given as follows. First, given the sequence $\{R_\nu\}$, the labor allocation $\{L_{i,\nu}, M_{i,\nu}\}_{i=1,2}$ (and hence the ratio of entrepreneurs between the two countries $\{m_\nu\}$) is given by (5), (6), and (7). Equilibrium wage ratio $\{w_\nu\}$ between two countries is solved from (19). Subsequently, $\{T_{i,\nu}, P_{i,\nu}\}$ are given by (1) and (4). Last, the sequence $\{R_\nu\}_{\nu=0}^{\infty}$ satisfies (10).

# 3 Trade and Sustained Growth

Our analysis focuses on a *balanced growth path* (BGP) equilibrium, which is a dynamic world equilibrium with a constant, common growth rate $g$.[11] Along a BGP, wages, total revenues all grow at the common rate $g(R)$, whereas all labor variables are constant.

## 3.1 Characterization of the Balanced Growth Path

Along the BGP, the subscript $\nu$ of all of the stationary variables will be dropped. Given $w_{1\nu} = 1$ by the choice of numeraire, it will be shown shortly that the wage ratio $w_\nu = 1/w_{2\nu}$ and hence $w_{2\nu}$ are both stationary. As a result, the nominal variables $\{X_{i\nu}, Y_{i\nu}\}$ are also

[11] We do not study the transitional path for this model mainly because any transitional path may likely involve ad hoc assumptions. To see this, observe (9) that the choice of $R_\nu$ depends on how the innovators expect the rate the destruction $\lambda\left(R_{\nu+1}\right)$. Let $R^*$ denote an initial BGP equilibrium $R$ and that $R^{*\prime}$ the new BGP equilibrium after a shock. GPT innovators can immediately choose a new $R_\nu = R^{*\prime}$ if they correctly expect $R_{\nu+1} = R^{*\prime}$, eliminating the transitional path. Thus, any transitional path requires assuming some adjustment frictions or information frictions, either of which can be ad hoc.

stationary. Using (11) and (12) and noting $w_1 = 1$, we have

$$X_1 + X_2 = \frac{1+\theta}{1-\beta+\theta}\left(N_1 - R + \frac{N_2}{w}\right). \tag{20}$$

Combining the above with (10), we obtain

$$\frac{r+\lambda(R)}{\lambda'(R)}\frac{P_{1,\nu+1}}{P_{1,\nu}} = \frac{\beta}{1-\beta+\theta}\left(N_1 - R + \frac{N_2}{w}\right). \tag{21}$$

For the growth rate of the price index, observe that

$$P_{n,\nu+1}^{-\theta} = \eta^{-\theta}\sum_{i=1,2} T_{i,\nu+1}\left(w_i\tau_{ni}\right)^{-\theta} = \gamma P_{n,\nu}^{-\theta},$$

which implies that for all $n$,

$$1 + g_p \equiv \frac{P_{n,\nu+1}}{P_{n,\nu}} = \gamma^{-\frac{1}{\theta}}. \tag{22}$$

Combine (20), (21), and (22), and we obtain

$$\frac{\beta\gamma^{\frac{1}{\theta}}\left(X_1+X_2\right)}{1+\theta} = \frac{\beta\gamma^{\frac{1}{\theta}}}{1-\beta+\theta}\left(N_1 - R + \frac{N_2}{w}\right) = \frac{r+\lambda(R)}{\lambda'(R)}. \tag{23}$$

Then, a dynamic equilibrium on a balanced growth path can be obtained by solving $\{R, w, m\}$ jointly using (18), (19), and (23); the wage ratio obtained from trade condition (19) is indeed stationary.

For our purpose, the central interest is to understand the growth of real income $y_{i,\nu} = Y_{i,\nu}/P_{i,\nu}$. As $Y_{i,\nu}$ is stationary, (22) implies that $1 + g_y \equiv y_{i,\nu+1}/y_{i,\nu} = \gamma^{\frac{1}{\theta}}$. Under the Poisson process of the GPT innovation and in continuous time, the exponential growth rate of real income is given by $\lambda(R)\ln\left(\gamma^{\frac{1}{\theta}}\right)$. The larger the GPT R&D effort, the faster real income grows.

Comparing with (10), it is clear that the two sides of the first equality in (23) are two ways to express the R&D multiplier, the endogenous part of which is reduced to the global sales of differentiated products (denominated in the North's wages). By observing the second equality in (23), one sees that had one ignored the effect of $R$ on relative wage $w$, a unique level of R&D activity $R$ would have been pinned down immediately. Yet, to assess the effect of trade liberalization, the general equilibrium object wage gap $w$ is crucial, thereby requiring more thorough analysis with which we now proceed.

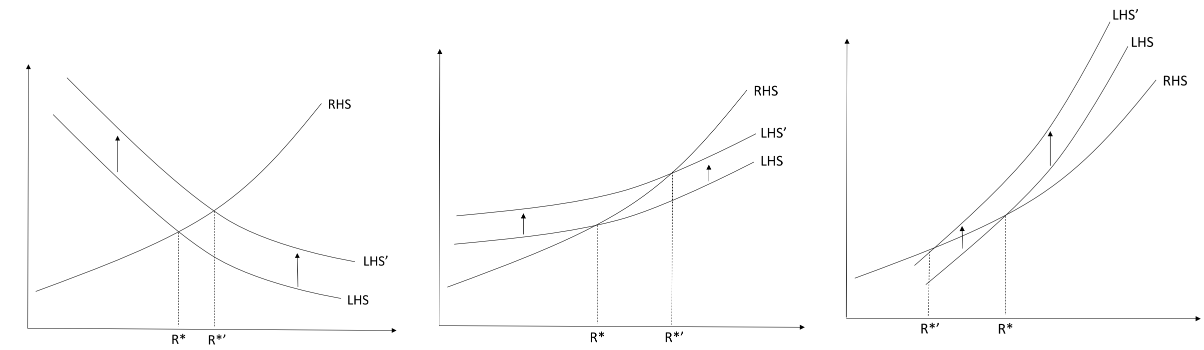
## 3.2 Trade and Sustained Growth

This subsection clarifies and explains the mechanism under which trade costs may affect long-run growth. The following analysis focuses on the second equality of (23). Observe that the R&D multiplier consists of two effects of GPT R&D effort: (i) a labor-reallocation effect as a result of occupational choice via the term $N_1 - R$, and (ii) a general equilibrium wage-gap effect via $w(R)$. In general, how the multiplier changes in $R$ is ambiguous.

Note that for a given $R$, a reduction in $\tau$ only affects the multiplier via the wage gap $w$, and the right-hand side of (23) is unaffected. Under the North-South structure, $t_0 = t_{1,0}/t_{2,0} > 1$, we generally expect that the wage gap $w > 1$.[12] Trade liberalization tends to reduce this gap, as it makes the global economy more integrated and spurs the labor demand in the South relative to that in the North. Thus, trade liberalization tends to increase the R&D multiplier for a given $R$. Because the right-hand side of (23) strictly increases in $R$, trade liberalization therefore would induce an increase in equilibrium $R$ as long as the multiplier decreases in $R$ or does not increase in $R$ faster than the right-hand side of (23) around the initial equilibrium $R$. This is Cases (a) and (b) in Figure 1. But is it possible that the multiplier increases in $R$ faster than the right-hand side as illustrated by Case (c) in Figure 1? This is implausible because it can be ruled out by checking whether equilibrium $R_\nu$ would increase when there is a transitory positive shock to the arrival rate, written as $\tilde{\lambda} = \lambda_0 \lambda(R_\nu)$ with $\lambda_0 > 1$ during period $\nu$ and $\lambda_0 = 1$ from $\nu + 1$ onward. For any given $R_\nu$, such a shock implies that the multiplier is unaffected, and the right-hand side decreases because the rate of creative destruction $\lambda(R_{\nu+1})$ is unaffected but $\lambda'(R_\nu)$ increases. Thus, equilibrium $R_\nu$ must decrease in Case (c), but this is implausible. That is, Case (c) can be ruled out by Samuelson's Correspondence Principle.

The key intuition here is that *aggregate world revenue and hence aggregate labor demand in the unit of the North's labor* $X_{1,\nu+1} + X_{2,\nu+1}$ *increase* in face of trade liberalization because a more integrated economy implies a stronger demand for the South's labor (relative to the North's) which reduces the wage gap. When $R$ increases as a result, the arrivals of new GPT become faster, and the real income of each country also grows faster.

[12] In addition to $t_0 > 1$, the differences in model primitives between the two countries include relative population size $\chi$ between the North and the South and the fact that GPT innovation occurs only in the North, the latter of which should also contribute to $w > 1$. Thus, the only possibility for $w > 1$ not to hold is a very large $\chi > 1$. In the sample for our quantitative analysis, the average population size (measured by employment) of the North countries is only $0.47$ of that of the South countries. In the benchmark equilibrium in the quantitative analysis, the employment-weighted average wage of the North countries is $9.7$ times larger than that of the South countries.




(a) Case (a) (b) Case (b) (c) Case (c)

Figure 1: Effect of Trade Liberalization on R&D

# 4 Quantitative Model and Welfare Gains from Trade

We now extend the theoretical model in the previous sections to a quantitative one that allows for an arbitrary number of countries and flexible trade costs between any pair of countries. We also investigate properties pertaining to welfare gains from trade.

## 4.1 Quantitative Model

### 4.1.1 Model generalization

There are $I$ countries, each of which is indexed by $i, n \in \{1, \ldots, I_N, ..., I\}$, where the first $I_N$ countries are the North countries, and there are $I - I_N$ South countries. Each country has a population $L_i$. To deliver a unit of a good to country $n$ from $i$, $\tau_{ni} \geq 1$ units need to be shipped from $i$. Assume that $\tau_{nn} = 1$. The model setup is the same as in Section 2 except for the following two features.

First, the production function of a product $\omega$ is now extended to incorporate intermediate inputs as in BEJK:

$$y_i(\omega) = B z_i(\omega)[\ell_i(\omega)]^{\alpha}[b_i(\omega)]^{1-\alpha}, \tag{24}$$

where $b_i$ denotes the bundles of intermediate inputs used, $\alpha$ the labor share for production, and $B = \left[\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\right]^{-1}$ a normalizing constant for a clean expression of the unit cost.

Second, assume that GPT innovators are multi-national corporations (MNCs) that set up branches in each North country. A GPT innovator hires $R$ units of composite labor input that combines researchers among the North countries via a Cobb-Douglas production function.

$$R = \frac{\prod_{i=1}^{I} R_i^{\mu_i}}{\prod_{i=1}^{I} \mu_i^{\mu_i}}$$

such that $\sum_{i=1}^{I} \mu_i = 1$, and $\mu_i = 0$ if $i$ is a South country. The Poisson arrival rate of a new GPT is given by $\lambda(R)$. As the GPT monopoly is an MNC, assume that the royalty payments received are distributed to its branch in country $i$ by its labor share $\mu_i$.

#### 4.1.2 Equilibrium conditions

We relegate the complete description of the equilibrium conditions to Appendix B.1. Here we describe some of the conditions derived from the two above-mentioned parts of the quantitative model. For the labor market, follow the same procedure as that in Section 2.4, one obtains similar labor allocation given $\{R_{i,\nu}\}_{i=1}^{I}$:

$$M_{i,\nu} = \frac{1-\beta}{1-\beta+\theta\alpha}(N_i - R_{i,\nu}) \tag{25}$$

$$L_{i,\nu} = \frac{\theta\alpha}{1-\beta+\theta\alpha}(N_i - R_{i,\nu}). \tag{26}$$

An innovating firm's cost minimization problem is

$$\min \sum_{i \in \text{North}} \frac{w_{i,\nu}}{P_{i,\nu}} R_{i,\nu}$$

$$s.t. \ \frac{\prod_{i=1}^{I} R_{i,\nu}^{\mu_i}}{\prod_{i=1}^{I} \mu_i^{\mu_i}} \geq R_\nu.$$

Let $w_\nu^R \equiv \prod_{i=1}^{I} w_{i,\nu}^{\mu_i}$ and $P_\nu \equiv \prod_{i=1}^{I} P_{i,\nu}^{\mu_i}$. The Cobb-Douglas structure implies that the aggregate price index for the composite input $R_\nu$ is given by

$$\frac{w_\nu^R}{P_\nu} = \prod_{i=1}^{I} \left(\frac{w_i}{P_{i,\nu}}\right)^{\mu_i}.$$

In equilibrium, the demand for researchers in each country satisfies

$$\frac{w_{i,\nu} R_{i,\nu}}{P_{i,\nu}} = \mu_i \frac{w_\nu^R R_\nu}{P_\nu}.$$

$$\sum_i \frac{w_{i,\nu} R_{i,\nu}}{P_{i,\nu}} = \frac{w_\nu^R R_\nu}{P_\nu} \tag{27}$$

The value of an innovation at generation $\nu+1$ is

$$V_{\nu+1} = \frac{\beta \sum_{i=1}^{I} X_{i,\nu+1}}{(1+\theta)[r+\lambda(R_{\nu+1})]}.$$

As assumed above, the GPT innovating firms are jointly owned by people in the North countries with the share of country $i$ given by $\mu_i$. So, the real value of a new innovation should be deflated by $P_{\nu+1} = \prod_{i=1}^{I} P_{i,\nu+1}^{\mu_i}$.[13] Thus, a GPT innovator's problem is

$$\max_{R_\nu} \frac{\lambda(R_\nu) V_{\nu+1}}{P_{\nu+1}} - \frac{w_\nu^R}{P_\nu} R_\nu. \tag{28}$$

Similar to (10), the first-order condition can be written as

$$\frac{\beta}{1+\theta} \frac{\left(\sum_{i=1}^{I} X_{i,\nu+1}\right)/P_{\nu+1}}{w_\nu^R/P_\nu} = \frac{r+\lambda(R_{\nu+1})}{\lambda'(R_\nu)}, \tag{29}$$

where the left-hand side is the R&D multiplier. As a fraction $\mu_i$ of royalty payment to the GPT firm is given to the branch at country $i$, country $i$'s national income is given by

$$Y_{i,\nu} = \frac{1-\beta}{1+\theta} X_{i,\nu} + w_{i,\nu} L_{i,\nu} + \frac{\beta \mu_i}{1+\theta} \sum_{n=1}^{I} X_{n,\nu}. \tag{30}$$

#### 4.1.3 Balanced growth path and growth rates

By Walras' law and for the general equilibrium in each generation, we set $w_\nu^R = 1$. Thus, the nominal variables $\{w_{i,\nu}, X_{i,\nu}, Y_{i,\nu}\}$ and the labor allocation variables $\{R_\nu, R_{i,\nu}, L_{i,\nu}, M_{i,\nu}\}$ are both stationary along the BGP; the subscript $\nu$ for these variables along the BGP can, therefore, be dropped. As the technology stock $T_{i,\nu}$ grows by a factor $\gamma$ for each new innovation, unit costs and price indices generally decline over time.

Complete equilibrium conditions along the BGP are relegated to Appendix B.2. The basic idea is that given an equilibrium composite research labor input $R$, the complete set of the above-mentioned stationary variables, along with *detrended* unit costs and price indices, can be solved in the static cross-sectional equilibrium as similar to BEJK. Then,

[13] Alternatively, one can assume that the MNC owner lives in a particular country $i$, and thus the problem can be written as

$$\max_{R_\nu} \frac{\lambda(R_\nu) V_{\nu+1}}{P_{i,\nu+1}} - \frac{w_\nu^R R_\nu}{P_\nu}.$$

It is readily verified that this alternative formulation is qualitatively similar to the benchmark formulation in (28).

following the procedure in the two-country model in Section 3.1, the (negative) growth rate of the price index along the BGP, $g_p$, is given by

$$1 + g_p \equiv \frac{P_{n,\nu+1}}{P_{n,\nu}} = \gamma^{-\frac{1}{\theta\alpha}}. \tag{31}$$

Combining the above with (29), equilibrium $R$ on the BGP can be solved from

$$\frac{\beta\gamma^{\frac{1}{\theta\alpha}}}{1+\theta}\sum_{i=1}^{I} X_i = \frac{r+\lambda(R)}{\lambda'(R)}, \tag{32}$$

where the left-hand side is the R&D multiplier.

For welfare calculations, we will need to know the growth rate in real income $y_{i,\nu} = Y_{i,\nu}/P_{i,\nu}$, denoted as $g_y$. As $Y_{i,\nu}$ is stationary, (B.15) implies that $1 + g_y \equiv y_{i,\nu+1}/y_{i,\nu} = \gamma^{\frac{1}{\theta\alpha}}$. Under the Poisson process of the GPT innovation and in continuous time, the exponential growth rate of real income is given by $k_y \equiv \lambda(R)\ln\left(\gamma^{\frac{1}{\theta\alpha}}\right)$. The corresponding annual growth rate is $[y(1) - y(0)]/y(0) = e^{k_y} - 1 = \gamma^{\frac{\lambda(R)}{\theta\alpha}} - 1$.

## 4.2 Welfare Gains from Trade

Assume that the economy is on the BGP from time zero, and thus the lifetime utility of country $i$'s representative agent on the BGP viewed at $t = 0$ is given by

$$U_i = \int_0^\infty Q_{i,0}e^{-(\rho-k_y)t}dt = \frac{1}{\rho-k_y} \times \frac{Y_{i,0}}{N_i} \times \frac{1}{P_{i,0}}.$$

To ensure finite lifetime utility, we impose a modified Brock-Gale condition in a fashion similar to that in Bond, Wang and Yip (1996): $\rho > \max_R \lambda(R)\ln(\gamma)$. This condition will be met if the maximal arrival rate is not too high and the step size of the ladder, $\gamma$, is not too large, which will be checked in our quantitative analysis. Also, note that if certain curvature were to be added to the flow utility function, the dynamic component $1/(\rho - k_y)$ remains the same.

Following the trade literature, we examine welfare changes in relative terms:

$$\begin{aligned}
\frac{U_i'}{U_i} &= \frac{\rho-k_y}{\rho-k_y'} \times \frac{Y_{i,0}'}{Y_{i,0}} \times \frac{P_{i,0}}{P_{i,0}'} = \frac{\rho-k_y}{\rho-k_y'} \times \frac{Y_{i,0}'/w_{i,0}'}{Y_{i,0}/w_{i,0}} \times \frac{w_{i,0}'/P_{i,0}'}{w_{i,0}/P_{i,0}} \\
&= \frac{\rho-k_y}{\rho-k_y'} \times \frac{Y_{i,0}'/w_{i,0}'}{Y_{i,0}/w_{i,0}} \times \left(\frac{\pi_{ii}'}{\pi_{ii}}\right)^{-\frac{1}{\theta\alpha}} \\
&\equiv GR_i \times IG_i \times ACR_i.
\end{aligned} \tag{33}$$

That is, the relative welfare changes can be decomposed into three terms. The first term, denoted by $GR_i$, is *the growth rate effect*. As the exponential growth rate of real income $k_y = \lambda(R)\ln\left(\gamma^{\frac{1}{\theta\alpha}}\right)$, this is a major channel via which the dynamic effect through R&D kicks in. Intuitively, a larger R&D effort advances technology faster and entails a larger welfare improvement. The mechanism illustrated in Section 3 shows that trade can induce more R&D effort and hence contribute to welfare improvement.

The last term in (33) captures the relative change in real wages, which is exactly the ACR formula for welfare gains from trade in a static model. To see this, simply use BEJK Results 1 and 3 and note that $\tau_{ii} = 1$ by assumption, and we have

$$\left(\frac{\pi'_{ii}}{\pi_{ii}}\right)^{-\frac{1}{\theta\alpha}} = \left(\frac{\sum_k T_k\left(C_k\tau_{ik}\right)^{-\theta}}{\sum_k T_k\left(C'_k\tau'_{ik}\right)^{-\theta}}\right)^{-\frac{1}{\theta\alpha}}\left(\frac{C'_i\tau'_{ii}}{C_i\tau_{ii}}\right)^{\frac{1}{\alpha}} = \frac{w'_i/P'_i}{w_i/P_i}.$$

Thus, the relevant trade elasticity is $\theta\alpha$. As is well-known, the static gains from trade in the ACR framework are reflected through changes in trade flows.

The second term in (33) is denoted as $IG_i$, the *income-gains effect*. This captures the relative changes in the *income-wage ratio*. In the ACR framework, this term disappears because of the assumptions that (1) trade is balanced, that (2) aggregate profit is a constant share of the total revenue, and that (3) labor is inelastically supplied. When trade is balanced, total income equals total revenue. Then, the latter two assumptions, along with the Cobb-Douglas production function, imply a constant income-wage ratio because the total labor costs $w_iL_i$ is a constant share of total income. By adopting the BEJK structure, assumptions (2) and (3) hold, but the first assumption is violated due to the royalty payments to the GPT monopoly. Thus, this income-gains effect reflects the North's gains from trade stemming from the North's ownership of GPT. To have a closer look at this effect, rewrite (30) using BEJK Result 4:

$$\frac{Y_{i,\nu}}{w_{i,\nu}} = \frac{1-\beta+\theta\alpha}{\theta\alpha}L_{i,\nu} + \frac{\beta\mu_i}{\theta\alpha}\sum_{n=1}^{I}\frac{w_{n,\nu}}{w_{i,\nu}}L_{n,\nu}. \tag{34}$$

For South countries, by $\mu_i = R_i = 0$ and (26), the above equation is reduced to $Y_{i,\nu}/w_{i,\nu} = N_i$, which means that per capita income equals the wage. Hence, there is no income-gains effect for South countries. For North countries, the income-gains effect is generally positive unless the terms of trade effect on relative wages are very unfavorable (see Section 6.1.1).

Note that the growth-rate effect does not fully reflect the true dynamic gains from trade because changes in the research effort $R$ and $R_i$ also affect the income-gains and the

ACR components through general equilibrium effects. To account for the true dynamic gains from trade, we compare our model with a purely static version of the model in which $\kappa = 0$. Obviously, both the equilibrium $R$ and growth rate are zero under such a parameter constraint. From (25) and (26), it is immediate that all labor-allocation variables $\{M_i, L_i\}$ become constant (which is a feature of the BEJK model). Also, growth rate $k_y = 0$ and hence the growth-rate effect disappear in this static model. To isolate the contribution of dynamics on the total gains from trade (called dynamic gains from trade), one could write:

$$\frac{U_i'}{U_i} = \frac{\frac{U_i'}{U_i}}{\left.\frac{U_i'}{U_i}\right|_{\text{static}}} \times \left.\frac{U_i'}{U_i}\right|_{\text{static}} = GR_i \times \frac{IG_i \times ACR_i}{IG_i|_{\text{static}} \times ACR_i|_{\text{static}}} \times \left(IG_i|_{\text{static}} \times ACR_i|_{\text{static}}\right), \quad (35)$$

where the sub-label of static indicates the corresponding value under the static model. The true dynamic gains are thus the first two multiplicative terms, which reflect the growth-rate effect in the dynamic model and the general equilibrium effects of trade on the income-gains and ACR components that are not captured by the static model. Equivalently, we can write (35) as

$$\frac{\ln\left(GR_i \times \frac{IG_i \times ACR_i}{IG_i|_{\text{static}} \times ACR_i|_{\text{static}}}\right)}{\ln\left(\frac{U_i'}{U_i}\right)} = 1 - \frac{\ln\left(IG_i|_{\text{static}} \times ACR_i|_{\text{static}}\right)}{\ln\left(\frac{U_i'}{U_i}\right)}, \quad (36)$$

and one can simply compute the share of the dynamic gains (which we sometimes call *dynamic share* in short) by calculating the right-hand side of the above equation after computing the gains from trade in the static model.

# 5 Quantification

We calibrate our model to match the world economy during 2000-2014. The first task in the calibration is to reasonably define the North and South countries. Our approach is to take the Penn World Tables (PWT9.0) data and select those countries with at least two-thirds of the U.S. real GDP per worker as the North.[14] The countries under consideration are based on the availability of other datasets used in the calibration, including Centre d'Etudes Prospectives et d'Informations Internationales (CEPII) data, Global Report of

[14]The GDP measure used is output at constant PPP in 2011 million US$ (rgdpo). The employment data is obtained from the WDI. All data are averaged over 2000-2014.

Table 1: Parameterization

| Panel A: Preset Parameters | | | |
|---|---|---|---|
| Parameter | Value | Meaning | Notes |
| $\rho$ | 0.03 | Time preference rate | Standard in the literature. |
| $r$ | 0.05 | Real interest rate | |
| $\gamma$ | 1.1017 | GPT innovation ladder size | Aghion and Howitt (1992) |
| $\theta$ | 5.03 | Trade elasticity | Head and Mayer (2014) |

| Panel B: Calibrated Parameters | | | |
|---|---|---|---|
| Parameter | Value | Meaning | Target |
| $\tau_{ni}$ | - | Trade costs | Calibrated by the fixed-effect gravity equation and the model structure. |
| $t_{i,0}$ | - | Initial technology stock | |
| $\beta$ | 0.074 | The bargaining power parameter of the GPT firm | Targeting profit rate of 15.4%. |
| $\kappa$ | 2.04 | Efficacy of the GPT innovation | Jointly calibrated by targeting BGP annual growth rate of 1.898%, and BGP R&D employment share in North countries to 0.58%. |
| $\varepsilon$ | 0.20 | Curvature parameter of the GPT innovation | |

Global Entrepreneurship Monitor (GEM), World Input Output Database (WIOD),[15] World Development Indicators (WDI), and Penn World Tables (PWT) 9.0. There are 52 countries in our sample. There are 19 North countries and 33 South countries. The complete list is provided in Table C.1 in the Appendix. Unless otherwise mentioned, all of the data values used in calibration are averages during 2000-2014.

Succinct information regarding the calibration is provided in Table 1. The following provides the details of calibration; some further details are relegated to the appendices.

**Data and preset parameters** We first retrieve a set of parameters from the literature. As the Fréchet shape parameter of the productivity distribution, $\theta$, is the trade elasticity, we set $\theta = 5.03$, which is the preferred value from Head and Mayer (2014) synthesis on the gravity equation literature. For the ladder size of productivity upgrading, we set $\gamma = 1.1017$, following Aghion et al. (2019).[16] Following much of the macro-growth literature, we set the time preference rate $\rho$ to $0.03$, and the real interest rate $r$ to $0.05$.

**Share of intermediate inputs** We use the Social Economic Account (SEA) from the World Input Output Database (WIOD) 2014 to calculate $1 - \alpha$. At the two-digit level, the SEA provides information about country-industry-specific gross output and the compensation for intermediate inputs at current basic prices. Taking the US for the benchmark value and aggregating across industries, we obtain $1 - \alpha = 0.44$. This calibration of $1 - \alpha$

[15]The 2016 release of the WIOD is used. See http://www.wiod.org/home.

[16]More specifically, this number is the average among the estimates of step size in three different periods in the data (1983-1993, 1993-2003, 2003-2013). See Table 11 of Aghion et al. (2019).

implies that we interpret the single fundamental input (labor) in the model broadly as the composite input consisting of both labor and capital in the data.

**R&D share and GPT's bargaining power** Based on (27), the R&D share $\mu_i$ for each North country $i$ is calibrated via:

$$\mu_i = \frac{\frac{w_i R_i}{P_i}}{\sum_{i=1}^{I_N} \frac{w_i R_i}{P_i}},$$

where $w_i/P_i$ is the country-specific real wages proxied by the real GDP per worker,[17] and $R_i$ is proxied by the number of researchers obtained from the WDI.

To calibrate $\beta$, note that this parameter directly affects the profit share of revenue of differentiated-product firms, as well as the royalty payments received by the GPT, which are the GPT's profit. Using the calibrated $\mu_i$'s, $\beta$ is calibrated by targeting the profit rate of $0.154$. We obtain $\beta = 0.074$. See further details in Appendix B.2.

**Trade costs and relative technology stocks** The complete details are relegated to Appendix B.2, and here we briefly describe how trade costs and relative technology stocks are estimated/calibrated. Using trade flows data and geography-related variables from CEPII[18] and given values of trade elasticity $\theta$, bilateral trade costs $\tau_{ni}$'s are estimated from the gravity equation implied by the model structure with a standard approach such as that in Head and Mayer (2014). Given estimated trade costs, together with values of $\theta$ and $\alpha$, the initial knowledge stock $t_{i,0}$ can be backed out from the model structure, as in Fieler (2011) and Ravikumar, Santacreu and Sposi (2019). In particular, combining the model structure with the gravity equation, the exporter fixed effects $D_i^{exp}$ is given by $T_i C_i^{-\theta}$, where the unit cost $C_i$ is recursively a function of technology stock $\{T_i\}_{i=1}^{I}$. Thus, $\{T_i\}_{i=1}^{I}$ can be solved using the following system of equations, given the exporter fixed effects $\{\hat{D}_i^{exp}\}_{i=1}^{I}$ and bilateral trade costs $\{\tau_{ni}\}$ estimated from the the gravity equation,

[17] Using the GDP per worker to proxy real wages is consistent with our calibration of the share of intermediate inputs which interpret the labor input in our model broadly.

[18] There are multiple sources of CEPII's trade flows data, and the UN Comtrade one is used here.

values of $(\theta, \alpha)$, and nominal wages $w_i^{data}$ for each country:[19]

$$T_i C_i^{-\theta} = \exp(\hat{D}_i^{exp}) \tag{37}$$

$$C_i = (w_i^{data})^{\alpha} P_i^{1-\alpha} \tag{38}$$

$$P_i = \left( \sum_{m=1}^{I} T_m [C_m \tau_{i,m}]^{-\theta} \right)^{-\frac{1}{\theta}}. \tag{39}$$

With data values of $M_i$ from the GEM, we then compute $t_{i0} \propto T_i/M_i$. In sum, the relative technology stocks $\{t_{i0}\}$ are how the model rationalizes the estimated exporter fixed effects, consistent with estimated trade costs and observed country-specific wages and adjusted for entrepreneurial activities.

**Innovation parameters** For our quantitative implementation, we specify the arrival rate as: $\lambda(R) = \kappa R^{\epsilon}$, with $\kappa > 0$, $\epsilon \in (0, 1)$, a form commonly used in the innovation and growth literature. Finally, the two parameters $\kappa$ and $\epsilon$ are solved simultaneously to match the following two targets. The first target is the annual real income growth rate. Specifically, we first calculate the annual growth rates of GDP per worker for all of the countries in our sample combined for each year during 1980–2014.[20] Then, we take the geometric average of these growth rates and obtain $1.8977\%$. The model counterpart of the annual growth rate is given by $\gamma^{\frac{\lambda(R)}{\theta\alpha}} - 1$ (see Section 4.1.3). The second target is the employment share of R&D researchers, $\sum_{i \in \text{North}} R_i^{\text{data}} / \sum_{i \in \text{North}} N_i^{\text{data}}$. Using the WDI data, the employment share of researchers is $0.58\%$. The resulting $(\kappa, \epsilon) = (2.04, 0.20)$.

# 6 Quantitative Analysis

## 6.1 Welfare Gains from Trade: Baseline Results

The primary goal of our quantitative analysis is to evaluate the welfare gains from trade, with a focus on the total gains and the importance of dynamic gains. Similar to many practices in the literature, we measure welfare gains from trade based on one of the following two scenarios: (i) *compared with autarky*: what are the gains from trade when the

[19] In the expression of the price index, we have renormalized the constant in the price index $\eta = 1$ (see BEJK Result 3). This is without loss of generality as what matters for equilibrium is the relative technology stock.

[20] To match the *long-run* growth rate and to avoid concerns about business cycles and decade-specific economic conditions, we lengthen the period beyond 2000–2014 for calculating the average annual growth rate.

global economy transits from autarkic economies to the benchmark model with estimated bilateral trade costs?; (ii) *compared with less-costly trade*: what are the gains from trade when the global economy transits from the benchmark model to the case where all bilateral trade costs are reduced by $25\%$. We first show the results of total gains from trade and decompose them into the growth-rate, income-gains, and the ACR effects. As mentioned in Section 4.2, the growth-rate effect in the simple decomposition is not necessarily the dynamic component of the total gains, if the general equilibrium effects are non-negligible. Thus, we compare with the static model in which $\kappa = 0$ (zero arrival rate) to find the "true" dynamic share of the gains from trade.

### 6.1.1 Gains from trade

We first study the gains from trade, which are measured by the gains from changing the trade costs from infinity (autarky) to the estimated levels in the benchmark model. We first conduct a simple decomposition by the three components in (33) to understand the separate effects through growth rate, income gains, and trade flows (i.e., the ACR statistic). The result of the first scenario (compared with autarky) is given in Table 2. In this scenario, the BGP annual growth rate of real income changes from $1.538\%$ under autarky to $1.8977\%$ under the benchmark. For each country in our sample, we report the total gains, gains due to each of the three components, and their respective shares in the total gains. We also report the average and population-weighted average of these figures for each set of countries (North and South) and for all countries. We also report the effect on global welfare, which is defined as the population-weighted average of country-specific individual utility.[21]

The weighted average of total gains is $30.6\%$, with the North gaining slightly more than the South ($32.1\%$ vs $30.2\%$). The impact of trade on global welfare is $36.0\%$. In the balanced growth model, the growth-rate gains are the same, at $27.4\%$, across all countries, and the weighted average of the share of the growth-rate component is $89.5\%$ with the respective weighted average for the North and the South being $85.5\%$ and $90.7\%$. In other words, the growth-rate component, on average, accounts for most of the gains, even though for certain countries, it can be exceeded by the ACR gains. The share of the growth rate component is higher for the South, as the North has higher ACR gains and income gains on average.

[21]Note that this is different from the population-weighted average of the gains from trade (or their components). Because the global welfare is already a weighted sum of individual countries' individual utility, the effect of trade on this measure cannot be decomposed in the same way as we decompose an individual country's welfare, which is multiplicative in the three components.

## Table 2: Simple Decomposition of Gains from Trade (moving from autarky to trade)

| Country | iso3 | North/South | Total Gains (%) | GR Gains (%) | IG Gains (%) | ACR Gains (%) | Share of GR Gains (%) | Share of IG Gains (%) | Share ACR Gains (%) |
|---|---|---|---|---|---|---|---|---|---|
| Australia | AUS | North | 33.20 | 27.4 | 3.37 | 2.43 | 82.54 | 10.15 | 7.31 |
| Belgium | BEL | North | 59.94 | 27.4 | 4.44 | 28.09 | 45.72 | 7.41 | 46.86 |
| Canada | CAN | North | 39.34 | 27.4 | 3.49 | 8.44 | 69.67 | 8.87 | 21.46 |
| Switzerland | CHE | North | 44.71 | 27.4 | 0.82 | 16.48 | 61.3 | 1.83 | 36.87 |
| Germany | DEU | North | 34.28 | 27.4 | 1.47 | 5.4 | 79.95 | 4.3 | 15.75 |
| Denmark | DNK | North | 43.59 | 27.4 | 3.59 | 12.59 | 62.88 | 8.24 | 28.88 |
| Spain | ESP | North | 37.13 | 27.4 | 4.49 | 5.24 | 73.8 | 12.08 | 14.12 |
| Finland | FIN | North | 46.01 | 27.4 | 5.0 | 13.6 | 59.57 | 10.87 | 29.56 |
| France | FRA | North | 41.98 | 27.4 | 7.46 | 7.11 | 65.28 | 17.77 | 16.94 |
| United Kingdom | GBR | North | 31.32 | 27.4 | -0.92 | 4.84 | 87.49 | -2.94 | 15.45 |
| Greece | GRC | North | 56.63 | 27.4 | 16.6 | 12.63 | 48.4 | 29.31 | 22.3 |
| Ireland | IRL | North | 44.91 | 27.4 | -1.34 | 18.85 | 61.02 | -2.98 | 41.96 |
| Italy | ITA | North | 31.75 | 27.4 | 0.02 | 4.33 | 86.32 | 0.06 | 13.62 |
| Japan | JPN | North | 28.06 | 27.4 | -0.24 | 0.89 | 97.68 | -0.87 | 3.18 |
| Netherlands | NLD | North | 45.44 | 27.4 | 6.03 | 12.01 | 60.3 | 13.26 | 26.44 |
| Norway | NOR | North | 35.74 | 27.4 | -1.14 | 9.47 | 76.67 | -3.18 | 26.5 |
| Singapore | SGP | North | 34.89 | 27.4 | -1.8 | 9.29 | 78.54 | -5.16 | 26.62 |
| Sweden | SWE | North | 35.07 | 27.4 | -0.94 | 8.61 | 78.14 | -2.68 | 24.54 |
| United States | USA | North | 26.42 | 27.4 | -1.89 | 0.9 | 103.72 | -7.14 | 3.42 |
| Argentina | ARG | South | 33.10 | 27.4 | 0 | 5.69 | 82.8 | 0 | 17.2 |
| Bosnia and Herzegovina | BIH | South | 102.68 | 27.4 | 0 | 75.27 | 26.69 | 0 | 73.31 |
| Brazil | BRA | South | 29.13 | 27.4 | 0 | 1.73 | 94.07 | 0 | 5.93 |
| Chile | CHL | South | 34.85 | 27.4 | 0 | 7.44 | 78.65 | 0 | 21.36 |
| China | CHN | South | 28.34 | 27.4 | 0 | 0.94 | 96.69 | 0 | 3.31 |
| Colombia | COL | South | 36.89 | 27.4 | 0 | 9.48 | 74.29 | 0 | 25.71 |
| Ecuador | ECU | South | 46.46 | 27.4 | 0 | 19.05 | 58.99 | 0 | 41.01 |
| Guatemala | GTM | South | 56.61 | 27.4 | 0 | 29.2 | 48.41 | 0 | 51.59 |
| Croatia | HRV | South | 65.89 | 27.4 | 0 | 38.49 | 41.59 | 0 | 58.41 |
| Hungary | HUN | South | 46.59 | 27.4 | 0 | 19.19 | 58.82 | 0 | 41.18 |
| India | IDN | South | 28.91 | 27.4 | 0 | 1.51 | 94.79 | 0 | 5.21 |
| Iran | IRN | South | 36.24 | 27.4 | 0 | 8.83 | 75.63 | 0 | 24.37 |
| Iceland | ISL | South | 62.50 | 27.4 | 0 | 35.1 | 43.85 | 0 | 56.15 |
| Jamaica | JAM | South | 81.30 | 27.4 | 0 | 53.89 | 33.71 | 0 | 66.29 |
| Korea | KOR | South | 30.27 | 27.4 | 0 | 2.86 | 90.55 | 0 | 9.45 |
| Latvia | LVA | South | 72.04 | 27.4 | 0 | 44.63 | 38.04 | 0 | 61.96 |
| Mexico | MEX | South | 30.72 | 27.4 | 0 | 3.32 | 89.21 | 0 | 10.79 |
| Malaysia | MYS | South | 36.24 | 27.4 | 0 | 8.84 | 75.62 | 0 | 24.38 |
| New Zealand | NZL | South | 33.73 | 27.4 | 0 | 6.33 | 81.25 | 0 | 18.76 |
| Panama | PAN | South | 53.94 | 27.4 | 0 | 26.54 | 50.81 | 0 | 49.19 |
| Peru | PER | South | 38.49 | 27.4 | 0 | 11.09 | 71.19 | 0 | 28.81 |
| Poland | POL | South | 38.40 | 27.4 | 0 | 11.0 | 71.37 | 0 | 28.63 |
| Portugal | PRT | South | 38.76 | 27.4 | 0 | 11.36 | 70.7 | 0 | 29.3 |
| Romania | ROU | South | 42.75 | 27.4 | 0 | 15.34 | 64.11 | 0 | 35.89 |
| Russia | RUS | South | 31.06 | 27.4 | 0 | 3.66 | 88.23 | 0 | 11.77 |
| Slovenia | SVN | South | 63.33 | 27.4 | 0 | 35.93 | 43.27 | 0 | 56.73 |
| Thailand | THA | South | 31.95 | 27.4 | 0 | 4.54 | 85.78 | 0 | 14.22 |
| Trinidad & Tobago | TTO | South | 63.63 | 27.4 | 0 | 36.23 | 43.07 | 0 | 56.93 |
| Turkey | TUR | South | 34.48 | 27.4 | 0 | 7.07 | 79.48 | 0 | 20.52 |
| Uganda | UGA | South | 72.12 | 27.4 | 0 | 44.71 | 38.0 | 0 | 62.0 |
| Uruguay | URY | South | 54.32 | 27.4 | 0 | 26.91 | 50.45 | 0 | 49.55 |
| Venezuela | VEN | South | 36.56 | 27.4 | 0 | 9.16 | 74.95 | 0 | 25.05 |
| South Africa | ZAF | South | 32.98 | 27.4 | 0 | 5.57 | 83.1 | 0 | 16.9 |
| Avg (weighted) | - | - | 30.62 | 27.4 | 0.14 | 3.07 | 89.51 | 0.46 | 10.03 |
| North Avg (weighted) | - | North | 32.07 | 27.4 | 0.67 | 3.99 | 85.45 | 2.1 | 12.45 |
| South Avg (weighted) | - | South | 30.23 | 27.4 | 0 | 2.82 | 90.66 | 0 | 9.34 |
| Avg | - | - | 43.76 | 27.4 | 0.93 | 15.42 | 62.62 | 2.13 | 35.25 |
| North Avg | - | North | 39.50 | 27.4 | 2.55 | 9.54 | 69.39 | 6.47 | 24.15 |
| South Avg | - | South | 46.22 | 27.4 | 0 | 18.81 | 59.29 | 0 | 40.71 |
| Global Welfare | - | - | 35.95 | - | - | - | - | - | - |
| North Welfare | - | North | 36.21 | - | - | - | - | - | - |
| South Welfare | - | South | 35.23 | - | - | - | - | - | - |

There is considerable variation in the ACR components across countries. The weighted average of the ACR gains among the North countries is $4.0\%$, larger than that among the South countries ($2.8\%$). The ACR theory suggests that gains from trade would be larger when the domestic expenditure shares are smaller, which tends to be the case if the country sizes or trade barriers are small. Hence, the larger ACR gains enjoyed by the North countries may be explained by the fact that the South countries' average population size is more than twice as large as the North countries' and that the North countries' trade barriers are on average smaller than the South ones'.

Recall from Section 4.2 that the income-gains component is zero by construction for the South countries. There is a large variation across the North countries in this component, but the importance of this component is only $2.1\%$ for the North countries. Some countries (seven out of 19 North countries) even suffer a loss in this term. A detailed analysis of the changes in the components of (34) reveals that most of the variation comes from the second component, $\dfrac{\beta\mu_i}{\theta\alpha}\sum_{n=1}^{I}\frac{w_{n,\nu}}{w_{i,\nu}}L_{n,\nu}$, which can be divided into a sum over the South countries and a sum over the North ones. A general effect of such a trade shock is that the South countries' wages relative to a North country $i$ rises; thus, the first sum generally benefits North countries in this component. Those North countries that suffer from income-gains loss (such as the US) are those whose relative wage to other North countries rises so much such that this negative relative-wage effect in the second sum dominates the positive relative-wage effect in the first sum. Despite these negative income gains for some North countries, both the weighted and unweighted averages of this component remain positive.

In short, the growth rate component plays a dominant role, followed by the ACR gains, and the income-gains component is the smallest among the three, though they are not negligible for quite a few North countries.

As mentioned, the simple decomposition exercises above do not reflect the *true dynamic gains* from trade because the GPT innovation ($R$) also affects wages that affect the IG and the ACR measures. To rectify this problem, we obtain pure static gains via a counterfactual exercise by shutting down the sole growth driver from GPT innovation. Specifically, this is by setting $\kappa = 0$, and hence $R = 0$ in equilibrium. In this case, GR = 1 by construction, but IG and ACR are also adjusted to exclude the R&D effect (see [35]) – so these components are generally different from those reported in Table 2. By subtracting pure static gains from the total gains, we obtain the true dynamic gains from trade, and the dynamic share can then be calculated by (36). The results are summarized in Table 3.

Unlike in the simple decomposition, the dynamic gains are not a constant, varying from $24.2\%$ to $27.4\%$. The weighted average of the dynamic share is $89.1\%$, with the re-

## Table 3: Dynamic vs. Static Gains (moving from autarky to trade)

| Country | iso3 | North/South | Total gains from trade (%) | Static gains from trade (%) | Dynamic Gains from Trade (%) | Dynamic share (%) |
|---|---|---|---|---|---|---|
| Australia | AUS | North | 33.20 | 6.71 | 26.49 | 79.79 |
| Belgium | BEL | North | 59.94 | 33.37 | 26.57 | 44.32 |
| Canada | CAN | North | 39.34 | 12.68 | 26.66 | 67.77 |
| Switzerland | CHE | North | 44.71 | 17.70 | 27.01 | 60.41 |
| Germany | DEU | North | 34.28 | 7.32 | 26.96 | 78.65 |
| Denmark | DNK | North | 43.59 | 17.10 | 26.48 | 60.76 |
| Spain | ESP | North | 37.13 | 10.79 | 26.35 | 70.95 |
| Finland | FIN | North | 46.01 | 19.77 | 26.24 | 57.03 |
| France | FRA | North | 41.98 | 15.96 | 26.01 | 61.97 |
| United Kingdom | GBR | North | 31.32 | 4.04 | 27.28 | 87.09 |
| Greece | GRC | North | 56.63 | 32.45 | 24.18 | 42.70 |
| Ireland | IRL | North | 44.91 | 17.63 | 27.28 | 60.74 |
| Italy | ITA | North | 31.75 | 4.60 | 27.15 | 85.51 |
| Japan | JPN | North | 28.06 | 0.81 | 27.25 | 97.13 |
| Netherlands | NLD | North | 45.44 | 19.24 | 26.20 | 57.65 |
| Norway | NOR | North | 35.74 | 8.43 | 27.31 | 76.41 |
| Singapore | SGP | North | 34.89 | 7.90 | 27.00 | 77.37 |
| Sweden | SWE | North | 35.07 | 7.81 | 27.26 | 77.74 |
| United States | USA | North | 26.42 | -0.25 | 26.68 | 100.96 |
| Argentina | ARG | South | 33.10 | 5.70 | 27.40 | 82.77 |
| Bosnia and Herzegovina | BIH | South | 102.68 | 75.45 | 27.22 | 26.51 |
| Brazil | BRA | South | 29.13 | 1.73 | 27.40 | 94.06 |
| Chile | CHL | South | 34.85 | 7.45 | 27.39 | 78.61 |
| China | CHN | South | 28.34 | 0.94 | 27.40 | 96.69 |
| Colombia | COL | South | 36.89 | 9.50 | 27.39 | 74.25 |
| Ecuador | ECU | South | 46.46 | 19.08 | 27.37 | 58.93 |
| Guatemala | GTM | South | 56.61 | 29.24 | 27.36 | 48.34 |
| Croatia | HRV | South | 65.89 | 38.60 | 27.29 | 41.41 |
| Hungary | HUN | South | 46.59 | 19.26 | 27.34 | 58.67 |
| India | IDN | South | 28.91 | 1.51 | 27.40 | 94.78 |
| Iran | IRN | South | 36.24 | 8.86 | 27.38 | 75.55 |
| Iceland | ISL | South | 62.50 | 35.20 | 27.30 | 43.68 |
| Jamaica | JAM | South | 81.30 | 53.97 | 27.33 | 33.61 |
| Korea | KOR | South | 30.27 | 2.87 | 27.40 | 90.53 |
| Latvia | LVA | South | 72.04 | 44.76 | 27.28 | 37.86 |
| Mexico | MEX | South | 30.72 | 3.32 | 27.40 | 89.18 |
| Malaysia | MYS | South | 36.24 | 8.85 | 27.39 | 75.59 |
| New Zealand | NZL | South | 33.73 | 6.34 | 27.39 | 81.19 |
| Panama | PAN | South | 53.94 | 26.58 | 27.36 | 50.73 |
| Peru | PER | South | 38.49 | 11.11 | 27.39 | 71.14 |
| Poland | POL | South | 38.40 | 11.04 | 27.36 | 71.26 |
| Portugal | PRT | South | 38.76 | 11.41 | 27.35 | 70.57 |
| Romania | ROU | South | 42.75 | 15.40 | 27.35 | 63.98 |
| Russia | RUS | South | 31.06 | 3.67 | 27.39 | 88.19 |
| Slovenia | SVN | South | 63.33 | 36.04 | 27.29 | 43.09 |
| Thailand | THA | South | 31.95 | 4.55 | 27.40 | 85.75 |
| Trinidad & Tobago | TTO | South | 63.63 | 36.29 | 27.34 | 42.97 |
| Turkey | TUR | South | 34.48 | 7.11 | 27.37 | 79.38 |
| Uganda | UGA | South | 72.12 | 44.81 | 27.31 | 37.87 |
| Uruguay | URY | South | 54.32 | 26.94 | 27.38 | 50.40 |
| Venezuela | VEN | South | 36.56 | 9.18 | 27.39 | 74.90 |
| South Africa | ZAF | South | 32.98 | 5.59 | 27.39 | 83.06 |
| Avg (weighted) | - | - | 30.62 | 3.35 | 27.27 | 89.05 |
| North Avg (weighted) | - | North | 32.07 | 5.29 | 26.78 | 83.51 |
| South Avg (weighted) | - | South | 30.23 | 2.83 | 27.40 | 90.63 |
| Avg | - | - | 43.76 | 16.66 | 27.10 | 61.93 |
| North Avg | - | North | 39.50 | 12.84 | 26.65 | 67.48 |
| South Avg | - | South | 46.22 | 18.86 | 27.36 | 59.20 |

Table 4: Simple Decomposition of Gains from Trade (Comparing Two Scenarios)

| Panel A: From Autarky to Benchmark Trade Costs | | | | |
|---|---|---|---|---|
| | Total Gains (%) | GR Gains (%) | IG Gains (%) | ACR Gains (%) |
| Avg (weighted) | 30.62 | 27.4 | 0.14 | 3.07 |
| North Avg (weighted) | 32.07 | 27.4 | 0.67 | 3.99 |
| South Avg (weighted) | 30.23 | 27.4 | 0 | 2.82 |
| | | Share of GR Gains (%) | Share of IG Gains (%) | Share ACR Gains (%) |
| Avg (weighted) | | 89.51 | 0.46 | 10.03 |
| North Avg (weighted) | | 85.45 | 2.1 | 12.45 |
| South Avg (weighted) | | 90.66 | 0 | 9.34 |
| **Panel B: 25% Reduction in Trade Costs from the Benchmark** | | | | |
| | Total Gains (%) | GR Gains (%) | IG Gains (%) | ACR Gains (%) |
| Avg (weighted) | 3.72 | 0.11 | 0.0 | 3.61 |
| North Avg (weighted) | 3.88 | 0.11 | 0.01 | 3.76 |
| South Avg (weighted) | 3.68 | 0.11 | 0 | 3.57 |
| | | Share of GR Gains (%) | Share of IG Gains (%) | Share ACR Gains (%) |
| Avg (weighted) | | 3.03 | 0.05 | 96.92 |
| North Avg (weighted) | | 2.9 | 0.24 | 96.85 |
| South Avg (weighted) | | 3.06 | 0 | 96.94 |

spective weighted average for the North and South being $83.5\%$ and $90.6\%$. These values are rather similar to the growth-rate component in Table 2, indicating that the general equilibrium effects via GPT innovation $R$ are small. This is intuitive because even though GPT innovation is critical to the overall welfare gains from trade, it accounts for only a small share of the labor force and therefore does not induce large effects via wages and trade. Thus, the growth-rate gains actually provide a reasonably good approximation to the dynamic gains.

### 6.1.2 Gains from trade liberalization

To implement the second scenario, a $25\%$ reduction in estimated trade costs is computed by $\tau'_{n,i} = 0.75 \times (\tau_{n,i} - 1) + 1$. We summarize the results of simple decomposition in Panel B of Table 4. For the ease of comparison, Panel A of Table 4 presents a similar summary for the first scenario (autarky to estimated trade costs).

In this scenario, the growth rate of real income increases from $1.8977\%$ under benchmark to $1.8989\%$. The weighted average of the total gains and growth-rate gains are $3.7\%$ and $0.11\%$, respectively, whereas the same numbers are $30.6\%$ and $27.4\%$ in the first scenario. The fact that the total gains are smaller is intuitive, as the changes in trade costs are smaller than in the first scenario. However, the decomposition exhibits an interesting pattern. The weighted average of the share of the growth rate component is drastically reduced from $89.5\%$ to $3.0\%$.

To understand why the share of the growth-rate component is larger when trade costs are larger, recall our theoretical analysis based on the two-country model in Section 3.2, which shows that trade promotes GPT innovation. The key mechanism can be observed from the second equality of (23). For a given $R$, the right-hand side is fixed, whereas the left-hand side changes because of the changes in the North-South wage ratio $w$; thus, the BGP equilibrium GPT R&D effort $R$ critically depends on the wage gap. Under a larger trade cost, the wage gap $w$ is larger. Moving from autarky to the benchmark trade costs thus reduces the wage gap $w$ drastically and results in a larger increase in $R$ compared with the second scenario. In other words, when trade barriers are high, there is less incentive to conduct GPT innovation because of the smaller market size in terms of the North's labor. Put differently, the ratio of marginal benefit to the marginal cost of GPT innovation becomes smaller when the trade barriers are high.

Second, the ACR gains are generally much larger in the second scenario than in the first. The weighted average of this component and its share in the total gains are $3.6\%$ and $96.9\%$, respectively, whereas the same numbers in the first scenario are $3.0\%$ and $10.0\%$. The intuition is that by Shephard's Lemma, the ACR statistic reflects essentially a direct effect of trade costs on prices, which operates mostly through imported goods. Compared with the first scenario, trade costs are lower in the second one, and thus demand for imported goods tends to be larger, amplifying the direct price effect. In addition, the South's wages also amplify the direct effect of trade costs on prices because they reflect the marginal costs in the South. As trade costs are lower in the second scenario, the South's wages tend to be closer to the North's, amplifying the direct price effect and resulting in a larger ACR statistic.

Table 5 reports the true dynamic gains for both scenarios. For the second scenario and on weighted-average terms, the dynamic share is $0.107\%$, which is rather close to the above-mentioned share of the growth rate component. Similar to the first scenario, this indicates the general equilibrium effects of GPT innovation effort via trade and wages are small.

## 6.2 The Role of Intermediate Inputs

This paper incorporates intermediate inputs in the production process in a "roundabout" fashion as in Krugman and Venables (1995) and Eaton and Kortum (2002). We do not incorporate more full-fledged approaches developed in the recent trade literature in order to make the interactions among intermediate inputs, trade, and dynamics clearer. For this

Table 5: Dynamic vs Static Gains (Comparing Two Scenarios)

| | Total gains (%) | Static gains (%) | Dynamic gains (%) | Dynamic share (%) |
|---|---|---|---|---|
| **Panel A: From Autarky to Benchmark Trade Costs** | | | | |
| Avg (weighted) | 30.62 | 3.35 | 27.27 | 89.05 |
| North Avg (weighted) | 32.07 | 5.29 | 26.78 | 83.51 |
| South Avg (weighted) | 30.23 | 2.83 | 27.4 | 90.63 |
| **Panel B: 25% Reduction in Trade Costs from the Benchmark** | | | | |
| | Total gains (%) | Static gains (%) | Dynamic gains (%) | Dynamic share (%) |
| Avg (weighted) | 3.722 | 3.614 | 0.107 | 2.883 |
| North Avg (weighted) | 3.882 | 3.764 | 0.118 | 3.049 |
| South Avg (weighted) | 3.678 | 3.574 | 0.104 | 2.836 |

purpose, we compare the benchmark results with an alternative model where there are no intermediate inputs, i.e., $\alpha = 1$. The model is recalibrated.

The results are presented in Panel B of Table 6, and Panel A replicates the benchmark result for ease of comparison. We report only the first scenario, as the pattern of larger dynamic gains in the first scenario than the second remains. We report only the simple decomposition (hence omit reporting the true dynamic gains) because the growth-rate gains still approximate the true dynamic gains well in this case.

Compared with the benchmark case, the ACR gains are reduced, and the growth-rate gains are increased when production does not require intermediate inputs. The increases in the growth-rate gains are much more than the reduction in the ACR gains, and hence the total gains and the dynamic share both increase. That the presence of intermediate inputs increases the ACR gains is well-understood in the literature. ACR statistic hinges on the trade flows, and the presence of intermediates increases trade flows and the importance of trade barriers. But why do the growth-rate gains increase so much?

The answer lies in how the recalibration changes the parameters. Following the procedure of quantification in Section 5, the recalibrated model features a higher technology gap between the North and South countries. The average $t_{i0}$ of the North countries is $14.7$ times larger than that of the South countries in the benchmark; this ratio grows to be $87.1$ in the model without intermediates. The recalibration also renders higher innovation parameters $(\kappa, \epsilon) = (7.46, 0.30)$, compared with $(2.04, 0.20)$. The higher technology gap implies a larger wage gap, which is instrumental to the dynamic gains, as illustrated in Section 3.2. Larger innovation parameters imply faster GPT innovation, which also contributes to larger dynamic gains.

So, we need to understand how the recalibration rationalizes the data targets. First, observe (37–39). With intermediates, the labor share $\alpha < 1$, and this allows countries to

Table 6: The Role of Intermediate Inputs

| Panel A: From Autarky to Benchmark Trade Costs (with Intermediates) | | | | |
|---|---|---|---|---|
| | Total Gains (%) | GR Gains (%) | IG Gains (%) | ACR Gains (%) |
| Avg (weighted) | 30.62 | 27.4 | 0.14 | 3.07 |
| North Avg (weighted) | 32.07 | 27.4 | 0.67 | 3.99 |
| South Avg (weighted) | 30.23 | 27.4 | 0 | 2.82 |
| | | Share of GR Gains (%) | Share of IG Gains (%) | Share ACR Gains (%) |
| Avg (weighted) | | 89.51 | 0.46 | 10.03 |
| North Avg (weighted) | | 85.45 | 2.1 | 12.45 |
| South Avg (weighted) | | 90.66 | 0 | 9.34 |
| **Panel B: From Autarky to Benchmark Trade Costs (without Intermediates)** | | | | |
| | Total Gains (%) | GR Gains (%) | IG Gains (%) | ACR Gains (%) |
| Avg (weighted) | 39.9 | 38.12 | 0.07 | 1.71 |
| North Avg (weighted) | 40.68 | 38.12 | 0.32 | 2.24 |
| South Avg (weighted) | 39.69 | 38.12 | 0 | 1.57 |
| | | Share of GR Gains (%) | Share of IG Gains (%) | Share ACR Gains (%) |
| Avg (weighted) | | 95.55 | 0.17 | 4.29 |
| North Avg (weighted) | | 93.71 | 0.78 | 5.51 |
| South Avg (weighted) | | 96.05 | 0 | 3.95 |

source from other countries and hence to utilize the labor forces and technology in other countries. This tends to smooth the differences in the production costs $C_i$. If international sourcing is shut down ($\alpha = 1$), $C_i = w_i^{data}$ and hence the differences in the production costs are enlarged. Because the wage levels in the North countries are higher than those in the South and the exporter fixed effects from the gravity-equation estimation are unaffected, the technology gap between the North and the South must become larger to rationalize the exporter fixed effects and data wages.

Second, to understand why $\kappa$ and $\varepsilon$ increase, first note that with $w_\nu^R$ normalized to 1, $P_\nu \equiv \prod_{i=1}^{I} P_{i,\nu}^{\mu_i}$, and (31), the R&D multiplier in (29) is proportional to $\sum_{i=1}^{I} X_{i,\nu+1} \times (P_\nu/P_{\nu+1}) = \sum_{i=1}^{I} X_{i,\nu+1} \times \gamma^{\frac{1}{\theta\alpha}}$. The presence of intermediate inputs increases gross output $\sum_{i=1}^{I} X_{i,\nu+1}$ and expedites the decline in the price index due to the recursive structure in the roundabout production. Thus, the presence of intermediate inputs implies a faster annual growth rate $\gamma^{\frac{\lambda(R)}{\theta\alpha}} - 1$ compared with the case without intermediates. However, to match the same annual growth rate in the data, the model without intermediate inputs must require a faster arrival rate and hence the increases in $\kappa$ and $\varepsilon$.

## 6.3 Sensitivity Analysis

We conduct the following three sensitivity analyses. First, we check what occurs if the trade elasticity is chosen to be $4$, as suggested by Simonovska and Waugh (2014). Second, we experiment with different sizes of the productivity ladder, $\gamma$. Third, we experiment with different sizes of the GPT's bargaining power parameter, $\beta$. In each case, the model is recalibrated. Table 7 reports the results. Again, the pattern of larger dynamic gains in the first scenario than in the second remains, as well as the pattern that the growth-rate gains approximate the true dynamic gains. Thus, we report only the simple decomposition in the first scenario and put the benchmark results in Panel A for ease of comparison.

The first observation is that the ACR gains and income gains are relatively similar across these cases, and most of the variation in the total gains from trade are explained by the growth-rate component.[22] Adopting $\theta = 4$, a lower value than the benchmark, entails larger ACR gains and much smaller growth-rate gains and total gains. It is well understood in the trade literature that lower trade elasticity entails larger ACR gains. To understand why the growth-rate gains are much smaller, again note that the model annual growth rate $\gamma^{\frac{\lambda(R)}{\theta\alpha}} - 1$ needs to match the same data counterpart. Thus, when the trade elasticity is smaller, the arrival rate parameters $(\kappa, \epsilon)$ have to be smaller. This results in smaller dynamic gains.

For alternative values of $\gamma$, we consider $50\%$ above and below the benchmark ladder size: $\gamma = (\gamma_0 - 1) \times 1.5 + 1$ and $\gamma = (\gamma_0 - 1) \times 0.5 + 1$. The results are that a larger $\gamma$ induces larger growth-rate gains, but only slightly. Note that $\gamma$ affects only the innovation parameters in the calibration procedure, which explains why the static gains are almost intact. A larger $\gamma$ implies smaller $\kappa$ and $\epsilon$ in order to rationalize the same data annual growth rate. However, the direct effect of a larger $\gamma$ slightly dominates the effect of smaller $\kappa$ and $\epsilon$ to entail a slight increase in the growth rate and total gains. The fact that the two effects are opposite to each other explains why the quantitative results are quite robust to the choice of $\gamma$.

For alternative values of $\beta$, we also consider $50\%$ above and below the benchmark value: $\beta = 1.5 \times \beta_0$ and $\beta = 0.5 \times \beta_0$. The result clearly shows that when $\beta$ is larger, the dynamic and total welfare gains become smaller. To see this, observe (32) and note that when $\beta$ increases, the equilibrium $R$ tends to increase as the right-hand side of (32) is an increasing function of $R$. This is intuitive as a larger bargaining power for the GPT firm incentivizes the GPT R&D. The fact that the model annual real income growth rate must match the data counterpart implies that the innovation parameters must decrease compared with the benchmark values, hence resulting in smaller dynamic and total gains from trade.

In summary, even in all these large-scale sensitivity exercises, total gains from trade

[22]Note that the analysis of the intermediate goods in the previous subsection does not show the same pattern, as shutting down intermediate inputs reduces the ACR gains substantially.

Table 7: Sensitivity Analysis

| **Panel A: From Autarky to Benchmark Trade Costs (Benchmark)** | | | | |
|---|---|---|---|---|
| | Total Gains (%) | GR Gains (%) | IG Gains (%) | ACR Gains (%) |
| Avg (weighted) | 30.62 | 27.4 | 0.14 | 3.07 |
| | | Share of GR Gains (%) | Share of IG Gains (%) | Share ACR Gains (%) |
| Avg (weighted) | | 89.51 | 0.46 | 10.03 |
| **Panel B: From Autarky to Benchmark Trade Costs ($\theta = 4$)** | | | | |
| | Total Gains (%) | GR Gains (%) | IG Gains (%) | ACR Gains (%) |
| Avg (weighted) | 11.79 | 8.05 | 0.42 | 3.32 |
| | | Share of GR Gains (%) | Share of IG Gains (%) | Share ACR Gains (%) |
| Avg (weighted) | | 68.29 | 3.56 | 28.14 |
| **Panel C: From Autarky to Benchmark Trade Costs ($\gamma = (\gamma_0 - 1) \times 1.1 + 1$)** | | | | |
| | Total Gains (%) | GR Gains (%) | IG Gains (%) | ACR Gains (%) |
| Avg (weighted) | 30.76 | 27.54 | 0.14 | 3.07 |
| | | Share of GR Gains (%) | Share of IG Gains (%) | Share ACR Gains (%) |
| Avg (weighted) | | 89.55 | 0.46 | 9.98 |
| **Panel D: From Autarky to Benchmark Trade Costs ($\gamma = (\gamma_0 - 1) \times 0.9 + 1$)** | | | | |
| | Total Gains (%) | GR Gains (%) | IG Gains (%) | ACR Gains (%) |
| Avg (weighted) | 30.48 | 27.27 | 0.14 | 3.07 |
| | | Share of GR Gains (%) | Share of IG Gains (%) | Share ACR Gains (%) |
| Avg (weighted) | | 89.46 | 0.47 | 10.08 |
| **Panel E: From Autarky to Benchmark ($\beta = 0.5 \times \beta_0$)** | | | | |
| | Total Gains (%) | GR Gains (%) | IG Gains (%) | ACR Gains (%) |
| Avg (weighted) | 49.23 | 46.08 | 0.04 | 3.11 |
| | | Share of GR Gains (%) | Share of IG Gains (%) | Share ACR Gains (%) |
| Avg (weighted) | | 93.6 | 0.08 | 6.32 |
| **Panel F: From Autarky to Benchmark ($\beta = 1.5 \times \beta_0$)** | | | | |
| | Total Gains (%) | GR Gains (%) | IG Gains (%) | ACR Gains (%) |
| Avg (weighted) | 18.48 | 15.18 | 0.25 | 3.06 |
| | | Share of GR Gains (%) | Share of IG Gains (%) | Share ACR Gains (%) |
| Avg (weighted) | | 82.13 | 1.33 | 16.54 |

remain sizable throughout, all above 10%. Moreover, the dynamic component continues to be most dominant, accounting for at least two thirds of the total gains.

# 7 Conclusion

This paper develops a general equilibrium model of trade, innovation, and growth based on the evolution of general purpose technology. We highlight the channel through which trade liberalization increases the GPT R&D and hence the long-run economic growth. It is fundamentally a market-size effect as an R&D multiplier, and the North-South structure plays a key role.

The welfare formula is a product of a growth-rate effect, an income-gains effect, and the ACR statistic. In our quantitative exercises, we find large total and dynamic gains from trade. We also find that the share of dynamic gains in total gains is larger when the initial trade costs are higher. Throughout different scenarios, we find that the dynamic share is well approximated by the growth-rate effect, suggesting that shutting down dynamics does not entail large general equilibrium effects on wages and revenues.

It is clear that GPT innovation in our model can be combined with other types of trade models with imperfect competition in which the profits/rents of firms can be shared with the GPT firms in a similar fashion. Hence, our model can be modified to incorporate other trade mechanisms that may be of interest, such as variety expansion *a là* Krugman (1980) or selection *a là* Melitz (2003). Nevertheless, an advantage of choosing the BEJK trade structure is that as the GPT advances spawn more innovations on the differentiated products, the BEJK structure implies the creative destruction (e.g., entry and exit of firms) as documented by Jovanovic and Rousseau (2005). Our model mechanism can also be generalized to incorporate social learning or international technology diffusion, such as Eaton and Kortum (1999), Buera and Oberfield (2020), and Perla, Tonetti and Waugh (2021). This may be an interesting avenue for future research because it will enable the decomposition of dynamic welfare gains from trade into various technology-related mechanisms.

# References

**Aghion, Philippe, and Peter Howitt.** 1992. "A Model of Growth Through Creative Destruction." *Econometrica*, 60(2): 323.

**Aghion, Philippe, Antonin Bergeaud, Timo Boppart, Peter J. Klenow, and Huiyu Li.** 2019. "Missing Growth from Creative Destruction." *American Economic Review*, 109(8): 2795–2822.

**Aghion, Philippe, Peter Howitt, and Giovanni L Violante.** 2002. "General purpose technology and wage inequality." *Journal of Economic Growth*, 7: 315–345.

**Alessandria, George, Horag Choi, and Kim J. Ruhl.** 2021. "Trade adjustment dynamics and the welfare gains from trade." *Journal of International Economics*, 131: 103458.

**Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare.** 2012. "New Trade Models, Same Old Gains?" *American Economic Review*, 102(1): 94–130.

**Atkeson, Andrew, and Ariel Tomás Burstein.** 2010. "Innovation, Firm Dynamics, and International Trade." *Journal of Political Economy*, 118(3): 433–484.

**Baldwin, Richard E.** 1992. "Measurable dynamic gains from trade." *Journal of Political Economy*, 100(1): 162–174.

**Bernard, Andrew B, Jonathan Eaton, J. Bradford Jensen, and Samuel Kortum.** 2003. "Plants and Productivity in International Trade." *American Economic Review*, 93(4): 1268–1290.

**Bloom, Nicholas, Paul Romer, Stephen J Terry, and John Van Reenen.** 2020. "Trapped Factors and China's Impact on Global Growth." *The Economic Journal*, 131(633): 156–191.

**Bond, Eric W., Ping Wang, and Chong K. Yip.** 1996. "A General Two-Sector Model of Endogenous Growth with Human and Physical Capital: Balanced Growth and Transitional Dynamics." *Journal of Economic Theory*, 68(1): 149–173.

**Bresnahan, Timothy F., and M. Trajtenberg.** 1995. "General purpose technologies 'Engines of growth'?" *Journal of Econometrics*, 65(1): 83–108.

**Buera, Francisco J., and Ezra Oberfield.** 2020. "The Global Diffusion of Ideas." *Econometrica*, 88(1): 83–114.

**Cai, Jie, Nan Li, and Ana Maria Santacreu.** 2022. "Knowledge Diffusion, Trade, and Innovation across Countries and Sectors." *American Economic Journal: Macroeconomics*, 14(1): 104–145.

**Caliendo, Lorenzo, Robert C Feenstra, John Romalis, and Alan M Taylor.** 2015. "Tariff reductions, entry, and welfare: Theory and evidence for the last two decades." National Bureau of Economic Research Working Paper No. 21768.

**Crafts, Nicholas.** 2004. "Steam as a General Purpose Technology: A Growth Accounting Perspective." *The Economic Journal*, 114(495): 338–351.

**De Loecker, Jan, Jan Eeckhout, and Gabriel Unger.** 2020. "The Rise of Market Power and the Macroeconomic Implications*." *The Quarterly Journal of Economics*, 135(2): 561–644.

**Eaton, Jonathan, and Samuel Kortum.** 1999. "International Technology Diffusion: Theory and Measurement." *International Economic Review*, 40(3): 537–570.

**Eaton, Jonathan, and Samuel Kortum.** 2002. "Technology, Geography, and Trade." *Econometrica*, 70(5): 1741–1779.

**Egger, Peter, and Nicole Loumeau.** 2018. "The Economic Geography of Innovation." *CEPR Discussion Papers*, DP13338.

**Fieler, Ana Cecilia.** 2011. "Nonhomotheticity and bilateral trade: Evidence and a quantitative explanation." *Econometrica*, 79(4): 1069–1101.

**Grossman, Gene M., and Elhanan Helpman.** 1991. "Trade, knowledge spillovers, and growth." *European Economic Review*, 35(2–3): 517–526.

**Head, Keith, and Thierry Mayer.** 2014. "Gravity Equations: Workhorse, Toolkit, and Cookbook." *Handbook of International Economics*, 131–195. Elsevier.

**Helpman, Elhanan, and Manuel Trajtenberg.** 1994. "A Time to Sow and a Time to Reap: Growth Based on General Purpose Technologies." National Bureau of Economic Research Working Paper No. 4854.

**Hsieh, Chang-Tai, Peter J. Klenow, and Ishan Nath.** 2023. "A Global View of Creative Destruction." *Journal of Political Economy Macroeconomics*, 1(2): 243–275.

**Impullitti, Giammario, and Omar Licandro.** 2017. "Trade, Firm Selection and Innovation: The Competition Channel." *The Economic Journal*, 128(608): 189–229.

**Jovanovic, Boyan, and Peter L. Rousseau.** 2005. "General Purpose Technologies." *Handbook of Economic Growth*, 1181–1224. Elsevier.

**Klette, Tor Jakob, and Samuel Kortum.** 2004. "Innovating Firms and Aggregate Innovation." *Journal of Political Economy*, 112(5): 986–1018.

**Krugman, Paul.** 1980. "Scale Economies, Product Differentiation, and the Pattern of Trade." *The American Economic Review*, 70(5): 950–959.

**Krugman, Paul, and Anthony J Venables.** 1995. "Globalization and the Inequality of Nations." *Quarterly Journal of Economics*, 110(4): 857–880.

**Melitz, Marc J.** 2003. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica*, 71(6): 1695–1725.

**Perla, Jesse, Christopher Tonetti, and Michael E. Waugh.** 2021. "Equilibrium Technology Diffusion, Trade, and Growth." *American Economic Review*, 111(1): 73–128.

**Ravikumar, B., Ana Maria Santacreu, and Michael Sposi.** 2019. "Capital accumulation and dynamic gains from trade." *Journal of International Economics*, 119: 93–110.

**Rivera-Batiz, Luis A., and Paul M. Romer.** 1991. "International trade with endogenous technological change." *European Economic Review*, 35(4): 971–1001.

**Sampson, Thomas.** 2015. "Dynamic Selection: An Idea Flows Theory of Entry, Trade, and Growth." *Quarterly Journal of Economics*, 131(1): 315–380.

**Segerstrom, Paul S, Thirumalai CA Anant, and Elias Dinopoulos.** 1990. "A Schumpeterian model of the product life cycle." *American Economic Review*, 1077–1091.

**Simonovska, Ina, and Michael E. Waugh.** 2014. "The elasticity of trade: Estimates and evidence." *Journal of International Economics*, 92(1): 34–50.

**Somale, Mariano.** 2021. "Comparative Advantage in Innovation and Production." *American Economic Journal: Macroeconomics*, 13(3): 357–396.

# Appendix

In the appendix, we provide detailed mathematical derivations of the benchmark model and the extensions, as well as detailed calibration strategies.

# A Joint Distribution of Top Two Productivities

The c.d.f. of a Fréchet distribution with scaling parameter $S$ and shape parameter $\theta$ is given by $F(z) = e^{-Sz^{-\theta}}$. Following standard procedures, the joint distribution of the top two order statistics $Z_1$ and $Z_2$ from $N$ draws from the Fréchet distribution is

$$\Pr\left[Z_1 \leq z_1, Z_2 \leq z_2\right] = e^{-NSz_2^{-\theta}} + N\left[e^{-S\left(z_1^{-\theta}-z_2^{-\theta}\right)} - 1\right] e^{-NSz_2^{-\theta}}. \tag{A.1}$$

In our model, $S = 1$ since $F_i^{\text{draw}}(z) = e^{-z^{-\theta}}$ and $N = \widetilde{T}_{i,\nu} = M_{i,\nu}\gamma^{\nu}\tilde{t}_{i,0}$. Recall from the main text that $\tilde{t}_{i,0} = t_{i,0}K$ and $T_{i,\nu} = \widetilde{T}_{i,\nu}/K$. Plug these into (A.1), and we have

$$\begin{aligned}\Pr\left[Z_1 \leq z_1, Z_2 \leq z_2\right] &= e^{-\widetilde{T}_{i,\nu}z_2^{-\theta}} + \widetilde{T}_{i,\nu}\left[e^{-\left(z_1^{-\theta}-z_2^{-\theta}\right)} - 1\right] e^{-\widetilde{T}_{i,\nu}z_2^{-\theta}} \\ &= e^{-T_{i,\nu}Kz_2^{-\theta}} + T_{i,\nu}K\left[e^{-\left(z_1^{-\theta}-z_2^{-\theta}\right)} - 1\right] e^{-T_{i,\nu}Kz_2^{-\theta}}.\end{aligned}$$

Consider the joint distribution of the re-scaled top two order statistics $K^{-1/\theta}Z_1$ and $K^{-1/\theta}Z_2$, and we have

$$\begin{aligned}\Pr\left[K^{-1/\theta}Z_1 \leq z_1, K^{-1/\theta}Z_2 \leq z_2\right] &= \Pr\left[Z_1 \leq K^{1/\theta}z_1, Z_2 \leq K^{1/\theta}z_2\right] \\ &= e^{-T_{i,\nu}z_2^{-\theta}} + \frac{e^{-\left(K^{-1}z_1^{-\theta}-K^{-1}z_2^{-\theta}\right)} - 1}{K^{-1}} T_{i,\nu} e^{-T_{i,\nu}z_2^{-\theta}}.\end{aligned}$$

As $\lim_{K\to\infty} \frac{e^{-\left(K^{-1}z_1^{-\theta}-K^{-1}z_2^{-\theta}\right)}-1}{K^{-1}} = z_2^{-\theta} - z_1^{-\theta}$, we have

$$\lim_{K\to\infty} \Pr\left[K^{-1/\theta}Z_1 \leq z_1, K^{-1/\theta}Z_2 \leq z_2\right] = \left[1 + T_{i,\nu}\left(z_2^{-\theta} - z_1^{-\theta}\right)\right] e^{-T_{i,\nu}z_2^{-\theta}}.$$

# B Mathematical Details of the Quantitative Model

## B.1 Equilibrium Conditions

The evolution of the number of ideas follows

$$T_{i,\nu} = M_{i,\nu}\gamma^{\nu}t_{i,0}. \tag{B.2}$$

From the production function (24), the cost for one unit of output is given by

$$C_{i,\nu} \equiv w_{i,\nu}^{\alpha} P_{i,\nu}^{1-\alpha}. \tag{B.3}$$

The BEJK structure implies that trade share and price index are given by

$$\pi_{ni,\nu} = \frac{T_{i,\nu}(C_{i,\nu}\tau_{ni})^{-\theta}}{\sum_k^I T_{k,\nu}(C_{k,\nu}\tau_{nk})^{-\theta}} \tag{B.4}$$

$$P_n = \eta \left[ \sum_{i=1}^{I} T_{i,\nu} \left(C_{i,\nu} d_{ni}\right)^{-\theta} \right]^{-\frac{1}{\theta}}, \tag{B.5}$$

where $\eta \equiv \left\{ \left[ 1 + \frac{\sigma-1}{1+\theta-\sigma} \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \right] \Gamma\left(\frac{1+2\theta-\sigma}{\theta}\right) \right\}^{\frac{1}{1-\sigma}}$ with $\Gamma$ denoting the gamma function. An innovating firm's cost minimization problem is

$$\min \sum_{i \in \text{North}} \frac{w_{i,\nu}}{P_{i,\nu}} R_{i,\nu}$$

$$s.t. \ \frac{\prod_{i=1}^{I} R_{i,\nu}^{\mu_i}}{\prod_{i=1}^{I} \mu_i^{\mu_i}} \geq R_{\nu}.$$

Let $w_{\nu}^{R} \equiv \prod_{i=1}^{I} w_{i,\nu}^{\mu_i}$ and $P_{\nu} \equiv \prod_{i=1}^{I} P_{i,\nu}^{\mu_i}$. The aggregate price index for the composite input $R_{\nu}$ is therefore given by

$$\frac{w_{\nu}^{R}}{P_{\nu}} = \prod_{i=1}^{I} \left( \frac{w_i}{P_{i,\nu}} \right)^{\mu_i}. \tag{B.6}$$

In equilibrium, the demand for researchers in each country satisfies

$$R_{i,\nu} \frac{w_{i,\nu}}{P_{i,\nu}} = \mu_i \frac{w_{\nu}^{R}}{P_{\nu}} R_{\nu}. \tag{B.7}$$

Note that for the South countries, $R_{i,\nu} = 0$ as $\mu_i = 0$. For the labor market, follow the same procedure as that in Section 2.4, one obtains similar labor allocation given $\{R_{i,\nu}\}_{i=1}^{I}$:

$$M_{i,\nu} = \frac{1-\beta}{1-\beta+\theta\alpha}(N_i - R_{i,\nu}) \tag{B.8}$$

$$L_{i,\nu} = \frac{\theta\alpha}{1-\beta+\theta\alpha}(N_i - R_{i,\nu}). \tag{B.9}$$

Let $E_{n,\nu}$ be the total expenditure on differentiated products in country $n$, and thus the total sales of differentiated products produced by country $i$ $X_{i,\nu}$ must satisfy $X_{i,\nu} =$

$\sum_{n=1}^{I} \pi_{ni,\nu} E_{n,\nu}$. Then, BEJK Result 4 ($w_{i,\nu} L_{i,\nu} = \alpha\theta X_{i,\nu} / (1+\theta)$) implies that

$$w_{i,\nu} L_{i,\nu} = \frac{\alpha\theta}{1+\theta} \sum_{n=1}^{I} \pi_{ni,\nu} E_{n,\nu}. \tag{B.10}$$

A country's income consists of entrepreneurs' payoffs, wage payments to production workers, and potential royalty revenue. As a fraction $\mu_i$ of GPT's royalty revenue is given to the branch at country $i$, country $i$'s income is given by

$$Y_{i,\nu} = \frac{1-\beta}{1+\theta} X_{i,\nu} + w_{i,\nu} L_{i,\nu} + \frac{\beta\mu_i}{1+\theta} \sum_{n=1}^{I} X_{n,\nu}. \tag{B.11}$$

Total expenditure consists of consumption expenditure on final goods and the expenditure on intermediate inputs:

$$\begin{aligned} E_{n,\nu} &= Y_{n,\nu} + \frac{\theta(1-\alpha)}{1+\theta} X_{n,\nu} \\ &= \frac{1-\beta+\theta}{\theta\alpha} w_{n,\nu} L_{n,\nu} + \frac{\beta\mu_n}{\theta\alpha} \sum_{i=1}^{I} w_{i,\nu} L_{i,\nu}, \end{aligned} \tag{B.12}$$

where the second equality make uses of (B.11) together with BEJK Result 4. Plugging (B.12) into (B.10), we obtain

$$w_{i,\nu} L_{i,\nu} = \sum_{n=1}^{I} \pi_{n,i,\nu} \left( \frac{1-\beta+\theta}{1+\theta} w_{n,\nu} L_{n,\nu} + \frac{\beta\mu_n}{1+\theta} \sum_{i=1}^{I} w_{i,\nu} L_{i,\nu} \right). \tag{B.13}$$

Conditioned on a sequence of $\{R_\nu\}_{\nu=0}^{\infty}$ and given a sequence of wages and research labor $\{w_{i,\nu}, R_{i,\nu}\}_{i,\nu}$, labor allocation and evolution of the technology stock $\{M_{i,\nu}, L_{i,\nu}, T_{i,\nu}\}$ are pinned down by (B.8), (B.9), and (B.2), unit cost and price index $\{C_{i,\nu}, P_{i,\nu}\}_{i,\nu}$ are solved from (B.3) and (B.5), and trade shares $\{\pi_{ni,\nu}\}_{n,i,\nu}$ are given by (B.4). The fixed point of $\{R_{i,\nu}\}_{i,\nu}$ is solved from (B.6) and (B.7). Then, the fixed point of wages $\{w_{i,\nu}\}_{i,\nu}$ can be solved from (B.13). Then, aggregate sales of differentiated products and national income are given by BEJK Result 4 and (B.11). At this stage, every variable is solved given a sequence of $\{R_\nu\}_{\nu=0}^{\infty}$ subject to a choice of numeraire for the general equilibrium at each generation. Without loss of generality, we set $w_\nu^R = 1$ for all $\nu$.

The value of an innovation at generation $\nu + 1$ is

$$V_{\nu+1} = \frac{\beta \sum_{i=1}^{I} X_{i,\nu+1}}{(1+\theta)[r + \lambda(R_{\nu+1})]}.$$

Assume that the GPT innovating firms are jointly owned by people in the North countries with the share of owners in country $i$ is given by $\mu_i$. So, the real value of a new innovation should be deflated by $P_{\nu+1} = \prod_{i=1}^{I} P_{i,\nu+1}^{\mu_i}$.[23] Thus, a GPT innovator's problem is

$$\max_{R_\nu} \frac{\lambda(R_\nu) V_{\nu+1}}{P_{\nu+1}} - \frac{w_\nu^R}{P_\nu} R_\nu.$$

Similar to (10), the first-order condition can be written as

$$\frac{\beta}{1+\theta} \frac{\left(\sum_{i=1}^{I} X_{i,\nu+1}\right)/P_{\nu+1}}{w_\nu^R/P_\nu} = \frac{r+\lambda(R_{\nu+1})}{\lambda'(R_\nu)}, \tag{B.14}$$

where the left-hand side is the R&D multiplier. This last equation is the condition for $\{R_\nu\}_{\nu=0}^{\infty}$.

## B.2 Balanced Growth Path and Growth Rates

In this subsection, we characterize the equilibrium conditions along the BGP and growth rates. As mentioned, we set $w_\nu^R = 1$ (but it is sometimes written out for easier interpretation); thus, the nominal variables $\{w_{i,\nu}, X_{i,\nu}, Y_{i,\nu}\}$ and the labor allocation variables $\{R_\nu, R_{i,\nu}, L_{i,\nu}, M_{i,\nu}\}$ are both stationary along the BGP. As the technology stock $T_{i,\nu}$ grows by a factor $\gamma$ for each new innovation, unit costs and price indices generally decline over time. The declining unit costs and price indices are detrended so that the detrended variables are stationary along the BGP:

$$\tilde{C}_i \equiv \frac{C_{i,\nu}}{\left(\sum_k \gamma^\nu t_{k,0}\right)^{-\frac{1-\alpha}{\alpha\theta}}}, \qquad \tilde{P}_i \equiv \frac{P_{i,\nu}}{\left(\sum_k \gamma^\nu t_{k,0}\right)^{-\frac{1}{\alpha\theta}}}.$$

Similar to the previous subsection, we first consider the equilibrium given $R$, (normalized) wages, and research labor $\{w_i, R_i\}_i$, the following equilibrium conditions allows us

[23] Alternatively, one can assume that the MNC owner lives in a particular country $i$, and thus, the problem can be written as

$$\max_{R_\nu} \frac{\lambda(R_\nu) V_{\nu+1}}{P_{i,\nu+1}} - \frac{w_\nu^R R_\nu}{P_\nu}.$$

It is readily verified that this alternative formulation is qualitatively similar to the benchmark formulation in (28).

to solve for the stationary variables $\{M_i, L_i, \tilde{C}_i, \tilde{P}_i, \pi_{ni}\}$:

$$\begin{aligned}
M_i &= \frac{1-\beta}{1-\beta+\theta\alpha}(N_i - R_i) \\
L_i &= \frac{\theta\alpha}{1-\beta+\theta\alpha}(N_i - R_i) \\
\tilde{C}_i &= w_i^{\alpha}\tilde{P}_i^{1-\alpha} \\
\tilde{P}_i &= \eta\left(\sum_{k=1}^{I} t_{k,0}\right)^{\frac{1}{\theta}}\left[\sum_{k=1}^{I} M_k t_{k,0}\left(\tilde{C}_{k,\nu} d_{ik}\right)^{-\theta}\right]^{-\frac{1}{\theta}} \\
\pi_{ni} &= \frac{M_{i,\nu}t_{i,0}(\tilde{C}_{i,\nu}\tau_{ni})^{-\theta}}{\sum_{k=1}^{I} M_{i,\nu}t_{i,0}(\tilde{C}_{k,\nu}\tau_{nk})^{-\theta}},
\end{aligned}$$

which also verify that $\tilde{C}_i$ and $\tilde{P}_i$ are indeed stationary. Then, $\{R_i\}_i$ is solved from

$$R_i w_i = \mu_i \frac{\tilde{P}_i}{\tilde{P}} R,$$

and the fixed point of wages $\{w_i\}$ can be obtained from

$$w_i L_i = \sum_{n=1}^{I} \pi_{ni}\left(\frac{1-\beta+\theta}{1+\theta} w_n L_n + \frac{\beta\mu_n}{1+\theta}\sum_{i=1}^{I} w_i L_i\right).$$

Then, using BEJK Result 4, $X_i = (1+\theta)\, w_i L_i / \alpha\theta$. Following the procedure in the two-country model in Section 3.1, the (negative) growth rate of the price index along the BGP, $g_p$, is given by

$$1 + g_p \equiv \frac{P_{n,\nu+1}}{P_{n,\nu}} = \gamma^{-\frac{1}{\theta\alpha}}. \tag{B.15}$$

Combining the above with (B.14), equilibrium $R$ on the BGP can be solved from

$$\frac{\beta\gamma^{\frac{1}{\theta\alpha}}}{1+\theta}\sum_{i=1}^{I} X_i = \frac{r + \lambda(R)}{\lambda'(R)},$$

where the left-hand is the R&D multiplier. Next, for welfare calculations, we will need to know the growth rate in real income $y_{i,\nu} = Y_{i,\nu}/P_{i,\nu}$, denoted as $g_y$. As $Y_{i,\nu}$ is stationary, (B.15) implies that $1 + g_y \equiv y_{i,\nu+1}/y_{i,\nu} = \gamma^{\frac{1}{\theta\alpha}}$.

# Calibration

## GPT's bargaining power

To calibrate $\beta$, note that this parameter directly affects the profit share of revenue of differentiated-product firms, as well as the royalty payments received by the GPT, which are the GPT's profit. That is, given calibrated $\mu_i$'s, we solve

$$(\text{Profit rate})_{\text{USA}} = \frac{(1-\beta)\frac{X_{\text{USA}}}{1+\theta} + \mu_{\text{USA}} \times \frac{\beta \sum_{n=1}^{I} X_n}{1+\theta}}{X_i + \mu_{\text{USA}} \times \frac{\beta \sum_{n=1}^{I} X_n}{1+\theta}},$$

where we proxy $X_i$ by country $i$'s gross output from the WIOD, averaged across years between 2000-2014. Using the profit rate of $0.154$, we obtain $\beta = 0.074$. The following explains how we obtain the profit rate.

In the data, firm revenues can be divided into variable costs, fixed costs, and profits. There are no fixed costs in our model, and hence there is a tension between the following two views for calibration. The first view is to attribute the fixed costs to be a part of the profits. Note that firm markups are, by definition, the ratio of prices to marginal costs and that De Loecker, Eeckhout and Unger (2020) document that the rise of markups over time is likely due to the rise of fixed costs. This supports viewing fixed costs as part of the profits in our model. The second view is to attribute the fixed costs to be a part of the variable costs, as everything can change in the long run. This view makes the concept of markup closer to "pure profit" or market power. De Loecker, Eeckhout and Unger (2020) compute markups using the Compustat North America for publicly listed firms, and there is an item SG&A in this data accounting for a firm's various overhead costs (such as R&D and advertising, etc.). The SG&A share of sales is $0.178$. Profit as a share of sales is $0.065$. When SG&A is part of the profit, then the profit rate is $0.243$. The simple average between the narrow sense of profit rate, $0.065$, and the broader sense of profit rate, $0.243$, is $0.154$.

## Trade Costs and Relative Technology Stock

Let $E_n$ denote the total expenditure of country $n$ and $E_{ni}$ the expenditure of country $n$ on country $i$'s goods. BEJK Results 1 and 2 imply that the following gravity equation:

$$X_{ni} = \pi_{ni} E_n = \frac{T_i \left(C_i \tau_{ni}\right)^{-\theta}}{\Phi_n} \times E_n.$$

Taking the logarithm of both sides of the above equation

$$\ln(X_{ni}) = \ln(T_i C_i^{-\theta}) + \theta \ln(\tau_{ni}^{-\theta}) - \ln(\Phi_n E_n)$$

Assuming the functional form of trade costs is a function of geography:

$$\ln(\tau_{ni}^{-\theta}) = b_1 \ln(Dist_{ni}) + b_2 \times \text{contig}_{ni} + b_3 \times \text{comlang\_off}_{ni} + e_{ni} \tag{B.16}$$

where $Dist_{ni}$ is the distance between $n$ and $i$, $\text{contig}_{ni}$ the dummy equal to 1 if countries are contiguous, $\text{comlang\_off}_{ni}$ the dummy for whether the two countries share a common official or primary language, and $e_{ni}$ the error term. The empirical specification of gravity equation therefore becomes

$$\ln(X_{ni,t}) = D_i^{exp} + D_n^{imp} + D_t + b_1 \ln(Dist_{ni}) + b_2 \times \text{contig}_{ni} + b_3 \times \text{comlang\_off}_{ni} + e_{ni,t},$$

where $D_i^{exp}$, $D_n^{imp}$ and $D_t$ are exporter, importer, and year fixed effects, respectively. To estimate the above specification, trade flow (in thousands current US$) during 2000-2014 from UN Comtrade as $X_{ni,t}$.[24] The geography variables are obtained from CEPII.

With $\theta = 5.03$, estimated trade costs can be computed with estimates of $b_1, b_2$, and $b_3$ via (B.16). Then, relative technology stock $T_{i,\nu}$ can be backed out from the model structure, as in Fieler (2011) and Ravikumar, Santacreu and Sposi (2019). We interpret the observed economy as on the balanced growth path so that $M_{i,\nu} = M_i$. The technology stock $T_{i,\nu} = M_i \gamma^\nu t_{i0}$ on the BGP scales up $\gamma$ times larger upon each successful innovation, but in each time point the relative technology stocks, i.e., the pair-wise ratios of $T_{i,\nu}$, remain constant. The technology stocks backed out by the following procedure are indeed relative technology stocks, as each country's technology stock $T_i$ is unique, subject to some scaling constant. With data values of $M_i$ (see Section 5), we can then back out the relative knowledge stock parameters $t_{i,0}$.

Given $\hat{D}_i^{exp}$, $(\theta, \alpha, \tau_{n,i})$ and nominal wages $w_i^{data}$ for each country,[25] we solve $\{T_i, P_i, C_i\}$

[24] The trade flows data are obtained from the BACI data in CEPII, and the original source is UN Comtrade. Intra-national flow $X_{ii,t}$ is constructed by $X_{ii,t} = GO_{i,t} - \sum_{i \neq n} X_{in,t}$, where $GO_{i,t}$ is country $i$'s gross output at year $t$. Data on gross output is available in CEPII during 2000-2006.

[25] We use the average values of GDP per capita in current US dollar from WDI to proxy nominal wages.

by the following system of equations

$$
\begin{aligned}
T_i C_i^{-\theta} &= \exp(\hat{D}_i^{exp}) \\
C_i &= (w_i^{data})^{\alpha} P_i^{1-\alpha} \\
P_i &= \left( \sum_{m=1}^{I} T_m [C_m \tau_{i,m}]^{-\theta} \right)^{-\frac{1}{\theta}},
\end{aligned}
$$

where in the last equation, we have renormalized the constant in the price index $\eta = 1$ (see BEJK Result 3). This is without loss of generality as what matters for equilibrium is the relative technology stock. With data values of $M_i$, we then compute $t_{i0} \propto T_i/M_i$.

# C Tables

Table C.1: List of Countries

| Country Name | iso code | Country Name | iso code |
|---|---|---|---|
| Australia | AUS | Guatemala | GTM |
| Belgium | BEL | Croatia | HRV |
| Canada | CAN | Hungary | HUN |
| Switzerland | CHE | India | IDN |
| Germany | DEU | Iran | IRN |
| Denmark | DNK | Iceland | ISL |
| Spain | ESP | Jamaica | JAM |
| Finland | FIN | Korea | KOR |
| France | FRA | Latvia | LVA |
| United Kingdom | GBR | Mexico | MEX |
| Ireland | IRL | Malaysia | MYS |
| Italy | ITA | New Zealand | NZL |
| Japan | JPN | Panama | PAN |
| Netherlands | NLD | Peru | PER |
| Norway | NOR | Poland | POL |
| Singapore | SGP | Portugal | PRT |
| Sweden | SWE | Romania | ROU |
| United States | USA | Russia | RUS |
| Argentina | ARG | Slovenia | SVN |
| Bosnia and Herzegovina | BIH | Thailand | THA |
| Brazil | BRA | Trinidad & Tobago | TTO |
| Chile | CHL | Turkey | TUR |
| China | CHN | Uganda | UGA |
| Colombia | COL | Uruguay | URY |
| Ecuador | ECU | Venezuela | VEN |
| Greece | GRC | South Africa | ZAF |