Income Inequality, Productivity, and International Trade

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October 14, 2021

Abstract

This paper discusses the effect of income inequality on selection and aggregate productivity in a general equilibrium model with nonhomothetic preferences and product quality. It highlights the negative effect of a mean-preserving spread of income on aggregate productivity through the softening of firms' selection. This effect is amplified in the presence of international trade. Furthermore, domestic expenditure shares and welfare gains from trade are constant across income groups.

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†We are grateful to the Co-Editor Costas Arkolakis and two anonymous referees for their comments and suggestions that substantially improved the paper. We also thank Madhav Aney, Pao-Li Chang, Fali Huang, Nicolas Jacquet, Jing Li, Andres Rodriguez-Clare, and Ping Wang for their helpful comments. We thank Xin Yi for his excellent research assistance.

1 Introduction

Income inequality has reappeared as a hot social and economic issue in many developed countries (Atkinson et al. 2011, Piketty 2014). The majority of the economics literature has focused on studying the causes of income inequality, with technological progress and trade liberalization being presented as two major forces.[1] In this paper, we ask a different question: how does income inequality affect aggregate economic performance and welfare? In particular, does there exist an equity-efficiency trade-off in the sense that an increase in income inequality increases efficiency as measured by aggregate productivity? Or, could it be the other way around? Finally, how does trade affect this relationship?

These questions have largely been ignored in the trade literature because of the usual premise of homothetic preferences (e.g. Krugman 1980, Melitz 2003[2]) or the lack of income effect in the consumption of traded goods (e.g. Melitz and Ottaviano 2008). As those premises make most aggregate economic variables invariant to income redistribution, there is no point discussing its effect there. In contrast, the assumption of nonhomothetic preferences allows us to shed light on the effect of income inequality on aggregate productivity in the frameworks of the recent trade literature with firm heterogeneity and endogenous product variety *à la* Melitz (2003) and Melitz and Ottaviano (2008).

We first motivate our theoretical investigation by noting the following stylized facts. First, *the rich consume a larger set of goods than the poor, and the set of their consumed goods largely contains that of the poor*; moreover, *the rich consume higher quality goods*. Second, *there exists a negative correlation between income inequality and aggregate productivity* (see Section 2 for details). These seemingly unrelated stylized facts can actually be linked in the presence of firm heterogeneity and selection. We use the first stylized facts to motivate our model setup and explain the second. To explain differences in consumption baskets across income groups, one must resort to a nonhomothetic demand. Our approach is to consider heterogeneous productivity per unit quality across firms with a nonhomothetic demand and various income groups. We sketch the model and how it explains these stylized facts as follows.

We study a general equilibrium model of trade and monopolistic competition in which firms have heterogeneous productivity and quality while individuals are endowed with different skills and the same Stone–Geary nonhomothetic preferences. The presence of

[1] For skill-biased technical change, see, for example, Berman, Bound, and Machin (1998) and Acemoglu (2002). On the effect of globalization, see, for example, Grossman and Rossi-Hansberg (2008), Costinot and Vogel (2010), Helpman, Itskhoki, and Redding (2010), Behrens, Pokrovsky, and Zhelobodko (2014), Grossman, Helpman, and Kircher (2017), Grossman and Helpman (2018), and Kim and Vogel (2020).

[2] In fact, this conclusion applies for all models in the model class characterized by Arkolakis, Costinot, and Rodriguez-Clare (2012).

various skill groups results in income inequality and leads to demand patterns varying with individuals' incomes. For analytical tractability, we first focus on an economy with two income groups (rich and poor); the model is extended to an arbitrary number of income groups in our quantitative analyses.

To clarify the basic properties of the model, we first analyze a closed economy where each firm enters and draws a differentiated good with specific quality and then decides to exit or produce its good according to its quality-adjusted production cost. Under the assumed preferences, the consumption choice of an individual is unambiguously represented by the *choke price* of her inverse demand function. This choke price is the maximum price at which she is willing to purchase a first unit of a good. In contrast to Melitz and Ottaviano (2008) where choke prices are the same across income groups due to the quasi-linear preferences, choke prices in our model differ in income. The choke prices of the rich and poor groups are then sufficient statistics of the demands for the whole set of goods in the economy. In this setting, the rich consume a larger set of goods than the poor, and the set of goods of the poor is contained in that of the rich.

That the rich purchase goods with higher cost per unit quality is more plausible than it seems at first glance. This is readily illustrated by the example of posters and paintings: while both goods have the same decorative functionality, the latter is much more costly to make (especially in terms of per unit quality). At the equilibrium, only richer individuals are willing to purchase the two goods to decorate their houses. Moreover, by assuming that higher-quality goods requires larger cost per unit quality, it is readily shown that the rich purchase products with higher quality.[3]

Income inequality affects the average productivity across firms through direct and general-equilibrium effects. We show that an increase in the rich group's income produces the direct effect of raising its own's choke price, but also produces a *cross effect* of reducing the poor's choke price. Furthermore, in this setup, the rich's choke price determines the selection cutoff on cost above which firms stop producing. Thus, when the rich are willing to consume a wider set of goods, new firms producing goods with higher costs and higher qualities are enticed to enter. At the same time, prices augment and the poor reduce the number and quantity of the goods that they purchase. On average, firms use more input to produce their goods, which decreases the average productivity. A similar effect emerges when the poor group becomes poorer because the cross effect implies that the rich's choke price increases. As a result, a mean-preserving spread of income distribution implies a lower average productivity because the rich's choke price unambiguously

[3]In a different vein, Wuergler (2010) studies how income inequality (also in terms of two income groups) affects the interactions between quality choices of firms and the number of varieties.

increases, and there are on average firms with higher costs and higher qualities in the economy. This provides an explanation to the negative correlation between income inequality and aggregate productivity.

Our analysis then moves on to the open economy to study how the negative effect of income inequality on average productivity and welfare is affected by trade integration. For this part of the analysis, we impose a Pareto productivity distribution, which is supported by the empirical literature on firm size distribution and entails more analytical tractability. Moreover, as shown by Arkolakis et al. (2019), a Pareto distribution yields important welfare properties which we further discuss.

We find that the negative effect of income inequality on aggregate productivity is amplified by trade integration. The intuition hinges on how trade affects the selection cutoff, which is given by the rich's choke price. As trade increases the pool of competitors and reduces the frictions for them to compete in the same market, only those with sufficiently high productivity survive. Thus, any effect of income inequality on average productivity through this selection cutoff is amplified by trade integration. Put differently, income redistribution from the rich to the poor improves each country's average productivity more under deeper trade integration.

We also show that welfare gains from trade are captured by the variations in consumers' expenditure share on domestic goods, exactly in the same form as in Arkolakis et al. (2019). This result stems from two theoretical results in our model: (1) each group's welfare gain from trade is captured by the same formula in Arkolakis et al. (2019); (2) domestic expenditure share does not differ across income groups. The two results together imply that welfare gains from trade are the same for all income groups. A key to these results is the Pareto productivity assumption, which is also imposed by Arkolakis et al. (2019). The important difference is that their study considers only homogeneous income within a country, whereas we show that their result generalizes to a setting with heterogeneous income.

To gauge the economic magnitude of the model mechanism, we conduct a quantitative analysis using a simple two-symmetric-country framework with the US being the representative. For this purpose, we extend the model for an arbitrary number of income groups and apply it to US income-quintile data. Our theoretical analysis focuses on the effect of inequality on the unweighted average productivity. The quantitative exercise confirms the same patterns for weighted average productivity (with weights on firm output). Moving from the US income distribution before taxes and transfers to a fully even distribution entails productivity gains of 23.7% and 2.5% for unweighted and weighted average productivity, respectively. The welfare gains from autarky to the current trade cost level are 4.8%, and as mentioned, it is the same for all income groups.

Our paper relates to the literature on heterogeneous firms and productivity pioneered by Eaton and Kortum (2002) and Melitz (2003). To our knowledge, our analysis is the first to offer new testable predictions about how income inequality affects firm selection and average productivity. In contrast to the traditional view of the equity-efficiency trade-off, Aghion et al. (1999) highlight that reducing income inequality may promote economic growth through saving, investment, and incentives; Murphy et al. (1989) find a similar conclusion via a market-size effect. Matsuyama (2002) proposes another mechanism through the dynamic effect of income inequality on productivity in the context of homogeneous firms and learning by doing. For this author, higher income inequality is detrimental to growth because it reduces the "mass of consumption" and therefore the dynamic productivity gains from learning by doing.[4] As in these three studies, our theoretical result shows the opposite of the equity-efficiency trade-off, but our mechanism differs by highlighting the role of heterogenous firms and the mechanism through which income inequality affects selection and aggregate productivity.

Our study is closely related to the literature on income heterogeneity and gains from trade trade. So far, the literature is inconclusive about the gains from trade that various income groups obtain through the expenditure channel. Behrens and Murata (2012) and Fajgelbaum and Khandelwal (2016) find that trade is pro-poor, meaning that trade liberalization benefits the poor more than the rich in percentage terms. Nigai (2016) finds that trade is actually pro-rich, while McCalman (2018) finds that trade can be either pro-poor or pro-rich, depending on a country's income distribution relative to the world income distribution. While the results are affected by the different mechanisms underlying the above-mentioned studies, it is important to note the recent empirical finding that the domestic expenditure share varies little across income groups (Hottman and Monarch, 2020, and Borusyak and Jaravel, 2021). To our knowledge, this paper provides the first model with a nonhomothetic preference that predicts this empirical pattern. Furthermore, Borusyak and Jaravel (2021) show that this empirical pattern implies that welfare gains from trade are income-neutral in a neoclassical framework. Our welfare result echoes that of Borusyak and Jaravel (2021). Our model matches this pattern but stems from a monopolistically competitive environment with a Pareto productivity distribution.

This paper also relates to the literature on nonhomothetic preferences. In particular,

[4]In a model with nonhomothetic preference, product innovation, and two income groups, Foellmi and Zweimüller (2006) show that more income inequality leads to faster growth as new products are sold exclusively to the rich. Galor and Moav (2004) develop a theory with human and physical capital competing to be the growth engine. They find a non-monotonic relationship between inequality and growth. At later stages of development, human capital replaces physical capital so that equality promotes growth.

our model features the same pro-competitive effects and pricing to markets as those in Simonovska (2015) and Bertoletti et al. (2018). Our analysis includes the study of production in monopolistically competitive markets, whose properties are extensively studied in Parenti et al. (2017) and Dhingra and Morrow (2019). This literature also studies structural changes, as in Comin et al. (2018), and patterns of trade, as in Fieler (2011), Matsuyama (2015), and Foellmi et al. (2018). Even though most of these papers are concerned with the impact of country incomes on production and trade, they do not study the effect of within-country income distribution as the present paper does.

The remainder of the paper is organized as follows. Section 2 discusses stylized facts motivating our theoretical investigation. Section 3 lays out the model in the closed economy, and provides various comparative statics, with a focus on the effect of income distribution. Section 4 extends the model to the open economy, and carries out a similar analysis with a focus on the effect of trade liberalization. Section 5 extends the model to multiple income groups and applies it to a quantitative analysis of income inequality and trade on productivity and welfare. Section 6 concludes.

2 Empirical Motivation

Our analysis is primarily motivated by four stylized facts reported by the empirical literature mainly based on micro-economic data. We describe them here and relate them to our theoretical framework.

First, *the rich consume a greater variety than the poor, and a large part of the set of goods of the poor is contained in that of the rich.* Using Indian data on household food consumption, Li (2021) documents a positive association between variety and expenditure, which is labeled the *variety Engel curve*. Although a purely deterministic hierarchy of goods is not supported, he finds that a large part of the poor's set of food varieties is contained in the rich's set, and that the richer households "diversify their consumption by adding varieties that are marginal along multiple dimensions" (p. 72). Using the US Consumer Expenditure Survey, Jackson (1984) shows evidence for the variety Engel curve for a broader commodity spectrum.

Second, *a country's economic growth is negatively correlated with its income inequality*; see, e.g., Aghion et al. (1999), Berg et al. (2018), and Cingano (2014). One possible cause is the fact that there exists a *negative correlation between a country's aggregate productivity and income inequality*; see Hayes et al. (1994), Fuentes et al. (2014), OECD (2016), Paul (2020), and Espoir et al. (2021).

Third, *the rich consume higher-quality goods*. For instance, Bils and Klenow (2001) provide evidence on richer households consuming higher-quality goods. Hummels and Klenow (2005) show that richer countries consume and export higher-quality goods.

Fourth, *firms with higher productivity set higher markups (pro-competitive pricing)*. De Loecker et al. (2016) show evidence that higher firm productivity leads to higher markups.

Our analysis mainly addresses the relationships between the first two stylized facts. Our model is based on heterogeneous firms and nonhomothetic preferences. On one hand, under such preferences, the rich's demand features a choke price that is higher than that of the poor. As a result, the rich's basket of goods contains that of the poor, which matches the first stylized fact.[5] On the other hand, in our model, a mean-preserving spread of income distribution entails general-equilibrium effects through choke prices. In particular, a stronger inequality raises the choke price in the rich's market segment and entices new firms to enter this segment, not only with higher quality goods but also higher cost per quality unit. This softens the selection of firms in the product market and also raises the average cost in the economy, which in turn reduces the average productivity. Our model therefore provides an explanation for the second stylized fact.

Our analysis then encompasses the last two stylized facts in a straightforward way. The third stylized fact is indeed a corollary of the first one in our framework where higher-quality goods require higher cost per unit quality to be produced. This plausible assumption has already been made in the literature, e.g., Foellmi et al. (2014). Finally, the last stylized fact simply follows from the Stone–Geary preferences discussed in this model.

Note that the first and third stylized facts focus on the extensive margin of a household's consumption basket — i.e., the number and types of goods included in the basket. These facts have also been explained by models featuring indivisible consumption as in Matsuyama (2002) and Foellmi et al. (2014). Such models mainly focus on the extensive margin of households' consumption baskets and are particularly suitable to explain the demand for durable goods such as automobiles and home appliances.

However, the intensive margin of a household's consumption basket — i.e., the quantity consumed per good — is also an important property to study, particularly for nondurable goods (such as food, beverages, and recreational services). This property is studied in Deaton and Maullbauer (1980), Matsuyama (2015), Behrens and Murata (2012), Fajgelbaum and Khandelwal (2016), Faber (2014), and Faber and Fally (2021). In these

[5] Note that, in this model, the rich consumes the full basket of the poor's goods, rather than a 'large share' of it as reported by Li (2021). This may not match the ideas that some 'inferior' goods may be purchased only by the poor or that lower versions of the same vertically differentiated good are purchased only by the poor. Nevertheless, our framework encompasses a similar property as it is shown that, within a same basket of goods, the rich balance their expenditure toward higher quality goods. See Section 3.2.

papers, the extensive margins are ignored as the number of goods is fixed.[6] One contribution of our analysis to the literature is that it embeds the property that the consumption patterns of the rich and poor differ in both their extensive and intensive margins, which contrasts with most of the above-mentioned studies focusing on one of the margins. We detail the consumption patterns in the next section.

3 Closed Economy

We present a model where a mass N of individuals are endowed with Stone–Geary preferences over a set of differentiated goods $\omega \in \Omega$. The goods are produced by firms with idiosyncratic costs and quality levels, and are sold in a monopolistically competitive market. Individuals are split in two groups according to the number of their labor efficiency units s_L and s_H (skill or human capital) with probabilities α_H and $\alpha_L \in (0,1)$ where $s_H > s_L$ and $\alpha_H + \alpha_L = 1$.[7] We choose labor efficiency unit as the numéraire so that s_h also measures individual h's income $h \in \{H, L\}$. Income inequality is captured by the income differential $v \equiv s_H - s_L$ so that incomes can be written as $s_H = \overline{s} + \alpha_L v$ and $s_L = \overline{s} - \alpha_H v$ where $\overline{s} \equiv \alpha_L s_L + \alpha_H s_H$ is the average income. A larger income differential v raises the usual measures of income inequality such as income variance, Gini index, and top-10% income shares.[8] In addition, in this context of two income groups, a rise in income differential v is also equivalent to a mean-preserving spread of the income distribution.

3.1 Demand

An individual in the income group h chooses the consumption profile $q(.)$ that maximizes her utility $\int_{\omega\in\Omega} \ln(1 + \beta(\omega) q(\omega))\, \mathrm{d}\omega$[9] subject to her budget constraint $\int_{\omega\in\Omega} p(\omega)\, q(\omega)\, \mathrm{d}\omega = s_h$, where $\beta(\omega)$ is a quality shifter reflecting the number of quality units embedded in

[6]In Faber (2014) and Faber and Fally (2021), the number of varieties for each sector can be endogenously determined, but the demand structure is such that both the rich and poor consume a positive amount of each variety, albeit at different intensities.

[7]The model with many income groups is studied in Section 5.

[8]With two income groups, the income variance is given by $\mathrm{var}(s_h) = v^2 \alpha_H \alpha_L$ and the Gini index $G = \left(\sum_l \sum_h \alpha_h \alpha_l |s_h - s_l|\right) / \left(2 \sum_h s_h \alpha_h\right)$ by $G = \alpha_L \alpha_H v / \overline{s}$. Setting $\alpha_H = 0.1$, the top-10% income share is given by $0.1 + 0.09 v/\overline{s}$. All three measures increase with larger income differential v.

[9]This is an affine transformation of the original Stone-Geary utility function $\int_{\omega\in\Omega} \ln(q(\omega) + \overline{q})\, \mathrm{d}\omega$.

good ω, and the price profile $p(\cdot)$ is taken as given.[10] Her demand is equal to

$$q_h(\omega) = \frac{\hat{p}_h}{p(\omega)} - \frac{1}{\beta(\omega)}, \tag{1}$$

where

$$\hat{p}_h = \frac{s_h + P_h}{|\Omega_h|} \tag{2}$$

is a choke price, Ω_h is the set of goods that she consumes, $|\Omega_h| \equiv \int_{\omega \in \Omega_h} \mathrm{d}\omega$ is the measure of this set, and

$$P_h \equiv \int_{\omega \in \Omega_h} \frac{p(\omega)}{\beta(\omega)} \mathrm{d}\omega$$

is her (personal) price index over her consumptions (see Appendix A). Note that the price index P_h and therefore the choke price $\hat{p}_h$ are adjusted for quality. The intercept of the individual demand curve is given by the choke price $\hat{p}_h$ times the quality shifter $\beta(\omega)$; this is the willingness to pay for the first unit of a good ω. For a given set of prices and consumed goods, the choke price increases with larger income, larger price index, and smaller set of consumed goods. Yet, for any endogenous set of consumed goods, it is readily shown that the choke price is larger for higher-income individuals: $\hat{p}_H > \hat{p}_L$.

The aggregate demand for each good ω with price $p(\omega) = p$ is given by

$$Q(\omega, p) \equiv \begin{cases} \alpha_H N \left(\frac{\hat{p}_H}{p} - \frac{1}{\beta(\omega)} \right) & \text{if} \quad \frac{p}{\beta(\omega)} \in [\hat{p}_L, \hat{p}_H) \\ N \left(\frac{\hat{p}_{HL}}{p} - \frac{1}{\beta(\omega)} \right) & \text{if} \quad \frac{p}{\beta(\omega)} \in [0, \hat{p}_L) \end{cases}, \tag{3}$$

where $\hat{p}_{HL} \equiv \alpha_H \hat{p}_H + \alpha_L \hat{p}_L$ is the average of the individual choke prices ($\hat{p}_H \geq \hat{p}_{HL} \geq \hat{p}_L$). Because of the presence of two income groups, the aggregate demand function has a convex kink at $p = \hat{p}_L \beta(\omega)$. The model extends Mussa and Rosen's (1978) quality framework with two income groups from one indivisible good to a continuum of divisible goods.

The price elasticity is

$$\varepsilon(p) = -\frac{\mathrm{d} \ln Q(\omega, p)}{\mathrm{d} \ln p} = \begin{cases} \frac{\hat{p}_H}{\hat{p}_H - p/\beta(\omega)} & \text{if} \quad \frac{p}{\beta(\omega)} \in [\hat{p}_L, \hat{p}_H) \\ \frac{\hat{p}_{HL}}{\hat{p}_{HL} - p/\beta(\omega)} & \text{if} \quad \frac{p}{\beta(\omega)} \in [0, \hat{p}_L) \end{cases}.$$

Because $\hat{p}_H \geq \hat{p}_{HL}$, for a same price p, the elasticity is lower in the rich consumer segment.

[10]We use an additive utility function that yields the Stone-Geary demand functions. Those are linear in income but do not exhibit expenditure proportionality (Pollak 1971). The linearity property is essential for the demand aggregation process below. Simonovska (2015) exploits this set-up to study international pricing-to-market under the assumption of homogenous income within a country.

3.2 Production

Each firm produces and sells a unique good ω under monopolistic competition. We assume the existence of a large pool of potential risk-neutral entrants. By hiring f fixed units of labor, each entrant obtains the technology to produce a distinct good ω with an amount of quality units $\beta(\omega)$ and marginal cost $\beta(\omega)c(\omega)$. In the latter expression, $c(\omega)$ measures the *quality-adjusted marginal cost*, defined as the labor inputs needed to produce each quality unit of the good ω. Without loss of generality, we can index all goods by this cost $c \in \mathbb{R}^+$ and assume that the quality is a function of cost $\beta(c)$. We assume that this cost is distributed according to a cumulative probability distribution G that has bounded support and finite mean $G : [0, c_M) \to [0, 1]$ such that $\mathrm{E}(c) = \int_0^{c_M} c\mathrm{d}G(c) < \infty$. The marginal cost of a good is given by $\beta(c)c$. We denote the mass of entrants by M. Therefore, each measure of goods $\mathrm{d}\omega$ is identical to the measure $M\mathrm{d}G(c)$.

Following the literature on product quality, we consider that producing higher-quality products requires higher marginal cost per unit quality. This occurs for instance because higher-quality goods have more numerous and more expensive components. We ensure this property by assuming that $\beta'(c) > 0$. Foellmi et al. (2014) use the same assumption in a model with indivisible goods. This assumption also implies that higher-quality goods have larger marginal costs $\beta(c)c$ since $\beta(c)$ rises with c. As will be emphasized below, it also ensures that richer people spend more on goods with higher quality $\beta(c)$.

Each firm with quality shifter β and quality-adjusted cost c maximizes its profit $(p - \beta c)$ $Q(c, p)$ taking the choke prices $\hat{p}_L$ and $\hat{p}_H$ as given. Under this specification, the profit turns out to be a function of the quality-adjusted price p/β and quality-adjusted cost c (but not the quality shifter β alone). Thus, it turns out that the optimal choice can be discussed only in terms of its quality-adjusted price and cost. Because the demand Q includes two segments, a firm can choose between targeting only the high-income group or both income groups. The profit-maximizing quality-adjusted price is given by

$$\frac{p^*(c)}{\beta(c)} = \begin{cases} (\hat{p}_{HL}c)^{1/2} & \text{if} \quad c \leq \hat{c} \\ (\hat{p}_H c)^{1/2} & \text{if} \quad c > \hat{c} \end{cases} \tag{4}$$

and

$$\hat{c}^{1/2} \equiv \frac{\hat{p}_{HL}^{1/2} - (\alpha_H \hat{p}_H)^{1/2}}{1 - \alpha_H^{1/2}} \tag{5}$$

(see Appendix B). Except at $c = \hat{c}$, the optimal price is a strictly concave increasing function of the quality-adjusted cost c. Observe that because $\hat{p}_H > \hat{p}_{HL}$, the price jumps upward for the firms with costs c rising about $\hat{c}$, reflecting a switch towards targeting high-income consumers. Note that, in a partial equilibrium where we change one choke price and take the other as fixed, we have

$$\frac{\partial \hat{c}}{\partial \hat{p}_H} < 0 \quad \text{and} \quad \frac{\partial \hat{c}}{\partial \hat{p}_L} > 0. \tag{6}$$

This means that the cutoff $\hat{c}$ falls when the rich earn a higher income and their choke price rises. This is because their willingness to pay improves and more firms find it profitable to target them. By contrast, the cutoff rises when the poor become richer and their choke price rises. Targeting the entire population becomes more profitable.

Firms' optimal markup is given by

$$m^* \equiv \frac{p^*(c)}{\beta(c)c} = \begin{cases} \hat{p}_H^{1/2} c^{-1/2} & \text{if} \quad c > \hat{c} \\ \hat{p}_{HL}^{1/2} c^{-1/2} & \text{if} \quad c \leq \hat{c} \end{cases}.$$

Within each market segment, markups fall with cost c, reflecting a pro-competitive effect as mentioned in the stylized facts described in Section 2. Also, the markup jumps up at $\hat{c}$, as a consequence of the price discrimination between the two segments.

We finally explain the difference in equilibrium consumption patterns between the rich and poor and the role of the assumption that producing higher-quality goods requires higher marginal costs per unit quality ($\beta'(c) > 0$). First, different income groups purchase different baskets of goods in equilibrium. The low-income consumers buy only the goods produced at quality-adjusted costs c in $[0, \hat{c}]$, while high-income individuals purchase goods produced at quality-adjusted costs in $[0, \hat{p}_H]$. As $\hat{p}_H > \hat{c}$, *higher-income individuals purchase a wider basket of goods*. Moreover, the assumption of $\beta'(c) > 0$ generates a one-to-one mapping between quality-adjusted cost c and product quality $\beta(c)$. As a consequence, *higher-income individuals' baskets include additional products with higher quality.* These are two stylized facts on the effect of income at the *extensive margin* of the consumption basket as we have mentioned in Section 2.

Second, we compare the consumption of the goods that both groups purchase. By (1), the rich buy a greater quantity of the same good than the poor. However, compared to the poor, they spend relatively more on higher-quality goods. Indeed, consider the individual h's expenditure share of a good with cost c, $\mu_h(c) = p^*(c) q_h [p^*(c), c] / s_h$, where $q_h(p, c)$ is determined by (1). Using (2), we successively get

$$\mu_h(c) = \frac{1}{s_h}\left[\hat{p}_h - \frac{p^*(c)}{\beta(c)}\right] = \frac{1}{s_h}\left[\frac{s_h + P_h^*}{|\Omega_h^*|} - \frac{p^*(c)}{\beta(c)}\right],$$

where P_h^* and $|\Omega_h^*|$ are her price index and number of goods. It is then easy to compare the consumption shares of two goods by two individuals with different incomes by inspecting the following expression:

$$\frac{\mathrm{d}^2\mu_h(c)}{\mathrm{d}s_h\mathrm{d}c} = \frac{1}{s_h^2}\frac{\mathrm{d}}{\mathrm{d}c}\left(\frac{p^*(c)}{\beta\,(c)}\right) > 0.$$

This is positive because the price per quality unit $p^*(c)/\beta\,(c)$ increases in c.[11] As a result, *a consumer with higher income purchases a larger share of goods with higher cost per quality unit c, i.e., with higher quality*. Another way to understand this result is to realize that higher-income individuals consume larger amounts of goods with higher income elasticities, which is a property held by goods with higher cost c. Indeed, it is readily verified that, at the equilibrium, the income elasticity of individual h's demand is given by

$$\frac{\mathrm{d}\ln q_h\,(c)}{\mathrm{d}\ln s_h} = \left(1 - \frac{p^*\,(c)}{\hat{p}_h\beta\,(c)}\right)^{-1}\frac{\mathrm{d}\ln\hat{p}_h}{\mathrm{d}\ln s_h} > 0,$$

which increases in c because $p^*\,(c)\,/\beta\,(c)$ also rises with it.

We also note that although high quality goods have high values for consumers, they also imply large costs for producers. In the present framework, the latter rises faster than the former. Firms with costs c above $\hat{p}_H$ have very high quality levels but also too large costs so that they do not break even and quit the market. As an example, compared to Boeing and Airbus, the Concorde passenger airplane provided a very high quality transport service with its supersonic speed connection between Europe and US but was unable to break even given its too-high cost and too-low resulting demand. By contrast, the firms with cost slightly above $\hat{c}$ are active in the high quality segment and make high profits.

Finally we make the remark that the assumption $\beta'(c) > 0$ is needed only for the interpretation about consumption of higher-quality goods by the richer group. It is however not required for the next results about equilibrium existence and average productivity.

3.3 Equilibrium

Given the above analysis, equilibrium choke prices can be written as

$$\hat{p}_L = \frac{s_L + P_L}{MG\,(\hat{c})}, \quad \hat{p}_H = \frac{s_H + P_H}{MG\,(\hat{p}_H)}, \quad \text{and} \quad \hat{p}_{HL} = \alpha_H\hat{p}_H + \alpha_L\hat{p}_L$$

[11] It indeed holds for any production system that supports an increasing schedule for the price per quality unit $p^*(c)/\beta(c)$. This property therefore mainly lies on preferences.

and the price indices as

$$P_L = \hat{p}_{HL}^{1/2} \int_0^{\hat{c}} c^{1/2} M \mathrm{d}G(c) \quad \text{and} \quad P_H = P_L + \hat{p}_H^{1/2} \int_{\hat{c}}^{\hat{p}_H} c^{1/2} M \mathrm{d}G(c).$$

Because choke prices and price indices are adjusted for quality, their equilibrium values depend only on the quality-adjusted costs c. Eliminating price indices, these equilibrium conditions can be expressed as

$$\frac{e_H(\hat{p}_H, \hat{p}_L)}{s_H} = \frac{e_L(\hat{p}_H, \hat{p}_L)}{s_L} = \frac{1}{M}, \tag{7}$$

where

$$e_H(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_H - \hat{p}_{HL}^{1/2} c^{1/2}\right) \mathrm{d}G(c) + \int_{\hat{c}}^{\hat{p}_H} \left(\hat{p}_H - \hat{p}_H^{1/2} c^{1/2}\right) \mathrm{d}G(c),$$
$$e_L(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_L - \hat{p}_{HL}^{1/2} c^{1/2}\right) \mathrm{d}G(c),$$

where $\hat{p}_{HL}$ and $\hat{c}$ are the functions of $(\hat{p}_H, \hat{p}_L)$ defined above. The latter expressions describe the consumers' average expenditures per available good. Thus, in equilibrium, the consumers' average expenditures per good and per unit of income are equalized across groups and made equal to the inverse of the mass of entrants. The product market equilibrium is defined by the solution of these two equations for the choke prices $(\hat{p}_H, \hat{p}_L)$. For a given M, the equilibrium choke prices are sufficient statistics of product market equilibrium consumption and production choices.

In the long run firms enter the market. Before entry, each entrant expects to cover her entry cost so that

$$\int_0^{\infty} \max\{\pi(c), 0\} \mathrm{d}G(c) = f,$$

where the profit $\pi(c)$ is given by $N\left(\hat{p}_{HL}^{1/2} - c^{1/2}\right)^2$ if $c \leq \hat{c}$ and by $\alpha_H N \left(\hat{p}_H^{1/2} - c^{1/2}\right)^2$ if $c > \hat{c}$. As stated earlier, profits are functions of quality-adjusted cost (but not the quality shifters $\beta(c)$). Then, the entry condition is expressed as

$$\pi(\hat{p}_H, \hat{p}_L) = \frac{f}{N}, \tag{8}$$

where we define

$$\pi(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{p}_H} \max\left\{\left(\hat{p}_{HL}^{1/2} - c^{1/2}\right)^2, \alpha_H \left(\hat{p}_H^{1/2} - c^{1/2}\right)^2\right\} \mathrm{d}G(c) \tag{9}$$

as the expected operational profit per capita and before entry.

The general equilibrium is defined by the variables $\hat{p}_H$, $\hat{p}_L$, and M solving the conditions in (7) and (8), which are expressed in terms of quality-adjusted cost c (but not quality shifters $\beta(c)$). Since these equations include continuous expressions, the condition for general equilibrium existence requires that the expressions change signs on their supports. We show in Appendix C that this holds true.

A condition for the uniqueness of the general equilibrium can be found as follows. First note that the expected operational profit $\pi(\hat{p}_H, \hat{p}_L)$ is an increasing function of both choke prices. Thus, the entry condition describes a decreasing relationship between the two choke prices. Second, it can be seen that the second equality in (7) describes an increasing relationship between the two choke prices if the conditions $\partial e_h/\partial \hat{p}_h > 0$ and $\partial e_h/\partial \hat{p}_l < 0$ hold for any $h \neq l \in \{H, L\}$. Under these conditions, it is clear that the two relationships cross at a single point $(\hat{p}_H, \hat{p}_L)$ that yields the unique equilibrium. The main question is verifying that these conditions are true.

Using (5), it is easy to verify that the poor's expenditure increases with their own choke price and falls with the rich's choke price: $\partial e_L/\partial \hat{p}_L > 0$ and $\partial e_L/\partial \hat{p}_H < 0$. The symmetric condition holds for the rich provided that firms do not change consumer segment targets, that is, if the cut-off cost $\hat{c}$ is fixed. However, by (5), the cut-off cost $\hat{c}$ falls ($\mathrm{d}\hat{c} < 0$) when $\hat{p}_H$ rises or $\hat{p}_L$ falls. Then, a mass $-g(\hat{c})\mathrm{d}\hat{c} > 0$ of firms shifts to the high-income segment target, which reduces the rich's expenditure by the amount $\left(\hat{p}_H^{1/2} - \hat{p}_{HL}^{1/2}\right)\hat{c}^{1/2}$ $(-g(\hat{c})\mathrm{d}\hat{c})$. The change in the firms' segment target therefore decreases the rich's expenditure and goes in the opposite direction of the effect of choke prices when $\hat{c}$ is fixed. Since this countervailing effect is proportional to the density $g(\hat{c})$, some smoothness properties are required to guarantee that G is not misbehaved about $c = \hat{c}$. Let

$$\partial e_H/\partial \hat{p}_H > 0 \quad \text{and} \quad \partial e_H/\partial \hat{p}_L < 0. \tag{A1}$$

We then have the following:

Proposition 1. *The equilibrium always exists. It is unique if the cost distribution G satisfies (A1).*

From now on, we assume that the cost distribution G satisfies (A1). This will ensure equilibrium uniqueness and the lemma and propositions of the following subsections.

3.4 Income Distribution

We are interested in understanding how demands and choke prices are affected by changes in income levels. Intuitively, an increase in the income of one group raises its willingness to pay, choke price, and product demands. Since demand elasticity falls with higher income, markups and prices increase. Facing higher prices, the other group is enticed to diminish its consumption, which should be reflected by lower choke prices. In Appendix D, we prove the following lemma:

Lemma 1. *A rise in the rich (resp. poor) group's income raises its choke price and demands whereas it reduces the poor's (resp. rich's). Formally,*

$$\frac{\mathrm{d}\ln \hat{p}_h}{\mathrm{d}\ln s_h} = -\frac{\mathrm{d}\ln \hat{p}_h}{\mathrm{d}\ln s_l} > 0, \quad h \in \{H, L\}, l \neq h. \tag{10}$$

This has implications about the effect of income distribution on the average productivity and set of consumption goods. First, in this model, income inequality is captured by the income differential v (recall that $v = s_H - s_L$). A mean-preserving spread of the income distribution is equivalent to a rise in v. Using (10), we have

$$\frac{\mathrm{d}\ln \hat{p}_H}{\mathrm{d}v} = \frac{\mathrm{d}\ln \hat{p}_H}{\mathrm{d}\ln s_H}\frac{\mathrm{d}\ln s_H}{\mathrm{d}v} + \frac{\mathrm{d}\ln \hat{p}_H}{\mathrm{d}\ln s_L}\frac{\mathrm{d}\ln s_L}{\mathrm{d}v} = \frac{\overline{s}}{s_H s_L}\frac{\mathrm{d}\ln \hat{p}_H}{\mathrm{d}\ln s_H} > 0.$$

As a result, a mean-preserving spread increases the high income and decreases the low income so that the choke price of the high-income group rises.[12] An important consequence of this is that income inequality expands the set of goods offered in the market $[0, \hat{p}_H]$.

Second, the expansion of the set of goods has implications on average cost and productivity in the economy. We define the (unweighted quality-adjusted) average cost in the economy by $\int_0^{\hat{p}_H} cM\mathrm{d}G(c) \,/\, \int_0^{\hat{p}_H} M\mathrm{d}G(c)$ and the (unweighted quality-adjusted) average productivity by its inverse value.[13] The former rises with the choke price $\hat{p}_H$. Indeed, $(\mathrm{d}/\mathrm{d}\hat{p}_H)\left(\int_0^{\hat{p}_H} c\mathrm{d}G(c) \,/\, \int_0^{\hat{p}_H} \mathrm{d}G(c)\right) > 0$ if and only if $\int_0^{\hat{p}_H} (\hat{p}_H - c)\,\mathrm{d}G(c) > 0$, which holds because $c \in (0, \hat{p}_H)$. From the previous paragraph, it is then clear that the above average cost rises with a stronger income difference v, and, as a result, the above average productivity falls with a spread of the income distribution. The point is that when the high-income group becomes richer, it consumes more goods with higher quality but also

[12] By the same argument, it can be shown that the mean-preserving spread decreases the choke price of the low income group.

[13] This definition follows the empirical literature that estimates productivity adjusting for product quality or controlling for unobserved characteristics that include product quality. See, for example, Collard-Wexler and De Loecker (2015) and Berry et al. (1995).

higher production cost, which raises the average cost and reduces the average productivity in the economy. The same conclusion holds for the unweighted average "unadjusted" cost and productivity.[14]

Finally, we investigate how income affects the set of goods consumed by the poor. The poor's consumption set $[0, \hat{c}]$ is given by the cut-off cost $\hat{c}$, which falls with the rich's income as we indeed have

$$\frac{\mathrm{d}\ln\hat{c}}{\mathrm{d}\ln s_H} = \frac{\partial \ln \hat{c}}{\partial \ln \hat{p}_H}\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_H} + \frac{\partial \ln \hat{c}}{\partial \ln \hat{p}_L}\frac{\mathrm{d}\ln\hat{p}_L}{\mathrm{d}\ln s_H} < 0, \tag{11}$$

where the inequality stems from (6) and (10). An increase in the rich's income raises their demand so that the profits increase in the rich's demand segment. This entices firms to switch from the other segment and target the rich's segment by raising their prices. This effect of reducing the set of goods offered to the poor, $[0, \hat{c}]$, is reflected by the first negative term in (11). Meanwhile, as the increase in the rich's income and demand raise the prices of all goods, the poor reduce their consumption for each good in order to maintain their budget constraints, which is reflected by a lower choke price. This further entices firms to target the rich's segment, resulting in a further reduction of the cutoff cost $\hat{c}$, as indicated in the second negative term in (11).[15]

It is readily verified that the opposite effect holds with a change in the poor's income: $\mathrm{d}\ln\hat{c}/\mathrm{d}\ln s_L > 0$. As a consequence, a mean-preserving spread of the income distribution reduces the cut-off cost $\hat{c}$. Indeed, it is easy to see that

$$\frac{\mathrm{d}\ln\hat{c}}{\mathrm{d}v} = \frac{\mathrm{d}\ln\hat{c}}{\mathrm{d}\ln s_H}\frac{\mathrm{d}\ln s_H}{\mathrm{d}v} + \frac{\mathrm{d}\ln\hat{c}}{\mathrm{d}\ln s_L}\frac{\mathrm{d}\ln s_L}{\mathrm{d}v} < 0.$$

This therefore reduces the relative measure of goods consumed by the poor to the rich: that is, it reduces the ratio

$$\frac{MG(\hat{c})}{MG(\hat{p}_H)} = \frac{G(\hat{c})}{G(\hat{p}_H)}.$$

Proposition 2. *A mean-preserving spread of the income distribution (i) expands the set of goods offered in the market, (ii) decreases the (unweighted quality-adjusted) average productivity in the economy, and (iii) reduces the set of goods consumed by the poor relative to that by the rich.*

An income redistribution policy has the opposite effect of a mean-preserving spread

[14] The unweighted "unadjusted" cost average rises with $\hat{p}_H$ because we have $(\mathrm{d}/\mathrm{d}\hat{p}_H)\left(\int_0^{\hat{p}_H}\beta(c)c\mathrm{d}G(c) \,/ \int_0^{\hat{p}_H}\mathrm{d}G(c)\right) > 0$ if and only if $\int_0^{\hat{p}_H}(\beta(\hat{p}_H)\hat{p}_H - \beta(c)c)\,\mathrm{d}G(c) > 0$, which is satisfied by $c \in (0, \hat{p}_H)$ and $\beta'(c) > 0$. The "unadjusted" average productivity goes in the opposite direction.

[15] When the rich's income increases, the poor also react in the intensive margin of their consumption basket. The poor shift their consumption intensity toward goods with lower c, as these goods are more cost-effective in generating quality units.

in the income distribution: it shrinks the set of goods offered in the market and raises average productivity. This model yields a clear-cut answer as to how a mean-preserving spread affects aggregate economic performance. The above result applies for any cost distribution and does not show up in models with homothetic or quasi-linear preferences (Melitz 2003; Melitz and Ottaviano, 2008).[16] Our analysis provides a plausible economic mechanism to explain the negative correlation between income inequality and average productivity as described in Section 2.

Note that our theoretical analysis focuses on the unweighted average productivity, as weighted average productivity is hardly tractable to analyze. We will resort to quantitative analysis in Section 5 for weighted average productivity. Thus, unless otherwise noted, before Section 5, average productivity/cost is used to refer to unweighted quality-adjusted average productivity/cost.

3.5 Pareto Productivity Distribution

To gain analytical tractability, we now assume a Pareto productivity distribution with c.d.f. $G(c) = (c/c_M)^\kappa$ for $c \in [0, c_M]$ where $c_M < \infty$ and $\kappa \geq 1$ are the distribution scale and shape parameters. This distribution is supported by the empirical literature on firm size distribution and has important implications on welfare properties (Arkolakis et al. 2019).

For the sake of conciseness, we further define the choke price ratio $r \equiv \hat{p}_H/\hat{p}_L$. The equilibrium quality-adjusted prices can be written as

$$\frac{p^*(c)}{\beta(c)} = \begin{cases} (\alpha_H r + \alpha_L)^{1/2} \hat{p}_L^{1/2} c^{1/2} & \text{if} \quad c \leq \hat{c} \\ r^{1/2} \hat{p}_L^{1/2} c^{1/2} & \text{if} \quad c > \hat{c} \end{cases}$$

and the cutoff cost as

$$\hat{c}^{1/2} = \frac{(\alpha_H r + \alpha_L)^{1/2} - \alpha_H^{1/2} r^{1/2}}{1 - \alpha_H^{1/2}} \hat{p}_L^{1/2}.$$

[16]For example, in Melitz (2003), the homothetic preference implies that all that matters for selection is the mean (or total) income. In Melitz and Ottaviano (2008), the quasi-linear preference also implies the income elasticity of demand for differentiated goods is zero. That is, richer individuals spend the same amount on differentiated products as poor individuals, and they only spend more in the numeraire good.

This gives the following three equilibrium conditions

$$\frac{s_H}{s_L} = \frac{\Gamma_H(r)}{\Gamma_L(r)}, \tag{12}$$

$$\hat{p}_L = \left(\frac{c_M^\kappa f}{\kappa N}\right)^{\frac{1}{\kappa+1}} \Phi(r)^{-\frac{1}{\kappa+1}}, \tag{13}$$

$$M = \frac{s_L N}{f}\frac{\Phi(r)}{\Gamma_L(r)}, \tag{14}$$

where Φ, Γ_H, and Γ_L are functions which are given in Appendix E and for which we make explicit the dependence on the choke price ratio r. The identity (12) yields the value of r as a function of income ratio s_H/s_L. Hence, income distribution affects only the ratios of choke prices rather than their levels. Given the value of r, identities (13) and (14) yield the low-income group's choke price $\hat{p}_L$ and the mass of entrants M. Under a Pareto productivity distribution, the income ratio s_H/s_L, entry cost f, and population size N have separate effects.

It is shown in Appendix E that Assumption (A1) holds under a Pareto productivity distribution, so that the general equilibrium exists and is unique while Proposition 2 holds. Hence, a mean-preserving spread of the income distribution reduces the average productivity in the economy and reduces the set of goods consumed by the poor relative to that consumed by the rich. Also, by Lemma 1, a mean-preserving spread increases the ratio of the equilibrium choke price ratio r. Income inequality therefore increases the discrepancy between the prices of the goods sold to the poor and to the rich.

A Pareto distribution brings additional equilibrium features that are reminiscent of Melitz and Ottaviano's (2008) model in which entering firms draw their productivity parameters from a Pareto probability distribution. For instance, it can be readily seen from (12) to (14) that the number of entrants M is proportional to the population size N and inversely proportional to entry cost f. In a similar spirit, the choke prices $(\hat{p}_L, \hat{p}_H)$, therefore the cut-off cost $\hat{c}$ and each equilibrium price $p^*(c)$, decrease with larger population size N and smaller entry cost f, which reflects the effect of increasing returns to scale and competition. Finally, a proportional increase in incomes (i.e., higher s_L and s_H holding s_H/s_L unchanged) raises the mass of entrants M, which reflects both increasing returns and love for variety.[17]

[17]With the Stone–Geary type preferences, proportional increases in income usually entail more complex effects than what we have here. For this reason, the literature on nonhomotheticity along the balanced growth path has avoided Stone–Geary; see, for examples, Boppart (2014) and Comin et al. (2018). The fact that the Pareto productivity distribution allows a tractable analysis for heterogenous income populations under the Stone–Geary preferences may be used for other research topics.

Under a Pareto productivity distribution, the equilibrium utility can be written as

$$U(s_H) = \frac{M\hat{p}_L^{\kappa}}{c_M^{\kappa}} \left[\frac{r^{\kappa}}{2\kappa} + \frac{a^{\kappa}}{2}\ln(r) - \frac{a^{\kappa}}{2}\ln(\alpha_H r + \alpha_L)\right] \tag{15}$$

$$U(s_L) = \frac{M\hat{p}_L^{\kappa}}{c_M^{\kappa}} \left[\frac{a^{\kappa}}{2\kappa} - \frac{a^{\kappa}}{2}\ln(a) - \frac{a^{\kappa}}{2}\ln(\alpha_H r + \alpha_L)\right], \tag{16}$$

where $a \equiv \hat{c}/\hat{p}_L = \left[(\alpha_H r + \alpha_L)^{1/2} - \alpha_H^{1/2} r^{1/2}\right]^2 / \left(1 - \alpha_H^{1/2}\right)^2$ with $1/r < a < 1 < r$.[18] Using the equilibrium conditions, we further get

$$\frac{M\hat{p}_L^{\kappa}}{c_M^{\kappa}} = s_L \left(\frac{N}{f\kappa^{\kappa} c_M^{\kappa}}\right)^{\frac{1}{\kappa+1}} \frac{\Phi(r)^{\frac{1}{\kappa+1}}}{\Gamma_L(r)}.$$

Thus, the ratio between high- and low-income consumers' utility values $U(s_H)/U(s_L)$ is a function of r and a, which depend only on income ratio s_H/s_L. In other words, the ratio of utility values depends only on the structure of income distribution. By contrast, utility levels depend negatively on the supply side parameters f and c_M and positively on the population mass N and income level indexed by s_L. A larger entry cost f entices fewer firms to enter the market, which generates less product diversity and weaker competition and ultimately harms consumers. A greater cost-distribution scale parameter c_M raises firms' costs and therefore reduces average productivity and utility levels. A larger population mass N increases product demands and entices more firms to enter. Product diversity increases and competition strengthens, which benefits consumers. Finally, for a given s_H/s_L, a higher s_L raises the endowments of labor efficiency units of all consumers. This raises the labor supply and allows firms to produce more goods, which increases consumption and utility.

4 Open Economy

We now study the implications of international trade and extend the above model to many trading countries and trade costs. We focus on the properties of income distribution and trade integration in the case of symmetric countries.

[18] Using this property, it can be shown that $U(s_H)/U(s_L) > 1$.

4.1 The Model in the Open Economy

We consider n symmetric countries with identical population mass N and distribution of labor efficiency units s_H and s_L with probability α_H and $\alpha_L \in (0,1)$ $(\alpha_H + \alpha_L = 1)$. Earnings in each country are respectively ws_H and ws_L where w is the local wage. In each country, firms produce under monopolistic competition. They pay an entry cost wf to enter the market and obtain a distinct good that can be produced at the quality-adjusted cost c, quality shifter $\beta(c)$, and marginal cost $w\beta(c)c$. The quality-adjusted cost c is drawn from the c.d.f. G. The measure of goods produced in each country is equal to $M\mathrm{d}G(c)$ where M is the mass of entrants. Firms now produce for local and foreign markets and incur an iceberg trade cost $\tau \geq 1$ per unit of exported good. That is, every unit of exported good costs $\tau w\beta(\omega)c$. Firms incur no trade cost on their local sales. Given the symmetric setting, all economic variables are equal and we can normalize all local wages to one.

Because of the symmetry, the aggregate demand for imports or local goods is given by the expression $Q\left(c,p\right)$ in (3). A firm producing a good with quality β and quality-adjusted cost c makes a profit $\left(p-\beta c\right)Q\left(c,p\right)$ on home sales and $\left(p-\tau\beta c\right)Q\left(c,p\right)$ on exports. Under monopolistic competition, the firm chooses the price that maximizes its profit in each local market taking as given the equilibrium choke prices at each local market. Optimal domestic quality-adjusted prices are written as before as

$$\frac{p^*\left(c\right)}{\beta(c)} = \begin{cases} \left(\hat{p}_{HL}c\right)^{1/2} & \text{if} \quad c \leq \hat{c} \\ \left(\hat{p}_{H}c\right)^{1/2} & \text{if} \quad c > \hat{c} \end{cases},$$

where $\hat{c}$ is given by (5), while optimal export quality-adjusted prices are given by

$$\frac{p^x\left(c\right)}{\beta(c)} = \begin{cases} \left(\hat{p}_{HL}\tau c\right)^{1/2} & \text{if} \quad \tau c \leq \hat{c} \\ \left(\hat{p}_{H}\tau c\right)^{1/2} & \text{if} \quad \tau c > \hat{c} \end{cases}.$$

The only difference is that the highest cost firm that sells to a foreign high (resp. low) income group has a cost equal to $\hat{p}_H/\tau$ (resp. $\hat{c}/\tau$). The equilibrium price levels and indices can be computed as before, and we have the following equilibrium condition:

$$\frac{e_H\left(\hat{p}_H,\hat{p}_L\right)}{s_H} = \frac{e_L\left(\hat{p}_H,\hat{p}_L\right)}{s_L} = \frac{1}{M}, \tag{17}$$

where

$$
\begin{aligned}
e_L\left(\hat{p}_H,\hat{p}_L\right) &= \int_0^{\hat{c}} \left(\hat{p}_L - \hat{p}_{HL}^{1/2} c^{1/2}\right) \mathrm{d}G\left(c\right) + \left(n-1\right) \int_0^{\hat{c}/\tau} \left(\hat{p}_L - \hat{p}_{HL}^{1/2} \left(\tau c\right)^{1/2}\right) \mathrm{d}G\left(c\right), \\
e_H\left(\hat{p}_H,\hat{p}_L\right) &= \int_0^{\hat{c}} \left(\hat{p}_H - \hat{p}_{HL}^{1/2} c^{1/2}\right) \mathrm{d}G\left(c\right) + \int_{\hat{c}}^{\hat{p}_H} \left(\hat{p}_H - \hat{p}_{H}^{1/2} c^{1/2}\right) \mathrm{d}G\left(c\right) \\
&\quad + \left(n-1\right)\left[\int_0^{\hat{c}/\tau} \left(\hat{p}_H - \hat{p}_{HL}^{1/2} \left(\tau c\right)^{1/2}\right) \mathrm{d}G\left(c\right) + \int_{\hat{c}/\tau}^{\hat{p}_H/\tau} \left(\hat{p}_H - \hat{p}_{H}^{1/2} \left(\tau c\right)^{1/2}\right) \mathrm{d}G\left(c\right)\right]
\end{aligned}
$$

express the consumers' average expenditure per available good.

A firm with cost c and quality shifter β gets the following profit from its home and foreign sales: $\pi\left(c\right) = \left(p^*(c) - \beta c\right) Q\left(c, p^*(c)\right) + \left(n-1\right)\left(p^x(c) - \tau\beta c\right) Q\left(c, p^x(c)\right)$. Free entry implies that $\mathrm{E}\left[\pi\left(c\right)\right] = f$. We write this as

$$
\pi\left(\hat{p}_H,\hat{p}_L\right) = \frac{f}{N}, \tag{18}
$$

where $\pi\left(\hat{p}_H,\hat{p}_L\right) = \mathrm{E}\left[\pi\left(c\right)\right]/N$, or equivalently,

$$
\begin{aligned}
\pi\left(\hat{p}_H,\hat{p}_L\right) &= \int_0^{\hat{c}} \left(\hat{p}_{HL}^{1/2} - c^{1/2}\right)^2 \mathrm{d}G\left(c\right) + \int_{\hat{c}}^{\hat{p}_H} \alpha_H \left(\hat{p}_{H}^{1/2} - c^{1/2}\right)^2 \mathrm{d}G\left(c\right) \\
&\quad + \left(n-1\right)\left[\int_0^{\hat{c}/\tau} \left(\hat{p}_{HL}^{1/2} - \left(\tau c\right)^{1/2}\right)^2 \mathrm{d}G\left(c\right) + \int_{\hat{c}/\tau}^{\hat{p}_H/\tau} \alpha_H \left(\hat{p}_{H}^{1/2} - \left(\tau c\right)^{1/2}\right)^2 \mathrm{d}G\left(c\right)\right].
\end{aligned}
$$

The mass of surviving firms in a country is equal to $MG(\hat{p}_H)$.

As in the closed economy, the three market conditions in (17) and (18) determine the choke prices $\left(\hat{p}_H,\hat{p}_L\right)$ and mass of entrants M.

Using a Pareto productivity distribution, we obtain the equilibrium identities

$$
\frac{s_H}{s_L} = \frac{\Gamma_H\left(r\right)}{\Gamma_L\left(r\right)}, \tag{19}
$$

$$
\hat{p}_L = \left[1 + \left(n-1\right)\tau^{-\kappa}\right]^{-\frac{1}{\kappa+1}} \left(\frac{c_M^{\kappa} f}{\kappa N}\right)^{\frac{1}{\kappa+1}} \Phi\left(r\right)^{-\frac{1}{\kappa+1}} \tag{20}
$$

$$
M = \frac{s_L N}{f}\frac{\Phi\left(r\right)}{\Gamma_L\left(r\right)}, \tag{21}
$$

where we use the definition $\hat{p}_H = r\hat{p}_L$ and Φ, Γ_L, and Γ_H are the same functions as in the closed-economy model. The difference with the closed economy lies in the presence of the term $\left(n-1\right)\tau^{-\kappa}$, which accounts for the number of trade partners and trade costs. Using the same argument as in Proposition 1, we conclude that the symmetric open economy also entails a unique equilibrium.

In the following subsections, we investigate the equilibrium properties.

4.2 Income Distribution and Trade Integration

The structure of equilibrium conditions (19) to (21) implies that Proposition 2 holds in the open economy. That is, a mean-preserving spread of the income distribution increases the choke price of each country's high-income group, expands the set of available goods, reduces the country's average productivity,[19] and shrinks the set of goods consumed by the poor relative to that consumed by the rich in each country. Those effects are confirmed by the expression of the rich's choke price $\hat{p}_H = r\hat{p}_L$, or equivalently,

$$\hat{p}_H = \left[1 + (n-1)\,\tau^{-\kappa}\right]^{-\frac{1}{\kappa+1}} \left(\frac{c_M^{\kappa} f}{\kappa N}\right)^{\frac{1}{\kappa+1}} \left[r\Phi\,(r)^{-\frac{1}{\kappa+1}}\right]. \tag{22}$$

By Lemma 1, a mean-preserving spread increases the rich's choke price $\hat{p}_H$ and reduces the poor's choke price $\hat{p}_L$, hence increasing r. Thus, $\mathrm{d}\left[r\Phi\,(r)^{-\frac{1}{\kappa+1}}\right]/\mathrm{d}r > 0$. At a given income distribution, a larger number of trade partners n or smaller trade cost τ decreases the rich's choke price $\hat{p}_H$. As a result, trade integration reduces the set of goods offered by each country while it reduces each country's average cost and raises the average productivity.

The question then becomes whether trade integration amplifies or attenuates the effect of a mean-preserving spread of the income distribution. Trade integration increases the term $[1 + (n-1)\,\tau^{-\kappa}]$ in (22). Since

$$\frac{\mathrm{d}^2\hat{p}_H}{\mathrm{d}r\mathrm{d}\left[1 + (n-1)\,\tau^{-\kappa}\right]} = -\frac{1}{\kappa+1}\frac{\hat{p}_H}{\left[1 + (n-1)\,\tau^{-\kappa}\right]}\frac{\mathrm{d}}{\mathrm{d}r}\ln\left[r\Phi\,(r)^{-\frac{1}{\kappa+1}}\right] < 0,$$

a mean-preserving spread increases the rich's choke price less in a more integrated trade network. Under a Pareto productivity distribution, the average cost can be shown to be proportional to $\hat{p}_H$ (see Appendix G). As a result, it increases less with a mean-preserving spread in a more integrated trade network. However, because average productivity is proportional to $1/\hat{p}_H$ and $\mathrm{d}^2\,(1/\hat{p}_H)\,/\mathrm{d}r\mathrm{d}\left[1 + (n-1)\,\tau^{-\kappa}\right] < 0$, it decreases more with a mean-preserving spread in a more integrated trade network. The multiplicative separability of the rich's choke price (22) implies that the percentage change of average productivity induced by a mean-preserving spread is independent of the level of trade openness. We have arrived at the following proposition:

[19] The (unweighted and quality-adjusted) average productivity is defined as the inverse of the average of quality-adjusted cost c across the firms in a country. Since all cost values of exporters and non-exporters lie in the interval $[0, \hat{p}_H]$, the average cost is given by $\int_0^{\hat{p}_H} cM\mathrm{d}G\,(c)\,/\,\int_0^{\hat{p}_H} M\mathrm{d}G\,(c)$, as in the closed economy. It increases with larger $\hat{p}_H$.

Proposition 3. *A mean-preserving spread of income distribution reduces each country's average productivity to a larger extent when trade integration is stronger (more trading partners or lower trade costs). However, the percentage decrease in average productivity induced by a mean-preserving spread is independent of the level of trade openness.*

Intuitively, trade increases the pool of competitors and reduces the friction for them to compete in the same arena. Only those with sufficiently high productivity survive. Thus, any effect of income inequality on average productivity is amplified by trade integration. Put differently, income redistribution from the rich to the poor improves each country's average productivity to a greater extent in a more integrated world.

4.3 Trade Integration and Welfare

The effects of trade integration/liberalization on consumption and number of goods are similar to those found in Melitz and Ottaviano (2008). Namely, trade integration induces both the rich and poor to consume more imported goods in both the quantity (intensive margin) and the number of goods (extensive margin). Moreover, the total set of goods expands for both groups. Due to pro-competitive effects of trade integration, product prices generally fall except for a few goods with productivity near $\hat{c}$ and $\hat{c}/\tau$.

However, the implications of trade on welfare differ greatly in our model from that of Melitz and Ottaviano due to the presence of two income groups and the income effect. From Appendix F, equilibrium utility levels can be written as

$$U(s_H) = \frac{M\left[1+(n-1)\tau^{-\kappa}\right]\hat{p}_L^{\kappa}}{c_M^{\kappa}}\left[\frac{r^{\kappa}}{2\kappa} + \frac{a^{\kappa}}{2}\ln(r) - \frac{a^{\kappa}}{2}\ln(\alpha_H r + \alpha_L)\right] \quad (23)$$

$$U(s_L) = \frac{M\left[1+(n-1)\tau^{-\kappa}\right]\hat{p}_L^{\kappa}}{c_M^{\kappa}}\left[\frac{a^{\kappa}}{2\kappa} - \frac{a^{\kappa}}{2}\ln(a) - \frac{a^{\kappa}}{2}\ln(\alpha_H r + \alpha_L)\right], \quad (24)$$

where $a = \hat{c}/\hat{p}_L$. By using (20) and comparing (15–16) and (23–24), we have

$$U(s_h) = \left[1+(n-1)\tau^{-\kappa}\right]^{\frac{1}{\kappa+1}} U^A(s_h), \quad (25)$$

where $h = H, L$, and $U^A(s_h)$ denotes the utility level under autarky. Trade integration therefore raises equilibrium utility levels of high- and low-income groups in the same proportion. The utility difference between these groups therefore rises in that same proportion.

The literature mainly highlights the effect of trade integration in countries with homogenous income. Our analysis also allows us to extend the welfare properties of changes in trade cost τ within a trade economy with countries with heterogenous income. In particular, we obtain from (25)

$$\frac{\mathrm{d}\ln U(s_h)}{\mathrm{d}\ln\tau} = -\frac{\kappa}{\kappa+1}\cdot\frac{(n-1)\tau^{-\kappa}}{1+(n-1)\tau^{-\kappa}}. \tag{26}$$

Hence an increase in trade cost not only raises the utility of each income group but augments it by the same percentage change.

As in Arkolakis et al. (2019) we can relate this formula to the domestic expenditure. The domestic expenditure of an individual with income s_h is the ratio λ_h of her domestic expenditure over her total expenditure. In Appendix F, we show that each income group's domestic expenditure share is the same, given by

$$\lambda_h = \lambda \equiv \frac{1}{1+(n-1)\tau^{-\kappa}}, \quad h = L, H; \tag{27}$$

thus λ is the overall population's domestic expenditure share. Accordingly, the share of consumption of home goods decreases with smaller trade cost and larger number of trading partners. Interestingly, this is the same for both income groups, even though these groups consume different amounts and types of goods. Hottman and Monarch (2020) and Borusyak and Jaravel (2021) provide empirical evidence on such a pattern of domestic expenditure shares across US income groups. This has important welfare implications as we further discuss in Section 5.4.

Differentiating the last equality with respect to τ and combining with (26), we get

$$\frac{\mathrm{d}\ln U(s_h)}{\mathrm{d}\ln\tau} = -\frac{1}{\kappa+1}\frac{\mathrm{d}\ln\lambda}{\mathrm{d}\ln\tau},$$

which can be rewritten in Arkolakis et al.'s (2019) format as

$$\mathrm{d}\ln U(s_h) = -(1-\eta)\frac{\mathrm{d}\ln\lambda}{\kappa}, \tag{28}$$

where κ is the trade elasticity, the last term measures the effect through domestic expenditure share, and $1-\eta$ captures the effect of variable markup with η equal to $1/(1+\kappa)$. Hence, the change in each group's utility can be assessed by the variation in the domestic expenditure share. Given the proportional impact of trade integration on each group, the above formula and conclusion apply to utilitarian welfare function and any linear social welfare function.[20] Hence, Arkolakis et al.'s (2019) welfare formula applies in the context

[20]Indeed, let a social welfare function be $W = \sum_h \nu_h U(s_h)$ where ν_h are welfare weights. A utilitarian welfare function imposes $\nu_h = \alpha_h$. Then, we have the following identities: $\mathrm{d}\ln W = \frac{1}{W}\sum_h \nu_h \mathrm{d}U(s_h)$

of several income groups.

The last paragraph discusses the effect of trade integration on utility levels. It is also interesting to study the effect in terms of equivalent income. Towards this aim, we compare the effect of a fall in trade cost with that of a global income rise. More precisely, we consider a common proportional rise in the income of each income group so that the ratio s_L/s_H and therefore the variables r and a remain constant. Therefore, a percentage fall in trade cost $\mathrm{d}\ln\tau$ and a percentage increase of income $\mathrm{d}\ln s_h$ yield the same change in utility if they satisfy the following relationship:

$$\left[\frac{\mathrm{d}\ln U\left(s_h\right)}{\mathrm{d}\ln\tau}\right]_{s_L\text{ fixed}}\mathrm{d}\ln\tau=-\left[\frac{\mathrm{d}\ln U\left(s_h\right)}{\mathrm{d}\ln s_L}\right]_{\tau\text{ fixed}}\mathrm{d}\ln s_h.$$

Computing the terms in squared brackets and reshuffling this equation, we obtain the equivalent income

$$\theta\equiv\frac{\mathrm{d}\ln s_h}{\mathrm{d}\ln\tau}=\frac{\kappa}{\kappa+1}\frac{(n-1)\,\tau^{-\kappa}}{1+(n-1)\,\tau^{-\kappa}}$$

(see Appendix F for details). In this expression, the value θ measures the percentage increase in income that is equivalent to a percentage fall in trade cost. The expression shows that the benefit of trade liberalization is stronger at lower trade costs ($\mathrm{d}\theta/\mathrm{d}\tau<0$) and in larger trade networks ($\mathrm{d}\theta/\mathrm{d}n>0$). For example, with $n=2$, $\tau=1.69$, and $\kappa=3.05$, we have $\theta=0.13$.[21] That is, a 10% fall in trade cost is equivalent to a 1.3% percent rise in the income of every group. In a network of ten countries, the fall in trade cost would be equivalent to a 4.8% rise in income ($\theta=0.48$ for $n=10$).

To sum up, we have the following proposition:

Proposition 4. *(i) Trade integration yields the same proportional increase in utility for all income groups despite their different baskets of consumptions. Utility and equivalent income rise faster at lower trade costs and in larger trade networks. (ii) The change in each group's utility and country welfare can be assessed by the same variation in domestic expenditure share, trade elasticity, and a variable-markup effect as in Arkolakis et al. (2019), regardless of the income distribution.*

5 Quantitative Analysis

In this section, we conduct a quantitative analysis of income distribution and trade integration. In order to bring more realism in the effect of income distributions, we first

$=\frac{1}{W}\sum_h\nu_hU\left(s_h\right)\mathrm{d}\ln U\left(s_h\right)=\left[\frac{1}{W}\sum_h\nu_hU\left(s_h\right)\left(-\left(1-\eta\right)\frac{\mathrm{d}\ln\lambda}{\kappa}\right)\right]=-\left(1-\eta\right)\frac{\mathrm{d}\ln\lambda}{\kappa}$ where we use (28).

[21] These parameter values are adopted in our quantitative analysis in Section 5. See the calibration in Section 5.2.

extend the model to multiple income groups and then fit the model to the income distribution in the US. This calibration allows us to quantify the effects on productivity and welfare measures.

5.1 Multiple Income Groups

In this subsection we consider many income groups $h \in \{1, 2, ..., H\}$, $H \geq 2$, with population share α_h ($\sum_{h=1}^{H} \alpha_h = 1$). In every group h, each individual is endowed with s_h labor efficiency units with $s_1 < s_2 < ... < s_H$ and has the same preferences as in the benchmark model. Each individual in group h has an individual demand function $q_h(\omega) = \hat{p}_h / p(\omega) - 1/\beta(\omega)$ where $\hat{p}_h$ is the group h's choke price. Choke prices are ordered by income such that $\hat{p}_1 < ... < \hat{p}_H$. The aggregate demand includes H segments that are given by

$$Q_h(p) = N\left(\frac{\sum_{k=h}^{H} \alpha_k \hat{p}_k}{p} - \frac{\sum_{k=h}^{H} \alpha_k}{\beta}\right) \quad \text{for } \frac{p}{\beta} \in [\hat{p}_{h-1}, \hat{p}_h), \tag{29}$$

$h \in \{1, 2, ..., H\}$, and where we define $\hat{p}_0 \equiv 0$. Each demand segment h includes the consumers with income larger than or equal to s_h.

As before, we can denote each firm by its cost per unit quality c. The firm's problem is to find the best profit out of all possible demand segments; that is, $\max_{h,p} \pi_h(p, c) \equiv (p - \beta(c)c)\, Q_h(p)$. On each demand segment h, the first-order condition gives the optimal quality-adjusted price

$$\frac{p_h^*(c)}{\beta(c)} = \left(\frac{\sum_{k=h}^{H} \alpha_k \hat{p}_k}{\sum_{k=h}^{H} \alpha_k}\right)^{1/2} c^{1/2} \tag{30}$$

and yields the profit $\pi_h^*(c) \equiv \pi_h(p_h^*(c), c)$. The firm chooses the demand segment h that maximizes its profit $\pi_h^*(c)$. It is shown that the firm chooses the segment h if its cost c belongs to the cost interval $[\hat{c}_{h-1}, \hat{c}_h)$ where each cutoff cost $\hat{c}_h$ is defined by $\pi_h^*(\hat{c}_h) = \pi_{h+1}^*(\hat{c}_h)$ that solves as

$$\hat{c}_h^{1/2} = \frac{\left(\sum_{k=h}^{H} \alpha_k \hat{p}_k\right)^{1/2} - \left(\sum_{k=h+1}^{H} \alpha_k \hat{p}_k\right)^{1/2}}{\left(\sum_{k=h}^{H} \alpha_k\right)^{1/2} - \left(\sum_{k=h+1}^{H} \alpha_k\right)^{1/2}}$$

for $h = 1, 2, ..., H-1$. For $h = 0$ and H it is defined that $\hat{c}_0 \equiv 0$ and $\hat{c}_H = \hat{p}_H$. Importantly, the firm's optimal price $p^*(c)$ jumps upwards when its cost rises and passes on each cutoff cost $\hat{c}_h$. Formally,

$$\lim_{c \to \hat{c}_h -} \frac{p^*(c)}{\beta(c)} < \hat{p}_h < \lim_{c \to \hat{c}_h +} \frac{p^*(c)}{\beta(c)} \quad \text{for all } h \in \{1, 2, ..., H-1\}.$$

This analysis is consistent with the case of two income groups.

In an open economy, the optimal domestic prices are defined as in (30) while the optimal export quality-adjusted prices must be adapted to the presence of trade cost τ as in Section 4. In particular,

$$\frac{p_h^x(c)}{\beta(c)} = \left(\frac{\sum_{k=h}^{H} \alpha_k \hat{p}_k}{\sum_{k=h}^{H} \alpha_k} \right)^{1/2} (\tau c)^{1/2} \quad \text{for } c \in [\hat{c}_{h-1}/\tau, \hat{c}_h/\tau).$$

The equilibrium conditions are given by the free-entry condition and each group's budget constraint. Let us denote the vector of choke prices by $\hat{p} \equiv \{\hat{p}_l\}_{l=1}^{H}$ and the vector of the ratios of each group's choke price to that of the poorest by $r \equiv \{\hat{p}_h/\hat{p}_1\}_{h=2}^{H}$. Under a Pareto productivity distribution, the equilibrium conditions for $\hat{p}$ and M can be written as

$$\frac{s_h}{s_1} = \frac{\Gamma_h(r)}{\Gamma_1(r)} \quad \text{for } h = 2, 3, ..., H, \tag{31}$$

$$\hat{p}_1 = \left[1 + (n-1)\tau^{-\kappa}\right]^{-\frac{1}{\kappa+1}} \times \left(\frac{c_M^{\kappa} f}{\kappa N}\right)^{\frac{1}{\kappa+1}} \Phi(r)^{-\frac{1}{\kappa+1}}, \tag{32}$$

$$M = \frac{s_1 N}{f} \frac{\Phi(r)}{\Gamma_1(r)}, \tag{33}$$

where the definitions of functions Γ_h and Φ are given in Appendix G. As in Section 4, it can be seen that the ratios of choke prices r and the mass of entrant M do not depend on trade parameters τ and n. These parameters affect only the lowest choke prices $\hat{p}_1$ and therefore alter all choke prices in the same proportion.

Finally, the individual utility in income group h is given by $U(s_h) = (1 + (n-1)\tau^{-\kappa})^{\frac{1}{\kappa+1}}$ $U^A(s_h)$ where $U^A(s_h)$ denotes the utility level under autarky ($\tau = 1$). Hence, trade integration affects the utility of each income group in the same proportion. The changes in utility with respect to τ and n can be related to the changes in the ratio of her domestic expenditure over total expenditure, λ_h, in the same way as in Section 4.3: that is,

$$\mathrm{d} \ln U(s_h) = -(1-\eta) \frac{\mathrm{d} \ln \lambda}{\kappa},$$

where $\eta = 1/(1+\kappa)$ again captures the effect of variable markup (see Appendix G). As a result, welfare gains are assessed by the same variations in domestic expenditure share, trade elasticity, and variable-markup effect as in Arkolakis et al. (2019), regardless of the income distribution.

We now apply this model with multiple income groups to the following calibration exercise.

5.2 Calibration

The multiple income group model is calibrated to an open economy with two symmetric countries with the US economy being the representative one. Our quantitative analysis presents the effects of income inequality and trade.

In the calibration of the baseline model, each country includes the US working population ($N = 148 * 10^6$) and its 2015 income quintiles ($s^o = \{s^o_h\}_{h=1}^H$ with $H = 5$) after federal taxes and transfers as reported by the Congressional Budget Office. The same data set also reports the income levels before taxes and transfers. Because the quality parameter β does not affect our main results, we normalize it to one.

We recover four baseline model parameters τ, κ, f, and c_M by matching four empirical targets of consumers' domestic expenditure share, firms' markups, survival rates, and employment rates (see details in Appendix G, Calibration). We use information on domestic absorption and imports from Penn World Table 9.0 to calculate the US 2014 domestic expenditure share $domshare = 0.828$. The relationship (27) then yields the trade cost value $\tau^o = 1.69$. This is very close to the US trade costs estimated in the literature.[22] In the spirit of Behrens and Murata (2012), we impose an (unweighted) average markup on local sales $mrkup^o \equiv \int_0^{\hat{p}_H} p^*(c)/c\mathrm{d}G$ of 115%.[23] We combine this relationship and (31) for the baseline income distribution s^o and markup $mrkup^o$ to pin down the choke price ratios r^o and productivity distribution shape parameter κ^o. We then use the value of firms' survival rate $surv^o = G(\hat{p}_H)$ of 90%[24] and average employment per firm $empl^o = N/(MG(\hat{p}_H))$ of 66 workers as reported in the 2015 US census data ($148 * 10^6$ workers in $2.22 * 10^6$ firms having more than 5 employees). Applying the identities (32) and (33) to those values yields the entry cost f^o and the productivity distribution scale parameter c^o_M.[25]

[22] For example, Novy (2013) estimates the trade costs τ in 2000 between the US and Germany and between the US and the UK to be 1.70 and 1.63, respectively. Using the same approach, the same set of estimates in 2014 reported by the World Bank's *International Trade Costs* data set are 1.723 and 1.704.

[23] For example, using Taiwanese manufacturing data and the markup-estimation approach by De Loecker and Warzynski (2012), Edmond, Midrigan, and Xu (2015) find an unweighted average markup of 1.13.

[24] We take the average exit rate as 0.1. See, for example, Klepper and Thompson (2006).

[25] Recovered parameters are $\kappa^o = 3.05$, $c^o_M = 0.344$, and $f^o = 1.31 * 10^6$.

5.3 Effects of Income Inequality

Table 1 presents the values of economic variables when workers' incomes increasingly spread about their mean. The three columns present various instances of income distribution. Column 2 reproduces the baseline situation of income distribution after federal taxes and transfers. Column 3 presents the mean-preserving contraction of the income distribution so that the average income is uniformly redistributed across quintiles (full redistribution). Finally, Column 1 displays the case of the mean-preserving spread of the income distribution so that income levels are spread in the same proportions as they were before taxes and transfers but the average income is kept equal to that after taxes and transfers. That is, $s_h = (s_h^{\text{before tax}}/\text{average}(s_h^{\text{before tax}}))*\text{average}(s_h^o)$ where $s_h^{\text{before tax}}$ is the 2015 distribution of income before federal tax and transfers reported by the Congressional Budget Office. The first six rows of Table 1 indicate the Gini coefficient and the income level in each quintile (2015 USD).

The second stylized fact and the theoretical result in Sections 2 and 3.4 state that average productivity and income dispersion are negatively correlated. The last two rows of Table 1 quantify these effects on two average productivity measures. The first one is the unweighted average productivity measured by the inverse of the average cost across firms. The second one is the weighted average productivity, which is given by the inverse of the average cost weighted by the total output of each firm. This is indeed the ratio of total cost of production over the total output in the economy (see Appendix G). The second measure is larger than the first because higher-productivity firms produce more output and hence carry heavier weights. As predicted by the theory, these measures decrease as the income distribution spreads around its mean. Ceteris paribus, as their income rises, rich consumers purchase goods with higher prices and also higher costs, which increases firms' average cost.

How large are the productivity gains resulting from a more even income distribution and given by the economic mechanism presented in this paper? Federal taxes and transfers smooth the income distribution and yield productivity gains of 3.3% ($= 2.75/2.66 - 1$) for the unweighted average productivity and 0.7% ($= 4.50/4.47 - 1$) for the weighted one. Going from no redistribution to full redistribution gives the maximal productivity gains possible of 23.7% and 2.5% for unweighted and weighted average productivity, respectively.

	No redistribution	Federal redistribution	Full redistribution
Gini	0.45	0.36	0.00
s_1	17100	33400	90100
s_2	37100	46800	90100
s_3	59900	64700	90100
s_4	91000	90600	90100
s_5	245500	215000	90100
Unweighted ave. prod.	2.66	2.75	3.29
Weighted ave. prod.	4.47	4.50	4.58

Table 1: Effect of Income Redistribution.

5.4 Effects of Trade Costs

Table 2 presents main economic indicators for alternative trade costs, keeping the other parameters at the baseline model levels. From left to right, the columns successively report the cases of free-trade ($\tau = 1$), a 10% fall in baseline trade cost ($\tau = 1.51$), the baseline trade cost ($\tau = 1.68$), and autarky ($\tau \to \infty$). As predicted by the model (Section 4.2), unweighted average productivity increases with deeper trade integration as selection becomes tougher. Weighted average productivity follows a similar pattern. Lowering trade cost by 10% from the baseline model augments unweighted productivity by 1.8% ($= 2.80/2.75 - 1$). Going from autarky to free trade increases average productivity by 18.6% ($= 3.12/2.63 - 1$). The result that trade liberalization induces higher average productivity is consistent with the firm-selection literature *à la* Melitz (2003). The expenditure share on domestic goods falls as the economy moves from autarky to free trade.

Trade cost	1	1.51	1.68	∞
λ_h	0.500	0.782	0.831	1.00
Unweighted ave. prod.	3.12	2.80	2.75	2.63
Weighted ave. prod.	5.10	4.57	4.50	4.30
$\Delta U_h/U_h$	0.134	0.015	0.00	-0.045

Table 2: Effects of Trade Costs

The last row of Table 2 shows the relative changes in utility $\Delta U_h/U_h$, which is the same across groups, as shown in Sections 4.3 and 5.1. The relative changes in utility predicted by the theory match those computed numerically. For instance, lowering trade cost by 10% to $\tau = 1.51$ reduces domestic expenditure over total expenditure λ_h by 5.9% (=

$0.782/0.831 - 1$). Using this value and κ^o in expression (28) yields a loss of 1.6% of utility, which corresponds to the relative utility loss that is computed and reported in the table. In terms of welfare gains from trade, the literature has mainly conducted the comparison based on the welfare gains from autarky to the baseline model. Our model mechanism and parameterization implies welfare gains from trade of 4.7% ($= 1/(1 - 0.045) - 1$).

5.5 Inequality and Trade Costs

Table 3 reports the average productivity for various combinations of trade cost and income distribution. It displays the unweighted (top) and weighted (bottom) average productivities. A mean-preserving spread of income distribution reduces average productivity levels whereas trade integration increases them. However, the mean-preserving spread of income has a larger impact on unweighted average productivity with more trade integration, as predicted by Proposition 3. The same pattern holds true for weighted average productivity. Under autarky, the unweighted and weighted average productivities decline by 0.61 and 0.11, respectively when one moves from full redistribution to no redistribution. Under free trade, the unweighted and weighted average productivities decline by 0.72 and 0.13, respectively, for the same change in income distribution. Thus, the impacts of trade cost on the effect of income distribution on average productivity seem quantitatively small. In contrast, the separate impacts of trade and income distribution are much larger.

Proposition 3 also states that the percentage decrease in unweighted average productivity induced by a mean-preserving spread of income is independent of the level of trade openness. Appendix G shows that this statement is also true for weighted average productivity. These properties can be verified using the figures in Table 3 (e.g. $3.01/3.72 = 2.70/3.34$, etc. for unweighted average productivity and $5.06/5.19 = 4.54/4.66$, etc. for the weighted one). When the income distribution changes from full redistribution to no redistribution, the decrease in average productivity is 19.3% and 2.5% for unweighted and weighted average productivity, respectively.

	No redistribution	Federal redistribution	Full redistribution
$\tau = 1$	3.01	3.12	3.73
	5.06	5.10	5.19
$\tau = 1.51$	2.70	2.80	3.34
	4.54	4.57	4.66
$\tau = 1.68$	2.66	2.75	3.29
	4.47	4.50	4.58
$\tau = \infty$	2.53	2.63	3.14
	4.26	4.30	4.37

Table 3: Unweighted/weighted Average Productivity.

A final way to grasp the impact of income inequality and trade cost is to maintain the average productivity at its baseline value. For instance, say that we maintain the unweighted average productivity at 4.50. What then would be the trade cost that would maintain this productivity after we eliminate federal taxes and benefits? The first column in Table 3 suggests that this trade cost lies between $\tau = 1.51$ and 1.68, which represents a reduction of 10% and 0% in the baseline trade cost. We actually compute that the trade cost should be reduced by 5.6%. Similarly, implementing full redistribution and maintaining productivity is equivalent to a rise in trade cost of 18%. We can do the same exercise by maintaining the weighted average productivity at 2.75. Then, eliminating redistribution is equivalent to a drop in trade cost of 19%. Implementing full redistribution has a greater impact than raising trade cost to the autarchic level.

The above calculations show that the required changes in trade cost needed to offset the negative impact of income inequality on average productivity are non-negligible. This can be explained by combining the following two effects. First, Table 3 indicates that the effect of income distribution on productivity is substantial. Second, the large home bias, as implied by the high value of domestic expenditure share at 0.828, reveals the existence of strong barriers between the two markets. This implies a large room for improving average productivity insofar as a trade cost decline introduces fiercer competition and selection. In short, the large home bias implies that the separate effect of trade cost on productivity can also be important.

6 Conclusion

In this paper, we study the impact of income inequality on the individual consumption and firm selection. We investigate its impact on aggregate productivity and welfare in a global economy where consumers have nonhomothetic preferences for goods with various quality levels. We find a negative effect of the rich's income on the poor's consumption and vice-versa. We also find that stronger inequality, measured as a mean-preserving spread of income distribution, reduces average productivity by softening firm selection and reshuffling consumption from low-quality to high-quality goods.

We further show that trade openness amplifies the negative effect of income inequality on average productivity. Under a Pareto productivity distribution, our model predicts that domestic expenditure share is constant across income groups, which is empirically validated by Hottman and Monarch (2020) and Borusyak and Jaravel (2021). As the latter authors suggest, we also theoretically show that gains from trade are income-neutral. To the best of our knowledge, this paper presents the first model with nonhomothetic preferences that generates such a property.

Note that the income neutrality result in this paper operates only through the consumers' expenditure channel. Trade may have different welfare effect on different income groups in a richer setting that incorporates a channel through workers' earnings.

Appendix A: Consumers' Demands

Individuals are endowed with utility function $U = \int_{\omega\in\Omega} \ln\left(1+\beta(\omega)q\left(\omega\right)\right) \mathrm{d}\omega$ over the commodity space $\Omega \subset \mathbb{R}$. Note that firm entry limits the mass of commodities that are offered. Let $\overline{\Omega}$ be the set of commodities that are actually offered and associated with a price $p\left(\omega\right)$, $\omega \in \overline{\Omega}$. Other commodities $\omega \in \Omega\backslash\overline{\Omega}$ are not offered and cannot be consumed so that $q(\omega) = 0$ for $\omega \in \Omega\backslash\overline{\Omega}$. An individual in the income group h chooses the consumption $q\left(\omega\right)$, $\omega \in \overline{\Omega}$ that maximizes her utility U subject to her budget constraint $\int_{\omega\in\overline{\Omega}} p\left(\omega\right) q\left(\omega\right) \mathrm{d}\omega = s_h$. The Lagrangian function of individual h with income s_h is therefore defined as

$$\mathcal{L}_h = \int_{\omega\in\overline{\Omega}} \ln\left(1+\beta\left(\omega\right)q\left(\omega\right)\right) \mathrm{d}\omega + \lambda_h \left(s_h - \int_{\omega\in\overline{\Omega}} p\left(\omega\right) q\left(\omega\right) \mathrm{d}\omega\right),$$

$\Omega_h \subseteq \mathbb{R}$. This is a concave function so that the following first-order condition yields the consumer's best consumption choice:

$$\begin{array}{lll} \frac{\partial\mathcal{L}}{\partial q(\omega)} = \frac{1}{q(\omega)+1/\beta(\omega)} - \lambda_h p\left(\omega\right) = 0 & \text{if} & q\left(\omega\right) > 0 \\ \frac{\partial\mathcal{L}}{\partial q(\omega)} = \frac{1}{q(\omega)+1/\beta(\omega)} - \lambda_h p\left(\omega\right) < 0 & \text{if} & q\left(\omega\right) = 0. \end{array}$$

The set of consumed goods is given by $\Omega_h \equiv \{\omega : q\left(\omega\right) > 0\} = \{\omega : p\left(\omega\right) < 1/\lambda_h\}$. For $\omega \in \Omega_h$, the first-order condition entails

$$q_h\left(\omega\right) = \frac{1}{\lambda_h p\left(\omega\right)} - \frac{1}{\beta\left(\omega\right)},$$

and thus

$$\lambda_h = \frac{\int_{\omega\in\Omega_h} \mathrm{d}\omega}{s_h + \int_{\omega\in\Omega_h} \frac{p(\omega)}{\beta(\omega)} \mathrm{d}\omega}.$$

Plugging λ_h back into the demand function, we obtain individual demand function

$$q_h\left(\omega\right) = \frac{\hat{p}_h}{p\left(\omega\right)} - \frac{1}{\beta\left(\omega\right)},$$

where

$$\hat{p}_h \equiv \frac{1}{\lambda_h} = \frac{s_h + P_h}{|\Omega_h|}$$

is the choke price of consumer with income s_h, $P_h \equiv \int_{\omega\in\Omega_h} \left[p\left(\omega\right)/\beta\left(\omega\right)\right] \mathrm{d}\omega$ is the aggregate price index for the goods consumed by s, and $|\Omega\left(s\right)| = \int_{\omega\in\Omega_h} \mathrm{d}\omega$ is the measure of the set of goods consumed by individual h. Combining the above results, we obtain (1) and (2).

Note that $\beta(\omega)\,\hat{p}_h$ is the highest price that h is willing to pay to purchase any nonnegative amount of a good ω. When s_h increases, λ_h falls and $\hat{p}_h$ rises so that Ω_h expands. As a result, one gets $s_H \geq s_L \iff \hat{p}_H \geq \hat{p}_L$.

Finally, given that $q(\omega) = 0, \omega \notin \Omega_h$, the consumer's utility can successively be rewritten as

$$U_h = \int_{\omega\in\Omega_h} \ln\left(1+\beta(\omega)q\left(\omega\right)\right)\mathrm{d}\omega + \int_{\omega\in\Omega\backslash\Omega_h} \ln\left(1\right)\mathrm{d}\omega = \int_{\omega\in\Omega_h} \ln\left(1+\beta(\omega)q\left(\omega\right)\right)\mathrm{d}\omega.$$

The indirect utility is thus equal to

$$V_h = \int_{\omega\in\Omega_h} \ln\left(\frac{s_h+P_h}{|\Omega_h|}\frac{\beta(\omega)}{p\left(\omega\right)}\right)\mathrm{d}\omega. \tag{34}$$

Appendix B: Firms' Choices

Let us denote $\beta(c)$ by β. Note that it is often convenient to express conditions in terms of quality-adjusted prices p/β. The problem for a firm with cost c is

$$\begin{aligned}\max_p\ \pi &= \left(p-\beta c\right)Q\left(p\right)\\ &= \begin{cases} \left(p-\beta c\right)\alpha_H N\left(\frac{\hat{p}_H}{p}-\frac{1}{\beta}\right) & \text{if} \quad p/\beta \in [\hat{p}_L, \hat{p}_H) \\ \left(p-\beta c\right)N\left(\frac{\hat{p}_{HL}}{p}-\frac{1}{\beta}\right) & \text{if} \quad p/\beta \in [0, \hat{p}_L) \end{cases}.\end{aligned}$$

For $p/\beta \in [0, \hat{p}_L)$, the firm sells to both groups and chooses the quality-adjusted price $p^*(c)/\beta = c^{1/2}\hat{p}_{HL}^{1/2}$ and quality-adjusted markup $(\hat{p}_{HL}/c)^{1/2}$. The quality-adjusted price increases in c, and the quality-adjusted markup decreases in c, showing a pro-competitive effect. The firm gets a profit equal to $\pi^*_{HL}(c) = N\left[\hat{p}_{HL}^{1/2} - c^{1/2}\right]^2$. For $p/\beta \in [\hat{p}_L, \hat{p}_H)$, a firm sells only to high-income consumers and sets a quality-adjusted prices $p^*(c)/\beta = c^{1/2}\hat{p}_H^{1/2}$ and quality-adjusted markup $(\hat{p}_H/c)^{1/2}$. These prices increase in c, and markups decrease in c. The firm gets a profit equal to $\pi^*_H(c) = \alpha_H N\left[(\hat{p}_H)^{1/2} - c^{1/2}\right]^2$. The firm chooses to charge $p^*(c)/\beta = c^{1/2}\hat{p}_{HL}^{1/2}$ if and only if $\pi^*_{HL}(c) \geq \pi^*_H(c)$, which is equivalent to

$$c^{1/2} \leq \hat{c}^{1/2} \equiv \frac{\hat{p}_{HL}^{1/2} - (\alpha_H\hat{p}_H)^{1/2}}{1-\alpha_H^{1/2}}.$$

This argument yields (4) and (5). Observe that $p_H > p_{HL}$. Hence, there is an upward jump of the price schedule $p^*(c)$ at $\hat{c}$.

In the product market equilibrium, it must be that each income group purchases the goods that are targeted to them. In particular, the low-income consumers should buy only the goods produced at cost in the range $[0, \hat{c}]$. This means that their choke price $\hat{p}_L$ should satisfy $p^*(\hat{c}-0)/\beta(\hat{c}-0) < \hat{p}_L < p^*(\hat{c}+0)/\beta(\hat{c}+0)$. We show that this condition holds. Indeed, since $p^*(\hat{c}-0)/\beta(\hat{c}-0) = \hat{c}^{1/2}\hat{p}_{HL}^{1/2}$ and $p^*(\hat{c}+0)/\beta(\hat{c}+0) = \hat{c}^{1/2}\hat{p}_H^{1/2}$ the previous condition becomes $\hat{c}^{1/2}\hat{p}_{HL}^{1/2} < \hat{p}_L < \hat{c}^{1/2}\hat{p}_H^{1/2}$. Plugging in the value of $\hat{c}$ and defining $r = \hat{p}_H/\hat{p}_L$ with $r > 1$ since $\hat{p}_H > \hat{p}_L$, we get the following inequalities:

$$(\alpha_H r + \alpha_L) - (\alpha_H r(\alpha_H r + \alpha_L))^{1/2} < 1 - \alpha_H^{1/2} < ((\alpha_H r + \alpha_L) r)^{1/2} - (\alpha_H)^{1/2} r.$$

Because $\alpha_H + \alpha_L = 1$, we have that the left-hand side and right-hand side are equal to the middle term for $r = 1$. It can be shown that the left-hand side falls with higher r while the right-hand side rises with it. Hence the inequalities are always satisfied.

For all goods to be supplied by firms with quality-adjusted cost c to poor individuals, it must also be that $c < \hat{p}_L$. This is obtained if $\hat{c} < \hat{p}_L$. Plugging in the value of $\hat{c}$ and using $r = \hat{p}_H/\hat{p}_L$ we get the condition:

$$((\alpha_H r + \alpha_L))^{1/2} - (\alpha_H r)^{1/2} < 1 - \alpha_H^{1/2},$$

where the left-hand side decreases with larger r and is equal to the right-hand side at $r = 1$. Thus, the condition is always satisfied.

Appendix C: Existence

The equilibrium is represented by the vector of variables $(\hat{p}_H, \hat{p}_L, M)$ with $\hat{p}_H \geq \hat{p}_L \geq 0$ and $M > 0$ that satisfy the market conditions (7), $e_H(\hat{p}_H, \hat{p}_L)/s_H = e_L(\hat{p}_H, \hat{p}_L)/s_L = 1/M$, and entry condition (9), $\pi(\hat{p}_H, \hat{p}_L) = f/N$ where

$$e_H(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_H - (\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2} c^{1/2}\right) \mathrm{d}G(c) + \int_{\hat{c}}^{\hat{p}_H} \left(\hat{p}_H - \hat{p}_H^{1/2} c^{1/2}\right) \mathrm{d}G(c),$$
$$e_L(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{c}} \left(\hat{p}_L - (\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2} c^{1/2}\right) \mathrm{d}G(c),$$

and

$$\pi(\hat{p}_H, \hat{p}_L) = \int_0^{\hat{p}_H} \max\left\{\left((\alpha_H \hat{p}_H + \alpha_L \hat{p}_L)^{1/2} - c^{1/2}\right)^2, \alpha_H \left(\hat{p}_H^{1/2} - c^{1/2}\right)^2\right\} \mathrm{d}G(c)$$

In these equations, $\hat{c}$ is implicitly given by the solution of

$$\hat{c}^{1/2} = \frac{\hat{p}_{HL}^{1/2} - (\alpha_H \hat{p}_H)^{1/2}}{1 - \alpha_H^{1/2}}$$

with $\partial\hat{c}/\partial\hat{p}_H < 0 < \partial\hat{c}/\partial\hat{p}_H$. It can readily be shown that $\pi_H > 0, \pi_L > 0$ and $e_{LL} > 0 > e_{LH}$ where $e_{hl} = \partial e_h/\partial\hat{p}_l$ and $\pi_l = \partial\pi/\partial\hat{p}_l$, $h, l \in \{H, L\}$.

We can rewrite the equilibrium conditions as

$$H(\hat{p}_H, \hat{p}_L, M) \equiv M - \frac{\alpha_H s_H + \alpha_L s_L}{\alpha_H e_H + \alpha_L e_L} = 0, \tag{35}$$

$$F(\hat{p}_H, \hat{p}_L) \equiv \frac{e_H}{s_H} - \frac{e_L}{s_L} = 0, \tag{36}$$

$$\Pi(\hat{p}_H, \hat{p}_L) \equiv \pi(\hat{p}_H, \hat{p}_L) - \frac{f}{N} = 0. \tag{37}$$

The equilibrium is then given by the vector $(\hat{p}_H, \hat{p}_L, M)$ that solves (35), (36), and (37). Note that the choke prices are solutions of (36) and (37) while the mass of entrants is the solution of (35) at equilibrium choke prices.

We now show the existence of equilibrium. Note that since the e_H, e_L and π are continuous functions of $(\hat{p}_H, \hat{p}_L, M)$, the expressions in conditions (37), (35), and (36) are also continuous on $\mathbb{R}^{+3}$. It then suffices to prove that each expression has opposite signs on two points in the support of $(\hat{p}_H, \hat{p}_L, M) \in \mathbb{R}^{+3}$ with $\hat{p}_H \geq \hat{p}_L \geq 0$ and $M > 0$.

First, suppose that $(\hat{p}_H, \hat{p}_L, M) = (y, 0, M)$. Then, $\hat{c} = 0$ so that $e_H(y, 0) = \int_0^y \left(y - y^{1/2}c^{1/2}\right) \mathrm{d}G(c) > 0$ and $e_L(y, 0) = 0$. We compute

$$\Pi(y, 0) = \alpha_H \int_0^y \left(y^{1/2} - c^{1/2}\right)^2 \mathrm{d}G(c) - \frac{f}{N},$$

$$H(y, 0, M) = M - \frac{\alpha_H s_H + \alpha_L s_L}{\alpha_H \int_0^y (y - y^{1/2}c^{1/2})\, \mathrm{d}G(c)},$$

$$F(y, 0) = \frac{1}{s_H} \int_0^y \left(y - y^{1/2}c^{1/2}\right) \mathrm{d}G(c).$$

If y is small enough, we have $\Pi(y, 0) < 0$, $H(y, 0, M) < 0$, and $F(y, 0) > 0$.

Second, we have assumed that $G(c)$ has a bounded support and finite mean. That is, $G : [0, c_M] \to [0, 1]$ such that $\mathrm{E}(c) = \int_0^{c_M} c\mathrm{d}G(c) < \infty$. We define $\hat{p}_L = x$, $\hat{p}_H = rx$, $\hat{p}_{HL} = (\alpha_H r + \alpha_L)\, x$, and $\hat{c} = ax$ where $1 \leq r < \infty$ and $a^{1/2} \equiv \left[(\alpha_H r + \alpha_L)^{1/2} - \alpha_H^{1/2} r^{1/2}\right] / \left(1 - \alpha_H^{1/2}\right) \in (0, 1]$. We further set x such that $c_M < ax < x < rx$. This implies that $\int_0^{rx} \mathrm{d}G = \int_0^{c_M} \mathrm{d}G = 1$, $\int_0^{ax} c^{1/2}\mathrm{d}G = \int_0^{c_M} c^{1/2}\mathrm{d}G(c) = \mathrm{E}(c^{1/2})$, and $\int_{ax}^{rx} c^{1/2}\mathrm{d}G(c) = \int_{c_M}^{c_M} c^{1/2}\mathrm{d}G(c) = 0$. Thus when

$(\hat{p}_H, \hat{p}_L) = (rx, x)$, we have

$$e_H(rx, x) = rx - (\alpha_H r + \alpha_L)^{1/2} x^{1/2} \mathrm{E}(c^{1/2}),$$
$$e_L(rx, x) = x - (\alpha_H r + \alpha_L)^{1/2} x^{1/2} \mathrm{E}(c^{1/2})$$

while

$$\Pi(rx, x) = \left[(\alpha_H r + \alpha_L) x - 2(\alpha_H r + \alpha_L)^{1/2} x^{1/2} \mathrm{E}(c^{1/2}) + \mathrm{E}(c)\right] - \frac{f}{N},$$
$$H(rx, x, M) = M - \frac{\alpha_H s_H + \alpha_L s_L}{(\alpha_H r + \alpha_L) x - (\alpha_H r + \alpha_L)^{1/2} x^{1/2} \mathrm{E}(c^{1/2})},$$
$$F(rx, x) = \left(\frac{r}{s_H} - \frac{1}{s_L}\right) x + \left(\frac{1}{s_L} - \frac{1}{s_H}\right) (\alpha_H r + \alpha_L)^{1/2} x^{1/2} \mathrm{E}(c^{1/2}).$$

For x sufficiently large, we have $\Pi(rx, x) > 0$ and $H(rx, x, M) > 0$ while $F(rx, x) < 0$ if $r < s_H/s_L$.

We can then choose three scalars, x large enough, y small enough and $r < s_H/s_L$ such that functions Π, H, and F have opposite signs at the points $(\hat{p}_H, \hat{p}_L, M) = (rx, x, M')$ and $(y, 0, M'')$. This proves the existence of an equilibrium.

As expressed in the main text, the condition for the uniqueness of the general equilibrium requires that $\partial e_h/\partial \hat{p}_h > 0$ and $\partial e_h/\partial \hat{p}_l < 0$ hold for any $h \neq l \in \{H, L\}$. The expected operational profit $\pi(\hat{p}_H, \hat{p}_L)$ is a strictly increasing function of $(\hat{p}_H, \hat{p}_L)$ so that the entry condition describes a strictly decreasing relationship between the two choke prices. The second equality in (7) also describes a strictly increasing relationship between $\hat{p}_H$ and $\hat{p}_L$ if $\partial e_h/\partial \hat{p}_h > 0$ and $\partial e_h/\partial \hat{p}_l < 0$ hold for any $h \neq l \in \{H, L\}$. Under these conditions, the two relationships cross at a single point $(\hat{p}_H, \hat{p}_L)$ that yields a unique equilibrium.

Appendix D: Income and Demand

In this appendix, we show how changes in income affect choke prices. Differentiating totally (37) and (36), we get

$$\begin{bmatrix} e_{HH} s_H^{-1} - e_{LH} s_L^{-1} & e_{HL} s_H^{-1} - e_{LL} s_L^{-1} \\ \pi_H & \pi_L \end{bmatrix} \cdot \begin{bmatrix} \mathrm{d}\hat{p}_H \\ \mathrm{d}\hat{p}_L \end{bmatrix} = \begin{bmatrix} -e_H \mathrm{d}s_H^{-1} + e_L \mathrm{d}s_L^{-1} \\ 0 \end{bmatrix}$$

where $e_{hl} \equiv \partial e_h/\partial \hat{p}_l$ and $\pi_l \equiv \partial \pi/\partial \hat{p}_l$, $h, l \in \{H, L\}$. In Appendix C, it has been shown that $\pi_H > 0$, $\pi_L > 0$, and $e_{LL} > 0 > e_{LH}$. Under the assumption (A1), i.e., $e_{HH} > 0 > e_{HL}$, the determinant of the matrix in the above LHS, $\Delta = \left(e_{HH} s_H^{-1} - e_{LH} s_L^{-1}\right) \pi_L -$

$\left(e_{HL}s_H^{-1} - e_{LL}s_L^{-1}\right)\pi_H$, is strictly positive. We have

$$\begin{bmatrix} \mathrm{d}\hat{p}_H/\mathrm{d}s_H^{-1} \\ \mathrm{d}\hat{p}_L/\mathrm{d}s_H^{-1} \end{bmatrix} = \frac{e_H}{\Delta}\begin{bmatrix} -\pi_L \\ \pi_H \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \mathrm{d}\hat{p}_H/\mathrm{d}s_L^{-1} \\ \mathrm{d}\hat{p}_L/\mathrm{d}s_L^{-1} \end{bmatrix} = \frac{e_L}{\Delta}\begin{bmatrix} \pi_L \\ -\pi_H \end{bmatrix}.$$

Noting that $e_h = s_h/M$ by (7) so that $\left(\mathrm{d}\hat{p}_h/\mathrm{d}s_h^{-1}\right) = e_h\hat{p}_hM\left(\mathrm{d}\ln\hat{p}_h/\mathrm{d}\ln s_h^{-1}\right)$, $h = H, L$, we can rewrite the above expression as

$$\begin{bmatrix} \mathrm{d}\ln\hat{p}_H/\mathrm{d}\ln s_H \\ \mathrm{d}\ln\hat{p}_L/\mathrm{d}\ln s_H \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} \pi_L \\ -\pi_H \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \mathrm{d}\ln\hat{p}_H/\mathrm{d}\ln s_L \\ \mathrm{d}\ln\hat{p}_L/\mathrm{d}\ln s_L \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} -\pi_L \\ \pi_H \end{bmatrix}.$$

We then get

$$\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_H} = -\frac{\mathrm{d}\ln\hat{p}_H}{\mathrm{d}\ln s_L} = \frac{1}{\Delta}\frac{\pi_L}{\hat{p}_H} > 0 \quad \text{and} \quad \frac{\mathrm{d}\ln\hat{p}_L}{\mathrm{d}\ln s_L} = -\frac{\mathrm{d}\ln\hat{p}_L}{\mathrm{d}\ln s_H} = \frac{1}{\Delta}\frac{\pi_H}{\hat{p}_L} > 0.$$

Appendix E: Pareto Productivity Distribution

We here assume Pareto productivity, which translates to cost distribution with the c.d.f given by $G(c) = (c/c_M)^\kappa$ for $c \in [0, c_M]$ and $\kappa \geq 1$ and $c_M < \infty$.

Define $a \equiv \hat{c}/\hat{p}_L = \left[(\alpha_H r + \alpha_L)^{1/2} - \alpha_H^{1/2} r^{1/2}\right]^2 / \left(1 - \alpha_H^{1/2}\right)^2$. The equilibrium is the vector $(\hat{p}_H, \hat{p}_L, M)$ that solves (37), (35), and (36). With some algebraic manipulations, these conditions are translated to

$$\begin{aligned} \frac{s_H}{s_L} &= \frac{\Gamma_H(r)}{\Gamma_L(r)}, \\ \hat{p}_L &= \left(\frac{c_M^\kappa f}{\kappa N}\right)^{\frac{1}{\kappa+1}} \Phi(r)^{-\frac{1}{\kappa+1}}, \\ M &= \frac{s_L N}{f}\frac{\Phi(r)}{\Gamma_L(r)}, \end{aligned}$$

where

$$\Gamma_L\left(r\right)=\frac{a^{\kappa}}{\kappa}-\frac{a^{\kappa+1/2}\left(\alpha_L+\alpha_H r\right)^{1/2}}{\kappa+\frac{1}{2}}$$

$$\Gamma_H\left(r\right)=\frac{r^{\kappa+1}}{\kappa}-\frac{a^{\kappa+1/2}\left(\alpha_L+\alpha_H r\right)^{1/2}+r^{\kappa+1}-r^{1/2}a^{\kappa+1/2}}{\kappa+\frac{1}{2}}$$

$$\Phi\left(r\right)=\frac{\alpha_H r^{\kappa+1}+\alpha_L a^{\kappa}}{\kappa}+\frac{\alpha_H r^{\kappa+1}+\alpha_L a^{\kappa+1}}{\kappa+1}$$
$$-\frac{2\left[a^{\kappa+1/2}\left(\alpha_L+\alpha_H r\right)^{1/2}+\alpha_H r^{1/2}\left(r^{\kappa+1/2}-a^{\kappa+1/2}\right)\right]}{\kappa+1/2}.$$

The Pareto distribution allows us to separate the effects of some parameters and sufficient statistics such as s_H/s_L and f/N. In the following, we collect the properties under Pareto that we mention in the main text and prove them in sequence. Note that the equilibrium always exists because the Pareto distribution has bounded support $c_M<\infty$ and therefore a finite mean, $\mathrm{E}(c)<\infty$.

Next, we show that Assumption (A1), $\partial e_H/\partial\hat{p}_H>0$ and $\partial e_H/\partial\hat{p}_L<0$, holds under a Pareto productivity distribution. As a result, there exists a unique equilibrium, and both Lemma 1 and Proposition 2 hold.

We can rewrite $e_H\left(\hat{p}_H,\hat{p}_L\right)$ as

$$\begin{aligned}
e_H\left(\hat{p}_H,\hat{p}_L\right)&=\frac{\kappa}{c_M^{\kappa}}\left[\int_0^{\hat{c}}\left(\hat{p}_H-\left(\alpha_H\hat{p}_H+\alpha_L\hat{p}_L\right)^{1/2}c^{1/2}\right)c^{\kappa-1}\mathrm{d}c+\int_{\hat{c}}^{\hat{p}_H}\left(\hat{p}_H-\hat{p}_H^{1/2}c^{1/2}\right)c^{\kappa-1}\mathrm{d}c\right]\\
&\propto\hat{p}_H\int_0^{\hat{p}_H}c^{\kappa-1}\mathrm{d}c-\left[\left(\alpha_H\hat{p}_H+\alpha_L\hat{p}_L\right)^{1/2}\int_0^{\hat{c}}c^{\kappa-1/2}\mathrm{d}c+\hat{p}_H^{1/2}\int_{\hat{c}}^{\hat{p}_H}c^{\kappa-1/2}\mathrm{d}c\right]\\
&\propto\frac{1/2}{\kappa\left(\kappa+1/2\right)}\hat{p}_H^{\kappa+1}-\frac{1}{\kappa+1/2}\frac{\left(\hat{p}_{HL}^{1/2}-\hat{p}_H^{1/2}\right)\left[\hat{p}_{HL}^{1/2}-\left(\alpha_H\hat{p}_H\right)^{1/2}\right]^{2\kappa+1}}{\left(1-\alpha_H^{1/2}\right)^{2\kappa+1}}.
\end{aligned}$$

Thus,

$$\begin{aligned}
&\partial e_H/\partial\hat{p}_L\\
&\propto-\frac{\alpha_L\hat{p}_{HL}^{-1/2}}{2}\left\{\left[\hat{p}_{HL}^{1/2}-\alpha_H^{1/2}\hat{p}_H^{1/2}\right]^{2\kappa+1}+\left(2\kappa+1\right)\left(\hat{p}_{HL}^{1/2}-\hat{p}_H^{1/2}\right)\left[\hat{p}_{HL}^{1/2}-\alpha_H^{1/2}\hat{p}_H^{1/2}\right]^{2\kappa}\right\}\\
&<0
\end{aligned}$$

and

$$\partial e_H/\partial \hat{p}_H$$
$$\propto \frac{1/2\left(\kappa+1\right)}{\kappa\left(\kappa+1/2\right)}$$
$$-\frac{\left(\frac{\alpha_H}{2}\hat{p}_{HL}^{-1/2}-\frac{1}{2}\hat{p}_H^{-1/2}\right)\times\left[\hat{p}_{HL}^{1/2}-\left(\alpha_H\hat{p}_H\right)^{1/2}\right]+\frac{\alpha_H(2\kappa+1)}{2}\left(\hat{p}_{HL}^{1/2}-\hat{p}_H^{1/2}\right)\left(\hat{p}_{HL}^{-1/2}-\left(\alpha_H\hat{p}_H\right)^{-1/2}\right)}{\left(\kappa+1/2\right)\left(1-\alpha_H^{1/2}\right)^{2\kappa+1}\left[\hat{p}_{HL}^{1/2}-\left(\alpha_H\hat{p}_H\right)^{1/2}\right]^{-2\kappa}\hat{p}_H^{\kappa}}.$$

The above is positive if the numerator of the second term is negative. Using $y\equiv\hat{p}_{HL}^{1/2}/\hat{p}_H^{1/2}=\left(\alpha_H+\alpha_L\hat{p}_L/\hat{p}_H\right)^{1/2}\in(0,1)$, the numerator is negative if and only if

$$\left[1+\left(2\kappa+1\right)\alpha_H^{1/2}\right]y^2-\left(2\kappa+2\right)\left(\alpha_H+\alpha_H^{1/2}\right)y+\left[\alpha_H^{3/2}+\alpha_H\left(2\kappa+1\right)\right]>0.$$

This is a convex quadratic function of y that takes only positive values because it is positive at $y=0$ and has the negative determinant

$$\Delta=-2\alpha_H\left[\left(2+6\kappa+8\kappa^2\right)\sqrt{\alpha_H}+\left(1+3\kappa\right)\alpha_H+3\kappa+1\right]<0.$$

The equilibrium utility can be inferred from Appendix F for the open economy, as it is the special case of $n=1$.

Appendix F: Trade Integration and Welfare

Indirect utility is given by (34) or

$$U\left(s_h\right)=\int_{\omega\in\Omega_h}\ln\left(\frac{\hat{p}_h}{p^*\left(\omega\right)/\beta\left(\omega\right)}\right)\mathrm{d}\omega.$$

The low-income worker has a set of consumed goods Ω_L that includes the ranges $[0,M]\times[0,\hat{c}]$ and $[0,M]\times[0,\hat{c}/\tau]$ for local and imported goods. Using equilibrium prices p^*, $p^*\left(c\right)/\beta\left(c\right)=\left(\hat{p}_{HL}c\right)^{1/2}$, and $p^x\left(c\right)/\beta\left(c\right)=\left(\hat{p}_{HL}\tau c\right)^{1/2}$ for local and imported consumption, we get

$$U\left(s_L\right)=\int_0^{\hat{c}}\ln\left(\frac{\hat{p}_L}{(\hat{p}_{HL}c)^{1/2}}\right)M\mathrm{d}G(c)+\left(n-1\right)\int_0^{\hat{c}/\tau}\ln\left(\frac{\hat{p}_L}{(\hat{p}_{HL}\tau c)^{1/2}}\right)M\mathrm{d}G(c).$$

One can compute $\int \ln\left(Ac^{-1/2}\right) \mathrm{d}G(c) = \frac{1}{2}\left(\frac{c}{c_M}\right)^{\kappa}\left[2\ln(A) + \frac{1}{\kappa} - \ln(c)\right]$ where A is a positive constant. Applying this to the above expression and simplifying, we get

$$U(s_L) = M\left[1 + (n-1)\tau^{-\kappa}\right]\frac{\hat{p}_L^{\kappa}}{c_M^{\kappa}}\left[\frac{a^{\kappa}}{2\kappa} - \frac{a^{\kappa}}{2}\ln(a) - \frac{a^{\kappa}}{2}\ln(\alpha_H r + \alpha_L)\right]$$

where $r = \hat{p}_H/\hat{p}_L$ and $a = \hat{c}/\hat{p}_L$.

The high-income worker has a set of consumed goods Ω_H that includes the ranges $[0, M] \times [0, \hat{p}_H]$ and $[0, M] \times [0, \hat{p}_H/\tau]$ for local and imported goods. Using equilibrium prices, we get

$$\begin{aligned} U(s_H) = &\int_0^{\hat{c}} \ln\left(\frac{\hat{p}_H}{(\hat{p}_{HL}c)^{1/2}}\right) M\mathrm{d}G(c) + \int_{\hat{c}}^{\hat{p}_H} \ln\left(\frac{\hat{p}_H}{(\hat{p}_H c)^{1/2}}\right) M\mathrm{d}G(c) \\ &+ (n-1)\left[\int_0^{\hat{c}/\tau} \ln\left(\frac{\hat{p}_H}{(\hat{p}_{HL}\tau c)^{1/2}}\right) M\mathrm{d}G(c) + \int_{\hat{c}}^{\hat{p}_H/\tau} \ln\left(\frac{\hat{p}_H}{(\hat{p}_H\tau c)^{1/2}}\right) M\mathrm{d}G(c)\right]. \end{aligned}$$

Using the same procedure as above, this simplifies to

$$U(s_H) = M\left[1 + (n-1)\tau^{-\kappa}\right]\frac{\hat{p}_L^{\kappa}}{c_M^{\kappa}}\left[\frac{r^{\kappa}}{2\kappa} + \frac{a^{\kappa}}{2}\ln(r) - \frac{a^{\kappa}}{2}\ln(\alpha_H r + \alpha_L)\right].$$

Next, we show that the domestic expenditure share of each income group is given by $\lambda_h = 1/\left(1 + (n-1)\tau^{-\kappa}\right)$, for $h = L, H$. This implies that the overall population's expenditure share (denoted by λ) is given by the same value.

From (7), the domestic expenditure share of a poor individual is

$$\lambda_L = \frac{\int_0^{\hat{c}}\left(\hat{p}_L - \hat{p}_{HL}^{1/2}c^{1/2}\right)\mathrm{d}G(c)}{\int_0^{\hat{c}}\left(\hat{p}_L - \hat{p}_{HL}^{1/2}c^{1/2}\right)\mathrm{d}G(c) + (n-1)\int_0^{\hat{c}/\tau}\left(\hat{p}_L - \hat{p}_{HL}^{1/2}(\tau c)^{1/2}\right)\mathrm{d}G(c)},$$

where the numerator is the expenditure on domestic goods and the denominator the total expenditure. Using the Pareto assumption and a change of variable $\tilde{c} = \tau c$, the expenditure on the goods from one foreign country is

$$\begin{aligned} \int_0^{\hat{c}/\tau}\left(\hat{p}_L - \hat{p}_{HL}^{1/2}(\tau c)^{1/2}\right)\mathrm{d}G(c) &= \int_0^{\hat{c}/\tau}\left(\hat{p}_L - \hat{p}_{HL}^{1/2}(\tau c)^{1/2}\right)\frac{\kappa c^{\kappa-1}}{c_M^{\kappa}}\mathrm{d}c \\ &= \tau^{-\kappa}\int_0^{\hat{c}}\left(\hat{p}_L - \hat{p}_{HL}^{1/2}\tilde{c}^{1/2}\right)\frac{\kappa\tilde{c}^{\kappa-1}}{c_M^{\kappa}}\mathrm{d}\tilde{c} \end{aligned}$$

Thus,

$$\lambda_L = \frac{1}{1 + (n-1)\,\tau^{-\kappa}}.$$

Following similar steps, it is readily verified that the rich's domestic expenditure share $\lambda_H = \lambda_L = \frac{1}{1+(n-1)\tau^{-\kappa}}$.

How does the utility increase due to trade integration compared to an increase in income? Let us fix income ratio s_H/s_L so that r and a remain constant. A percentage decrease in trade cost yields the same change in utility resulting from a percentage increase in average income if it satisfies the following relationship:

$$\left[\frac{\mathrm{d}\ln U\left(s_h\right)}{\mathrm{d}\ln\tau}\right]_{s_L \text{ fixed}} \mathrm{d}\ln\tau = -\left[\frac{\mathrm{d}\ln U\left(s_h\right)}{\mathrm{d}\ln s_L}\right]_{\tau \text{ fixed}} \mathrm{d}\ln s_L.$$

We have

$$\left[\frac{\mathrm{d}\ln U\left(s_h\right)}{\mathrm{d}\ln\tau}\right]_{s_L \text{ fixed}} = \frac{\mathrm{d}\ln\left(1+(n-1)\tau^{-\kappa}\right)}{\mathrm{d}\ln\tau} + \frac{\mathrm{d}\ln\hat{p}_L^{\kappa}}{\mathrm{d}\ln\tau} = -\frac{\kappa}{\kappa+1}\frac{(n-1)\,\tau^{-\kappa}}{1+(n-1)\,\tau^{-\kappa}}.$$

From (21) and (23), we observe that $U\left(s_h\right)$ is proportional to M, which, in turn, is proportional to s_L. Hence, we get

$$\left[\frac{\mathrm{d}\ln U\left(s_h\right)}{\mathrm{d}\ln s_L}\right]_{\tau \text{ fixed}} = 1.$$

Using the above results, we obtain

$$\theta \equiv \frac{\mathrm{d}\ln s_L}{\mathrm{d}\ln\tau} = \frac{\left[\frac{\mathrm{d}\ln U(s_h)}{\mathrm{d}\ln\tau}\right]_{s_L \text{ fixed}}}{-\left[\frac{\mathrm{d}\ln U(s_h)}{\mathrm{d}\ln s_L}\right]_{\tau \text{ fixed}}} = \frac{\kappa}{\kappa+1}\frac{(n-1)\,\tau^{-\kappa}}{1+(n-1)\,\tau^{-\kappa}}.$$

Appendix G: Multiple Income Groups

We first establish the firms' equilibrium price schedule as a function of cost per unit quality c, equilibrium choke prices $\{\hat{p}_h\}_{h=1}^{H}$, and mass of entrants M. We then express equilibrium conditions and finally the welfare properties with respect to trade parameters. We also provide the formula used for the calibration and quantitative analysis.

Equilibrium Price Schedule

The firm's problem is to find the best profit out of all possible demand segments; that is, $\max_{h,p} \pi_h(p,c) \equiv (p - \beta(c)c)\, NQ_h\,(p)$. We here prove the following lemma.

Lemma 2. *The equilibrium price $p^*(c)/\beta(c)$ of the firms with cost $c \in [\hat{c}_{h-1}, \hat{c}_h)$ is given by*

$$\frac{p_h^*(c)}{\beta(c)} = \left(\frac{\sum_{k=h}^H \alpha_k \hat{p}_k}{\sum_{k=h}^H \alpha_k}\right)^{1/2} c^{1/2}, \tag{38}$$

where the cutoff costs $\hat{c}_h$ are given by

$$\hat{c}_h^{1/2} = \frac{\left(\sum_{k=h}^H \alpha_k \hat{p}_k\right)^{1/2} - \left(\sum_{k=h+1}^H \alpha_k \hat{p}_k\right)^{1/2}}{\left(\sum_{k=h}^H \alpha_k\right)^{1/2} - \left(\sum_{k=h+1}^H \alpha_k\right)^{1/2}} < \hat{p}_h^{1/2} \tag{39}$$

for $h = 1, 2, ...H-1$ and by $\hat{c}_0 = 0$ and $\hat{c}_H = \hat{p}_H$. The optimal price jumps upward at each $\hat{c}_h$ for $h = 1, 2, ...H-1$. Moreover, $\hat{c}_h < \hat{p}_h$ and

$$\lim_{c \to \hat{c}_h -} \frac{p^*(c)}{\beta(c)} < \hat{p}_h < \lim_{c \to \hat{c}_h +} \frac{p^*(c)}{\beta(c)} \quad \textit{for all } h \in \{1, 2, ..., H-1\}. \tag{40}$$

Proof. Expression (38) results from the first-order condition of the profit on demand segment h with prices $p/\beta \in [\hat{p}_{h-1}, \hat{p}_h)$. This define the optimal profit on the segment h by

$$\pi_h^*(c) \equiv \pi_h\left(p_h^*(c), c\right) = N\left[\left(\sum_{k=h}^H \alpha_k \hat{p}_k\right)^{1/2} - \left(\sum_{k=h}^H \alpha_k\right)^{1/2} c^{1/2}\right]^2. \tag{41}$$

We first prove that $\pi_h^*(c) \geq \pi_{h'}^*(c)$ for $h' \neq h$ and $c \in [\hat{c}_{h-1}, \hat{c}_h)$. Note that the square root of the profit on each segment $[\pi_h^*(c)]^{1/2}$ is a linear and decreasing function in $c^{1/2}$ whose intercept and slope decrease with higher h. So, the upper envelope of the graphs of the functions $[\pi_h^*(c)]^{1/2}$ is a convex, decreasing and piece-wise linear function of $c^{1/2}$. Then, there exists a cutoff cost $\hat{c}_h$ with $h = 1, ..., H-1$ that equalizes profits between two adjacent segments, $[\pi_h^*(\hat{c}_h)]^{1/2} = \left[\pi_{h+1}^*(\hat{c}_h)\right]^{1/2}$ and such that $[\pi_h^*(c)]^{1/2} \geq \left[\pi_{h+1}^*(c)\right]^{1/2}$ if and only if $c \leq \hat{c}_h$. The binding equality solves as (39). By construction $\hat{c}_h < \hat{c}_{h+1}$. We will show that (40) holds, which implies that $p^*(c)/\beta(c)$ given by (38) on $c \in [\hat{c}_{h-1}, \hat{c}_h)$ indeed falls in $[\hat{p}_{h-1}, \hat{p}_h)$. Combining these properties for all $h = 1, ..., H-1$, we obtain that optimal price $p^*(c)/\beta(c)$ is given by (38) and the associated profit by (41) for any $c \in [\hat{c}_{h-1}, \hat{c}_h)$.

Next, we check whether $\pi_h^*(c) \geq 0$ for $c \in [\hat{c}_{h-1}, \hat{c}_h)$ for all $h = 1, 2, ..., H$. Since the upper envelope of the graphs of $[\pi_h^*(c)]^{1/2}$ is decreasing, the highest profit is obtained at $c = 0$ on the segment $h = 1$ while the lowest profit is given on the segment H by the highest cost firm with $c = \hat{p}_H$, that is, $\pi_H^*(\hat{p}_H)$, which can be readily shown to be zero. In this analysis, we suppose that firm selection always exists so that $\hat{p}_H \leq c_M$. So, we set

$\hat{c}_H = \hat{p}_H$ and define $\hat{c}_0 = 0$ for convenience. (Otherwise, we must bound those values by c_M.)

We now come back to prove (40). Define $r_k^h \equiv \hat{p}_k/\hat{p}_h \geq 1$ for all $k \geq h$, and thus $r_h^h = 1$ and $r_k^h > 1$ for all $k > h$. Then, after rearrangement, the first inequality of (40) is given by

$$\sum_{k=h}^{H} \alpha_k r_k^h - \left[\alpha_h \sum_{k=h+1}^{H} \alpha_k r_k^h + \left(\sum_{k=h+1}^{H} \alpha_k r_k^h\right)^2\right]^{1/2} < \sum_{k=h}^{H} \alpha_k - \left[\alpha_h \sum_{k=h+1}^{H} \alpha_k + \left(\sum_{k=h+1}^{H} \alpha_k\right)^2\right]^{1/2}.$$

Note that the RHS is independent of r_k^h and the LHS is equal to the RHS at $r_k^h = 1$. It is readily shown that the LHS is a decreasing function of every r_k^h. By $r_h^h = 1$ and $r_k^h > 1$ for all $k > h$, the above inequality holds. Similarly, the second inequality can be rearranged as

$$\left[\alpha_h \sum_{k=h+1}^{H} \alpha_k + \left(\sum_{k=h+1}^{H} \alpha_k\right)^2\right]^{1/2} - \sum_{k=h+1}^{H} \alpha_k < \left[\alpha_h \sum_{k=h+1}^{H} \alpha_k r_k^h + \left(\sum_{k=h+1}^{H} \alpha_k r_k^h\right)^2\right]^{1/2} - \sum_{k=h+1}^{H} \alpha_k r_k^h.$$

The LHS is independent of r_k^h while the RHS is equal to the LHS at $r_k^h = 1$. As the RHS is readily shown as an increasing function of every r_k^h, the above inequality also holds.

Finally, we show that $\hat{c}_h < \hat{p}_h$. From (39) this is equivalent to

$$\left(\sum_{k=h}^{H} \alpha_k r_k^h\right)^{1/2} - \left(\sum_{k=h+1}^{H} \alpha_k r_k^h\right)^{1/2} < \left(\sum_{k=h}^{H} \alpha_k\right)^{1/2} - \left(\sum_{k=h+1}^{H} \alpha_k\right)^{1/2}.$$

The LHS is independent of r_k^h while the RHS is equal to the LHS for $r_k^h = 1$. As the LHS is a decreasing function of every r_k^h, the above inequality holds. ■

Equilibrium Conditions

Here, we derive the equilibrium conditions as shown in (31)-(33). Let $\hat{p} \equiv \{\hat{p}_l\}_{l=1}^H$ denote the vector of choke prices. From budget constraints and the free-entry condition, the equilibrium conditions for $\hat{p}$ and mass of entrants M are given by

$$\frac{e_h(\hat{p})}{s_h} = \frac{1}{M} \quad \text{for } h = 1, 2, ..., H$$
$$\pi(\hat{p}) = \frac{f}{N},$$

where

$$e_h(\hat{p}) = \sum_{k=1}^{h} \int_{\hat{c}_{k-1}}^{\hat{c}_k} \left(\hat{p}_h - \left(\frac{\sum_{l=k}^{H} \alpha_l \hat{p}_l}{\sum_{l=k}^{H} \alpha_l} \right)^{1/2} c^{1/2} \right) \mathrm{d}G(c)$$
$$+ (n-1) \sum_{k=1}^{h} \int_{\hat{c}_{k-1}/\tau}^{\hat{c}_k/\tau} \left[\hat{p}_h - \left(\frac{\sum_{l=k}^{H} \alpha_l \hat{p}_l}{\sum_{l=k}^{H} \alpha_l} \right)^{1/2} (\tau c)^{1/2} \right] \mathrm{d}G(c)$$

expresses the group-h consumers' average expenditure per available good, and

$$\pi(\hat{p}) = \sum_{h=1}^{H} \int_{\hat{c}_{h-1}}^{\hat{c}_h} \left[\left(\sum_{k=h}^{H} \alpha_k \hat{p}_k \right)^{1/2} - \left(\sum_{k=h}^{H} \alpha_k \right)^{1/2} c^{1/2} \right]^2 \mathrm{d}G(c)$$
$$+ (n-1) \sum_{h=1}^{H} \int_{\hat{c}_{h-1}/\tau}^{\hat{c}_h/\tau} \left[\left(\sum_{k=h}^{H} \alpha_k \hat{p}_k \right)^{1/2} - \left(\sum_{k=h}^{H} \alpha_k \right)^{1/2} (\tau c)^{1/2} \right]^2 \mathrm{d}G(c)$$

expresses the expected operational profit (per capita and before entry).

We denote by $r = \{\hat{p}_h/\hat{p}_1\}_{h=1}^{H}$ and $a = \{\hat{c}_h/\hat{p}_1\}_{h=0}^{H}$ the vectors of ratios of choke prices and cutoff costs to the poorest's choke price. The second vector is a function of the first as

$$a_h(r) \equiv \frac{\hat{c}_h}{\hat{p}_1} = \left[\frac{\left(\sum_{k=h}^{H} \alpha_k r_k \right)^{1/2} - \left(\sum_{k=h+1}^{H} \alpha_k r_k \right)^{1/2}}{\left(\sum_{k=h}^{H} \alpha_k \right)^{1/2} - \left(\sum_{k=h+1}^{H} \alpha_k \right)^{1/2}} \right]^2$$

for $h = 1, ..., H+1$, while $a_0(r) = 0$ and $a_H(r) = \hat{p}_H/\hat{p}_1 = r_H$.

Applying the Pareto productivity distribution, the above $H+1$ equilibrium conditions can be written as

$$\frac{s_h}{M} \frac{c_M^{\kappa}}{\kappa \left[1 + (n-1)\tau^{-\kappa}\right]} = \hat{p}_1^{\kappa+1} \Gamma_h(r), \qquad \text{for } h = 1, 2, ..., H$$
$$\frac{f}{N} \frac{c_M^{\kappa}}{\kappa \left[1 + (n-1)\tau^{-\kappa}\right]} = \hat{p}_1^{\kappa+1} \Phi(r)$$

where

$$\Gamma_h(r) = \sum_{k=1}^{h} \left[r_h \frac{a_k^{\kappa} - a_{k-1}^{\kappa}}{\kappa} - \left(\frac{\sum_{l=k}^{H} \alpha_l r_l}{\sum_{l=k}^{H} \alpha_l} \right)^{1/2} \frac{a_k^{\kappa+1/2} - a_{k-1}^{\kappa+1/2}}{\kappa + \frac{1}{2}} \right]$$

$$\Phi(r) = \sum_{h=1}^{H} \left[\begin{array}{c} \left(\sum_{k=h}^{H} \alpha_k r_k\right) \frac{a_h^{\kappa} - a_{h-1}^{\kappa}}{\kappa} + \left(\sum_{k=h}^{H} \alpha_k\right) \frac{a_h^{\kappa+1} - a_{h-1}^{\kappa+1}}{\kappa+1} \\ -2 \left(\sum_{k=h}^{H} \alpha_k r_k\right)^{1/2} \left(\sum_{k=h}^{H} \alpha_k\right)^{1/2} \frac{a_h^{\kappa+1/2} - a_{h-1}^{\kappa+1/2}}{\kappa+1/2} \end{array} \right]$$

which must be solved for the two variables $\hat{p}_1$ and M and the $H-1$ variables included in r (since $r_1 = 1$ by construction). The vector r is therefore found by the solution of the following system of equations:

$$\frac{s_h}{s_1} = \frac{\Gamma_h(r)}{\Gamma_1(r)} \quad \text{for } h = 2, 3, ..., H \tag{42}$$

The vector of ratios of choke prices r is independent of the trade costs τ or the number of trading partners. The variables $\hat{p}_1$ and M are given by

$$\hat{p}_1 = \left[1 + (n-1)\tau^{-\kappa}\right]^{-\frac{1}{\kappa+1}} \times \left(\frac{c_M^{\kappa} f}{\kappa N} \right)^{\frac{1}{\kappa+1}} \Phi(r)^{-\frac{1}{\kappa+1}}. \tag{43}$$

$$M = \frac{s_1 N}{f} \frac{\Phi(r)}{\Gamma_1(r)}. \tag{44}$$

Note that $\hat{p}_1$ depends on the trade cost and the number of trading partners only through the first multiplicative term in (43). These imply that trade affects the level of choke prices but not their ratios. The number of entrants is also independent of the trade cost or the number of trading partners. We have seen these properties in Appendix E for the two-group case.

Welfare

The indirect utility of an individual in income group h is

$$\begin{aligned} U(s_h) &= \int_{\omega \in \Omega} \ln \left(\frac{\hat{p}_h}{p(\omega)/\beta(\omega)} \right) \mathrm{d}\omega \\ &= M\left[1 + (n-1)\tau^{-\kappa}\right] * \sum_{k=1}^{h} \int_{\hat{c}_{k-1}}^{\hat{c}_k} \ln \left[\hat{p}_h \left(\frac{\sum_{l=k}^{H} \alpha_l \hat{p}_l}{\sum_{l=k}^{H} \alpha_l} \right)^{-1/2} c^{-1/2} \right] \mathrm{d}G(c). \end{aligned} \tag{45}$$

Under Pareto productivity, $\int \ln\left(Ac^{-1/2}\right) \mathrm{d}G(c) = \frac{1}{2}\left(\frac{c}{c_M}\right)^{\kappa}\left[2\ln(A) + \frac{1}{\kappa} - \ln(c)\right]$, where A is a positive constant. Applying this expression to (45) yields

$$U(s_h) = M\frac{[1+(n-1)\tau^{-\kappa}]}{c_M^{\kappa}}\hat{p}_1^{\kappa}\sum_{k=1}^{h}\left[\begin{array}{c}\left(a_k^{\kappa} - a_{k-1}^{\kappa}\right)\left[\ln\left(r_h^1\left(\frac{\sum_{l=k}^{H}\alpha_l r_l^1}{\sum_{l=k}^{H}\alpha_l}\right)^{-1/2}\right) + \frac{1}{2\kappa}\right] \\ -\left[a_k^{\kappa}\ln\left(a_k^{1/2}\right) - a_{k-1}^{\kappa}\ln\left(a_{k-1}^{1/2}\right)\right]\end{array}\right]. \quad (46)$$

Following the same procedure in Appendix F, it is readily shown that the domestic expenditure share of each income group λ_h and the overall domestic expenditure share are the same and given by $\lambda = \lambda_h = [1+(n-1)\tau^{-\kappa}]^{-1}$. Combining (43) and (46) and noting the fact that equilibrium r does not depend on trade cost, the same procedure in Section 4.3 yields that the welfare formula in Arkolakis et al. (2019) applies in this model with multiple income groups.

Calibration

The average markup of surviving firms is given by $mrkup^o \equiv \int_0^{\hat{p}_H} \frac{p^*(c)}{\beta(c)c}\mathrm{d}G(c|c<\hat{p}_H)$. Using optimal prices, Pareto productivity distribution, and the above definition of a, this gives

$$\begin{aligned} mrkup^o &= \frac{\kappa}{\hat{p}_H^{\kappa}}\sum_{h=1}^{H}\left(\frac{\sum_{k=h}^{H}\alpha_k\hat{p}_k}{\sum_{k=h}^{H}\alpha_k}\right)^{1/2}\frac{\hat{c}_h^{\kappa-1/2} - \hat{c}_{h-1}^{\kappa-1/2}}{\kappa-1/2} \\ &= \frac{\kappa}{r_H^{\kappa}}\sum_{h=1}^{H}\left(\frac{\sum_{k=h}^{H}\alpha_k r_k}{\sum_{k=h}^{H}\alpha_k}\right)^{1/2}\frac{a_h^{\kappa-1/2} - a_{h-1}^{\kappa-1/2}}{\kappa-1/2} \end{aligned}$$

Thus, markups depend only on the coefficient κ and the vector of choke-price ratios r. Plugging the vector of income s in this identity and the equations (42) yields (κ, r).

We calibrate the trade cost parameter τ to the domestic expenditure share $expshare$. Using (27), we get $\tau = \left(\frac{expshare^{-1}-1}{n-1}\right)^{-1/\kappa}$. For $expshare = 0.828$, this gives $\tau = 1.67$. Next, we calibrate to the survival rate and the employment per firm given by

$$\begin{aligned} surv^o &= G(\hat{p}_H), \\ empl^o &= N/(MG(\hat{p}_H)). \end{aligned}$$

These two equations, (43), and (44) entail

$$
\begin{aligned}
M &= \frac{N}{empl^o * surv^o} \\
f &= s_1 \frac{\Phi(r)}{\Gamma_1(r)} empl^o * surv^o \\
\hat{p}_1 &= \frac{s_1}{\Gamma_1(r)} \frac{r_H^{\kappa} empl^o}{\kappa N \left[1 + (n-1)\tau^{-\kappa}\right]} \\
c_M &= \frac{s_1}{\Gamma_1(r)} \frac{r_H^{\kappa+1} empl^o}{\kappa N \left(surv^o\right)^{\frac{1}{\kappa}} \left[1 + (n-1)\tau^{-\kappa}\right]},
\end{aligned}
$$

which allow us to solve for $\hat{p}_1$, M, f and c_M, given s, N and $n = 1$ and given the solutions (κ, r).

Average Productivity

In the quantitative analysis, we report both unweighted and weighted productivity. Here, we provide the mathematical details. The unweighted average productivity is given by the inverse of the unweighted average cost per quality unit c, which is equal to

$$
\int_0^{\hat{p}_H} c \mathrm{d}G(c|c < \hat{p}_H) = \frac{\int_0^{\hat{p}_H} c \mathrm{d}G(c)}{\int_0^{\hat{p}_H} \mathrm{d}G(c)} = \frac{\kappa}{\kappa+1}\hat{p}_H,
$$

where we use the definition of Pareto productivity for the second equality. This definition of unweighted average productivity applies to both closed and open economies.

The weighted average productivity is given by the inverse of the weighted average cost, with weights being the total output of the firms. This is indeed the ratio of the total cost of production over the total output produced in the economy. Using the Pareto assumption and considering that a firm's output is affected by its export status, the weighted average cost is given by

$$
\begin{aligned}
&\frac{\int_0^{\hat{p}_H} cQ(p^*(c)) \mathrm{d}G(c|c < \hat{p}_H)}{\int_0^{\hat{p}_H} Q(p^*(c)) \mathrm{d}G(c|c < \hat{p}_H)} \\
&= \frac{\sum_{h=1}^{H} \int_{\hat{c}_{h-1}}^{\hat{c}_h} cQ_h(p_h^*(c)) c^{\kappa-1} \mathrm{d}c + (n-1) \sum_{h=1}^{H} \int_{\hat{c}_{h-1}/\tau}^{\hat{c}_h/\tau} c\tau Q_h(p_h^x(c)) c^{\kappa-1} \mathrm{d}c}{\sum_{h=1}^{H} \int_{\hat{c}_{h-1}}^{\hat{c}_h} Q_h(p_h^*(c)) c^{\kappa-1} \mathrm{d}c + (n-1) \sum_{h=1}^{H} \int_{\hat{c}_{h-1}/\tau}^{\hat{c}_h/\tau} \tau Q_h(p_h^x(c)) c^{\kappa-1} \mathrm{d}c} \\
&= \frac{\sum_{h=1}^{H} \int_{\hat{c}_{h-1}}^{\hat{c}_h} cQ_h(p_h^*(c)) c^{\kappa-1} \mathrm{d}c}{\sum_{h=1}^{H} \int_{\hat{c}_{h-1}}^{\hat{c}_h} Q_h(p_h^*(c)) c^{\kappa-1} \mathrm{d}c},
\end{aligned}
$$

where the second equality follows from the fact that in both the numerator and denominator, the second term is $(n-1)\,\tau^{1-\kappa}$ times of the first term. Hence, the formula of the weighted average cost in the open economy is also the same as in the closed economy. Using (29) and (30), the weighted average cost can be written as

$$\hat{p}_1 \times \frac{\sum_{h=1}^{H}\left[\left(\sum_{k=h}^{H}\alpha_k r_k\right)^{1/2}\left(\sum_{k=h}^{H}\alpha_k\right)^{1/2}\frac{a_h^{\kappa+1/2}-a_{h-1}^{\kappa+1/2}}{\kappa+1/2}-\left(\sum_{k=h}^{H}\alpha_k\right)\frac{a_h^{\kappa+1}-a_{h-1}^{\kappa+1}}{\kappa+1}\right]}{\sum_{h=1}^{H}\left[\left(\sum_{k=h}^{H}\alpha_k r_k\right)^{1/2}\left(\sum_{k=h}^{H}\alpha_k\right)^{1/2}\frac{a_h^{\kappa-1/2}-a_{h-1}^{\kappa-1/2}}{\kappa-1/2}-\left(\sum_{k=h}^{H}\alpha_k\right)\frac{a_h^{\kappa}-a_{h-1}^{\kappa}}{\kappa}\right]},$$

where the second multiplicative term is a function of r only and is hence unaffected by the value of the trade openness term, $1+(n-1)\,\tau^{-\kappa}$, as implied by the equilibrium conditions (42) and (43). The multiplicative form of this expression also implies that the percentage change in the weighted average productivity induced by a mean-preserving spread of income is the same for all levels of trade openness.

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