Online Appendix to "Innovation, Firm Size Distribution, and Gains from Trade"

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This online appendix provides the derivations for Table 1. We shows that the demand structures listed in Table 1 satisfy our Assumption 1. These demand structures include Bipower Direct and Inverse demand (Mrázová and Neary 2017), Pollak Family demand (Pollak 1971, which is equivalent to the HARA [Hyperbolic Absolute Risk Aversion] preference [Merton 1971; Zhelobodko et al. 2012]), PIGL (Price-Independent Generalized Linear) demand (Muellbauer 1975), the QMOR (Quadratic Mean of Order r) expenditure function (Diewert 1976; Feenstra 2018), CEMR (Constant Elasticity of Marginal Revenue) demand, and CREMR (Constant Revenue Elasticity of Marginal Revenue) demand (Mrázová, Neary, and Parenti 2021).

Bipower direct demand $q = \widehat{a}p^{-\nu} + ap^{-\sigma}$**:**

Assume that $\sigma > 1$, $a > 0$, and $\sigma > \nu$; we immediately obtain

$$q = p^{-\sigma}\left(\widehat{a}p^{\sigma-\nu} + a\right) \equiv \boldsymbol{q}\left(p\right)$$

$$\lim_{p\to 0}\left(\widehat{a}p^{\sigma-\nu} + a\right) = a$$

$$\lim_{p\to 0} q = \infty,\ \lim_{p\to 0} pq = \infty.$$

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Moreover, $\partial p/\partial q = -p^{-\nu-1}\left(\nu\widehat{a} + \sigma a p^{\nu-\sigma}\right)$ and is strictly negative for small enough p. Thus for large enough q the inverse demand is well-defined as $p = q^{-\frac{1}{\sigma}}\left(\widehat{a}\left[\boldsymbol{q}^{-1}(q)\right]^{\sigma-\nu} + a\right)^{\frac{1}{\sigma}} \equiv q^{-\frac{1}{\sigma}}Q(q)$ where $\lim\limits_{q\to\infty} Q(q) = a^{\frac{1}{\sigma}}$.

Pollak demand $q = \widehat{a} + ap^{-\sigma}$**:**

Assume that $\sigma > 1$ and $a > 0$; we immediately obtain

$$q = \widehat{a} + ap^{-\sigma}$$

$$\lim_{p\to 0} q = \infty,\ \lim_{p\to 0} pq = \infty.$$

As a result, $p = q^{-\frac{1}{\sigma}} a^{\frac{1}{\sigma}}\left(1 - \frac{\widehat{a}}{q}\right)^{-\frac{1}{\sigma}} \equiv q^{-\frac{1}{\sigma}}Q(q)$ where $\lim\limits_{q\to\infty} Q(q) = a^{\frac{1}{\sigma}}$.

PIGL demand $q = \widehat{a}p^{-1} + ap^{-\sigma}$**:**

Assume that $\sigma > 1$ and $a > 0$; we immediately obtain

$$q = p^{-\sigma}\left(\widehat{a}p^{\sigma-1} + a\right) \equiv \boldsymbol{q}(p)$$

$$\lim_{p\to 0}\left(\widehat{a}p^{\sigma-1} + a\right) = a$$

$$\lim_{p\to 0} q = \infty,\ \lim_{p\to 0} pq = \infty.$$

Moreover, $\partial p/\partial q = -p^{-2}\left(\widehat{a} + \sigma a p^{1-\sigma}\right)$ and is strictly negative for small enough p. Thus for large enough q the inverse demand is well-defined as $p = q^{-\frac{1}{\sigma}}\left(\widehat{a}\left[\boldsymbol{q}^{-1}(q)\right]^{\sigma-1} + a\right)^{\frac{1}{\sigma}} \equiv q^{-\frac{1}{\sigma}}Q(q)$ where $\lim\limits_{q\to\infty} Q(q) = a^{\frac{1}{\sigma}}$.

QMOR demand $q = ap^{r-1} + \widehat{a}p^{\frac{r}{2}-1}$**:**

Assume that $\sigma \equiv 1 - r > 1$ and $a > 0$. This implies that $r < 0$ and $1 - r > 1 - \frac{r}{2} > 0$; thus

$$q = p^{r-1}\left(\widehat{a}p^{-\frac{r}{2}} + a\right) \equiv \boldsymbol{q}(p)$$

$$\lim_{p\to 0}\left(\widehat{a}p^{-\frac{r}{2}}+a\right)=a$$

$$\lim_{p\to 0}q=\infty,\ \lim_{p\to 0}pq=\infty.$$

Moreover, $\partial p/\partial q = p^{r-2}\left[\left(\frac{r}{2}-1\right)\widehat{a}p^{-\frac{r}{2}}+(r-1)\,a\right]$ and is strictly negative for small enough p. Thus for large enough q the inverse demand is well-defined as

$$p=q^{\frac{1}{r-1}}\left(a+\widehat{a}\left[\boldsymbol{q}^{-1}(q)\right]^{-\frac{r}{2}}\right)^{\frac{1}{1-r}}\equiv q^{\frac{1}{r-1}}Q(q),$$

where $\lim_{q\to\infty}Q(q)=a^{\frac{1}{1-r}}$.

Bipower inverse demand $p=\widehat{a}q^{-\nu}+aq^{-\frac{1}{\sigma}}$:

Assume that $\sigma>1$, $a>0$, and $\nu>\frac{1}{\sigma}$; we immediately obtain

$$p=q^{-\frac{1}{\sigma}}\left(\widehat{a}q^{\frac{1}{\sigma}-\nu}+a\right)\equiv q^{-\frac{1}{\sigma}}Q(q)$$

$$\lim_{q\to\infty}Q(q)=a,\ \lim_{q\to\infty}p=0,\ \lim_{q\to\infty}pq=\infty.$$

CEMR demand $p=\widehat{a}q^{-1}+aq^{-\frac{1}{\sigma}}$:

Assume that $\sigma>1$ and $a>0$; we immediately obtain

$$p=q^{-\frac{1}{\sigma}}\left(\widehat{a}q^{\frac{1}{\sigma}-1}+a\right)\equiv q^{-\frac{1}{\sigma}}Q(q)$$

$$\lim_{q\to\infty}Q(q)=a,\ \lim_{q\to\infty}p=0,\ \lim_{q\to\infty}pq=\infty.$$

CREMR demand $p=\frac{a}{q}(q-\widehat{a})^{\frac{\sigma-1}{\sigma}}$:

Assume that $\sigma>1$, $a>0$, and $q>\widehat{a}\sigma$; we immediately obtain

$$p=q^{-\frac{1}{\sigma}}a\left(1-\frac{\widehat{a}}{q}\right)^{\frac{\sigma-1}{\sigma}}\equiv q^{-\frac{1}{\sigma}}Q(q)$$

$$\lim_{q\to\infty}Q(q)=a,\ \lim_{q\to\infty}p=0,\ \lim_{q\to\infty}pq=\infty.$$

□

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