Online Appendix to "Innovation, Firm Size Distribution, and Gains from Trade"

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This online appendix is organized as follows. Section A provides a microfoundation for the innovation cost function assumed in the main paper. Section B provides the derivations for Table 1. Section C shows an alternative model with a perfectly-competitive and centralized financial market.

A Microfoundation for Innovation Cost Function

Each firm determines its productivity level by engaging in R&D activities in the following manner. The production process involves a continuum of procedures, and the firm chooses the size of the continuum, a. How well the firm performs in each procedure (which we term the quality of the procedure) depends on the outcome of a sequence of experiments that the firm conducts. For each procedure, every firm starts off with one quality unit. When the first experiment is successful, then the firm obtains one additional quality unit for this procedure, and can continue to conduct the second experiment. Recursively, every successful experiment results in one additional quality unit and the chance to conduct the next experiment. If the experiment fails, however, no more experiments will be performed and the quality of the procedure is finalized. Firms differ in their probabilities of failure, $\gamma \in (0, 1]$. The probability of obtaining quality $y = 1, 2, ...$ for a procedure is therefore

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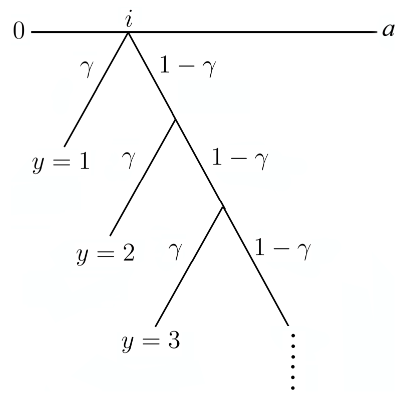

Notes: For each procedure i in the continuum of procedures of mesaure a, experiments for improving the quality of the procedure are Bernoulli trials in which the failure probability is firm-specific and given by γ.

Figure 1: A Sequence of Bernoulli Trials

$(1-\gamma)^{y-1}\gamma$, i.e., y is geometrically distributed. The process is illustrated in Figure 1.

Each procedure requires a worker, say a research assistant, to perform the experiments. Therefore, the mass of research assistants employed by the firm equals the size of the continuum of procedures, a. The productivity φ is a function of the total quality of all a procedures, $aE(y)$:

$$\varphi \equiv B(aE(y)) = B\left(a\sum_{y=1}^{\infty}(1-\gamma)^{y-1}\gamma y\right) = B\left(\frac{a}{\gamma}\right).$$

The function $B(\cdot)$ is strictly increasing and concave on $\mathbb{R}_+$, and $\lim_{a\to\infty} B\left(\frac{a}{\gamma}\right) = \infty$. The concavity of $B(\cdot)$ reflects the management burden for the firm to manage these research assistants. Inverting the above equation yields

$$k = \gamma V(\varphi) + \kappa_R,$$

where $V \equiv B^{-1}$ is strictly increasing and convex in φ and $\lim_{\varphi\to\infty} V(\varphi) = \infty$. For a firm to perform all these experiments, a fixed cost, κ_R, such as setting up a laboratory is required. Thus, the total labor requirement for a firm to acquire productivity φ is $k = \gamma V(\varphi) + \kappa_R$.

B Derivations for Table 1

This section shows that the demand structures listed in Table 1 satisfy our Assumption 1. These demand structures include Bipower Direct and Inverse demand (Mrázová and Neary 2017), Pollak Family demand (Pollak 1971, which is equivalent to the HARA [Hyperbolic Absolute Risk Aversion] preference [Merton 1971; Zhelobodko et al. 2012]), PIGL (Price-Independent Generalized Linear) demand (Muellbauer 1975), the QMOR (Quadratic Mean of Order r) expenditure function (Diewert 1976; Feenstra 2018), CEMR (Constant Elasticity of Marginal Revenue) demand, and CREMR (Constant Revenue Elasticity of Marginal Revenue) demand (Mrázová, Neary, and Parenti 2020).

Bipower direct demand $q = \widehat{a}p^{-\nu} + ap^{-\sigma}$**:**

Assume that $\sigma > 1$, $a > 0$, and $\sigma > \nu$; we immediately obtain

$$q = p^{-\sigma}\left(\widehat{a}p^{\sigma-\nu} + a\right) \equiv \boldsymbol{q}\left(p\right)$$

$$\lim_{p\to 0}\left(\widehat{a}p^{\sigma-\nu} + a\right) = a$$

$$\lim_{p\to 0} q = \infty,\ \lim_{p\to 0} pq = \infty.$$

Moreover, $\partial p/\partial q = -p^{-\nu-1}\left(\nu\widehat{a} + \sigma ap^{\nu-\sigma}\right)$ and is strictly negative for small enough p. Thus for large enough q the inverse demand is well-defined as $p = q^{-\frac{1}{\sigma}}\left(\widehat{a}\left[\boldsymbol{q}^{-1}\left(q\right)\right]^{\sigma-\nu} + a\right)^{\frac{1}{\sigma}} \equiv q^{-\frac{1}{\sigma}}Q\left(q\right)$ where $\lim_{q\to\infty} Q\left(q\right) = a^{\frac{1}{\sigma}}$.

Pollak demand $q = \widehat{a} + ap^{-\sigma}$**:**

Assume that $\sigma > 1$ and $a > 0$; we immediately obtain

$$q = \widehat{a} + ap^{-\sigma}$$

$$\lim_{p\to 0} q = \infty,\ \lim_{p\to 0} pq = \infty.$$

As a result, $p = q^{-\frac{1}{\sigma}} a^{\frac{1}{\sigma}} \left(1 - \frac{\widehat{a}}{q}\right)^{-\frac{1}{\sigma}} \equiv q^{-\frac{1}{\sigma}} Q(q)$ where $\lim_{q \to \infty} Q(q) = a^{\frac{1}{\sigma}}$.

PIGL demand $q = \widehat{a} p^{-1} + a p^{-\sigma}$**:**

Assume that $\sigma > 1$ and $a > 0$; we immediately obtain

$$q = p^{-\sigma} \left(\widehat{a} p^{\sigma - 1} + a\right) \equiv \boldsymbol{q}(p)$$
$$\lim_{p \to 0} \left(\widehat{a} p^{\sigma - 1} + a\right) = a$$
$$\lim_{p \to 0} q = \infty, \ \lim_{p \to 0} pq = \infty.$$

Moreover, $\partial p / \partial q = -p^{-2} \left(\widehat{a} + \sigma a p^{1-\sigma}\right)$ and is strictly negative for small enough p. Thus for large enough q the inverse demand is well-defined as $p = q^{-\frac{1}{\sigma}} \left(\widehat{a} \left[\boldsymbol{q}^{-1}(q)\right]^{\sigma - 1} + a\right)^{\frac{1}{\sigma}} \equiv q^{-\frac{1}{\sigma}} Q(q)$ where $\lim_{q \to \infty} Q(q) = a^{\frac{1}{\sigma}}$.

QMOR demand $q = a p^{r-1} + \widehat{a} p^{\frac{r}{2} - 1}$**:**

Assume that $\sigma \equiv 1 - r > 1$ and $a > 0$. This implies that $r < 0$ and $1 - r > 1 - \frac{r}{2} > 0$; thus

$$q = p^{r-1} \left(\widehat{a} p^{-\frac{r}{2}} + a\right) \equiv \boldsymbol{q}(p)$$
$$\lim_{p \to 0} \left(\widehat{a} p^{-\frac{r}{2}} + a\right) = a$$
$$\lim_{p \to 0} q = \infty, \ \lim_{p \to 0} pq = \infty.$$

Moreover, $\partial p / \partial q = p^{r-2} \left[\left(\frac{r}{2} - 1\right) \widehat{a} p^{-\frac{r}{2}} + (r - 1) a\right]$ and is strictly negative for small enough p. Thus for large enough q the inverse demand is well-defined as

$$p = q^{\frac{1}{r-1}} \left(a + \widehat{a} \left[\boldsymbol{q}^{-1}(q)\right]^{-\frac{r}{2}}\right)^{\frac{1}{1-r}} \equiv q^{\frac{1}{r-1}} Q(q),$$

where $\lim_{q \to \infty} Q(q) = a^{\frac{1}{1-r}}$.

Bipower inverse demand $p = \widehat{a}q^{-\nu} + aq^{-\frac{1}{\sigma}}$**:**

Assume that $\sigma > 1$, $a > 0$, and $\nu > \frac{1}{\sigma}$; we immediately obtain

$$p = q^{-\frac{1}{\sigma}} \left(\widehat{a}q^{\frac{1}{\sigma}-\nu} + a \right) \equiv q^{-\frac{1}{\sigma}} Q(q)$$

$$\lim_{q \to \infty} Q(q) = a, \ \lim_{q \to \infty} p = 0, \ \lim_{q \to \infty} pq = \infty.$$

CEMR demand $p = \widehat{a}q^{-1} + aq^{-\frac{1}{\sigma}}$**:**

Assume that $\sigma > 1$ and $a > 0$; we immediately obtain

$$p = q^{-\frac{1}{\sigma}} \left(\widehat{a}q^{\frac{1}{\sigma}-1} + a \right) \equiv q^{-\frac{1}{\sigma}} Q(q)$$

$$\lim_{q \to \infty} Q(q) = a, \ \lim_{q \to \infty} p = 0, \ \lim_{q \to \infty} pq = \infty.$$

CREMR demand $p = \frac{a}{q} (q - \widehat{a})^{\frac{\sigma-1}{\sigma}}$**:**

Assume that $\sigma > 1$, $a > 0$, and $q > \widehat{a}\sigma$; we immediately obtain

$$p = q^{-\frac{1}{\sigma}} a \left(1 - \frac{\widehat{a}}{q} \right)^{\frac{\sigma-1}{\sigma}} \equiv q^{-\frac{1}{\sigma}} Q(q)$$

$$\lim_{q \to \infty} Q(q) = a, \ \lim_{q \to \infty} p = 0, \ \lim_{q \to \infty} pq = \infty.$$

□

C Model with a Perfectly-Competitive and Centralized Financial Market

This section shows that the model in the main text can be easily extended to incorporate a perfectly competitive and centralized financial market. Recall that in the first two stages of the model each entrant must pay an entry cost to draw a distinct product and a failure rate γ, and then pay the innovation cost to acquire a desired productivity if the failure rate is sufficiently low. We now bring in the aspect of borrowing and lending, and assume that entrants and firms do not have any fund before they sell their products to the markets. Assume that all of the consumers are the financiers. Entrants/firms thus must borrow from the financiers to finance their entry and innovation costs. They will have to repay the debt with interest at the end of the third stage, or default.

As there are numerous financiers and borrowers, we assume that the financial market is perfectly competitive with a market interest rate r. Assume that all agents are risk neutral. Also assume that the financiers have no information regarding a borrower's (an entrant/firm's) individual failure rate γ. Recall from the main model that any firm which decides to innovate subsequently produces. Denote the set of firms which innovate and produce by Ω. Then, every entrant must borrow to finance entry costs, but only the firms in Ω must borrow to finance innovation cost. After the realization of their types, those firms outside Ω default and quit at the beginning of the second stage, whereas those firms in Ω must pay back the debts with interest payments once their outputs are sold to the markets in the third stage. A financier's expected profit from lending to a borrower in the market is given by

$$\begin{aligned}\overline{\Pi}_{\text{financier}} \equiv & (1+r)\,w\left\{\Pr(\gamma\in\Omega)\,\kappa_e + \int_{\Omega}\left[\kappa_R + \gamma V\left(\widetilde{\varphi}(\gamma)\right)\right]dF(\gamma)\right\} \\ & - w\left\{\kappa_e + \int_{\Omega}\left[\kappa_R + \gamma V\left(\widetilde{\varphi}(\gamma)\right)\right]dF(\gamma)\right\}.\end{aligned}$$

That the financial market is perfectly competitive implies that the expected profit $\overline{\Pi}_{\text{financier}}$ is driven to zero in equilibrium. Thus, the equilibrium interest rate is given by

$$r = \frac{\left[1-\Pr(\gamma\in\Omega)\right]\kappa_e}{\int_{\Omega}\left[\kappa_R + \gamma V\left(\widetilde{\varphi}(\gamma)\right)\right]dF(\gamma) + \Pr(\gamma\in\Omega)\,\kappa_e}. \tag{1}$$

In this model in which there is no time discounting among the three stages, the market interest rate is still strictly positive to compensate for the losses from default by those entrants which do not find it profitable to innovate and produce. The probability of an entrant defaulting is therefore the probability of exiting, $1 - \Pr(\gamma \in \Omega)$.

In the above derivation, we have implicitly assumed that any entrants which do not innovate (and hence do not produce) do not borrow at the beginning of Stage 2. For this to stand, it is further assumed that any firms which produce, obtain positive operating profits π, and subsequently default have their operating profits π forfeited with an additional default cost that equals $\epsilon > 0$ fraction of the size of the loan. At the beginning of the second stage and for the entrants outside Ω, there are three options. First, by definition they earn a $\Pi(\gamma) < 0$ if they choose to still innovate, subsequently produce, and not default. Second, if they innovate, produce, but default, then their expected return is $-\epsilon w \left[\kappa_e + \kappa_R + \gamma V\left(\widetilde{\varphi}\left(\gamma\right)\right)\right]$. Third, if they quit at the beginning of the second stage, the expected return is $-\epsilon w \kappa_e$, which dominates the second option. Here, ϵ is assumed to be an arbitrarily small number, and thus in equilibrium the subset of firms with $\Pi(\gamma) < 0$ which find it optimal to choose the first option is negligible.

How does incorporating a financial sector affect the equilibrium outcome? Recall that the third-stage profit is given by $\pi\left(\varphi\right) = \max_q pq - w\varphi^{-1}q - w\kappa_D$. Note that the total debt for a producing firm with productivity φ is $w\left[\kappa_e + \kappa_R + \gamma V\left(\varphi\right)\right]$. In the innovation stage, a firm with γ chooses φ to maximize its total profit:

$$\Pi\left(\gamma\right) \equiv \max_{\varphi} \pi\left(\varphi\right) - \left(1+r\right) w \left[\kappa_e + \kappa_R + \gamma V\left(\varphi\right)\right].$$

A firm innovates and produces subsequently if and only if $\Pi\left(\gamma\right) \geq 0$. Note that this problem is different from the one in the main text because the entry cost paid in the first stage is no longer sunk, as the firm must repay it in the third stage. For simplicity, we assume that ϵ is arbitrarily small so that the defaulting costs can be ignored in computing the expected profits viewed at the first stage. Thus the free-entry condition is still given by

$$E\left(\Pi\right) = \int_{\Omega} \Pi\left(\gamma\right) dF\left(\gamma\right) = 0.$$

Compared with the benchmark model in the main text, the existence of a positive interest rate and the fact that the entry cost κ_e is no longer sunk change the equilibrium productivity $\widetilde{\varphi}(\gamma)$ and the set of surviving firms Ω; thus the free-entry condition implies that the mass of entrants M_e is also different.

Note that one can also incorporate the financial market into the Melitz (2003) model in the same manner. As there is no innovation stage, (1) becomes

$$r = \frac{1 - \Pr(\varphi \in \Omega)}{\Pr(\varphi \in \Omega)}.$$

In the Melitz model, selection is determined by $\{\varphi \in \Omega : \pi(\varphi) \geq 0\}$, where the fixed cost of production $\kappa_D > 0$ inside π is key to the determination of selection. With a financial market, selection is instead determined by $\{\varphi \in \Omega : \pi(\varphi) \geq (1 + r) w\kappa_e\}$. Again, the entry cost κ_e is no longer sunk, and with a positive interest rate, the set of surviving firms Ω is also different from that in the Melitz model. The fact that entry cost κ_e is treated as sunk in the Melitz model suggests that the borrowing and lending problem is abstracted away in that model; a likely justification is that there is a large pool of entrants with their own fund for the entry cost.

It is now clear that incorporating a financial sector qualitatively changes neither Melitz (2003) nor our model in terms of the main messages, but it does matter in terms of the magnitudes of the equilibrium productivity, selection, and the mass of entrants. As Melitz (2003) and many other trade models ignore this aspect of the financial market, we follow them to keep the model in the main text simpler.

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