Comparative Advantage and Optimal Trade Policy with Strategic Interactions*

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Abstract

This paper introduces strategic interactions into a Ricardian model where two countries optimize trade policies. In a Nash Equilibrium, we find that optimal import tariffs are zero, while export taxes rise with comparative advantage, aligning with but extending the unilateral model of Costinot et al. (2015) by allowing both countries to act strategically. Our analysis reveals that welfare gains from trade policy diminish when both countries pursue optimal policies. Surprisingly, a smaller country can benefit from Nash Equilibrium through higher export taxes in goods with technological advantages. Applying the model to agriculture, we assess welfare under autarky, free trade, Nash Equilibrium, and unilateral policy, showing that small, competitive nations can outperform free trade through strategic policy, providing insights into optimal trade policy across goods.

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1 Introduction

Over two centuries have passed since David Ricardo proposed the groundbreaking concept of comparative advantage, a theory that has profoundly shaped international trade. Comparative advantage theory has since been expanded, refined, and successfully applied to address numerous questions in trade economics (See, for example, Dornbusch et al., 1977; Eaton and Kortum, 2002; Costinot et al., 2012; Caliendo and Parro, 2015). Yet, classical studies on optimal trade policies often concentrate on one or a limited number of goods, leaving open the question of how optimal trade policies might relate to comparative advantages across a broad array of goods. In their seminal work, Costinot et al. (2015) study a country's optimal trade policy in a canonical two-country Ricardian model *à la* Dornbusch et al. (1977). Focusing exclusively on the Home country's trade policy and assuming that the Foreign country remains passive (imposing no import tariffs or export taxes), they conclude that the optimal welfare-maximizing policy for the Home country involves uniform import tariffs and export taxes that are weakly monotonic in relation to its comparative advantage.

Building on the framework of Costinot et al. (2015), this paper introduces strategic interactions into the Ricardian model, allowing both countries to set optimal trade policies. In this model, a continuum of goods is produced using constant-returns-to-scale technology in a perfectly competitive market. Differences in productivity across goods generate comparative advantages, and each country's planner determines optimal import/export taxes or subsidies for each good, taking the trade policy of the other country's planner as given. This setup enables us to analyze a Nash Equilibrium of optimal trade policies—an extension from Costinot et al. (2015), who focused solely on unilateral trade policies. To achieve an analytical characterization of Nash Equilibrium properties, we assume quasilinear preferences, which simplifies the model structure relative to the original framework.

Given the differences in the setup, our model requires a nuanced interpretation. While a continuum of goods is often seen as representing distinct sectors within an economy, here it is more accurately viewed as representing individual products within a single sector. Consequently, our model adopts a partial equilibrium perspective, similar to approaches in the Industrial Organization literature. In reality, import tariffs vary across products within a sector, and industrial policies frequently impose differential taxes or subsidies on these products to influence export and import dynamics. Our model provides a valuable framework for examining these policy variations. For our quantitative application, we focus on the agricultural sector, where government intervention is extensive, making these considerations particularly relevant.

We demonstrate that, in a Nash Equilibrium, a country's optimal import tariffs are zero, while the optimal export taxes increase weakly with respect to its comparative advantage.

Consequently, the structure of optimal trade policies with respect to comparative advantages in this model aligns with that of Costinot et al. (2015). However, the welfare gains derived from these optimal trade policies are lower in our framework. The reasoning is as follows. For goods in which Home exhibits strong comparative disadvantages, a passive Foreign would lead Home to choose free trade, allowing production to shift to lower-cost firms in Foreign. In this scenario, the world prices—and thus Home's domestic prices—are determined solely by Foreign's marginal costs, enabling Home to capture a substantial consumer surplus. However, when Foreign is also strategic, it imposes strictly positive export taxes on these goods, resulting in increased world and domestic prices for Home. Consequently, Home's consumer surplus diminishes. Indeed, Costinot et al. suggest that Home's welfare could be quantitatively smaller in a Nash Equilibrium,[2] and our findings confirm this conjecture.

In terms of welfare implications, previous literature generally posits that a small country would fare worse under a Nash Equilibrium of optimal trade policies compared to a scenario of free trade (across all goods). However, our model suggests that this outcome may be reversed. Specifically, a small country can experience welfare gains under a Nash Equilibrium of optimal trade policies by imposing high export taxes on goods in which it has a significant technological edge over the other country. By doing so, the small country reaps substantial tax revenues, effectively capturing a portion of Foreign's consumer surplus.

In our quantitative analysis, we examine the welfare implications across four trade situations—autarky, free trade, Nash Equilibrium, and unilateral optimal trade policy (when the other country is passive)—using the agricultural sector as a case study. By setting the US and China as focal countries in different scenarios, we calculate welfare, tax revenue, and consumer surplus, normalizing each country's welfare to 100% under autarky for comparative clarity. Our results reveal that while both countries experience welfare gains when moving from autarky to free trade, unilateral optimal policies yield the highest welfare for each country when they alone control trade taxes. Interestingly, under Nash Equilibrium, the welfare of smaller, technologically superior country, such as the US relative to the Rest of the World (ROW) or China, improves compared with the free trade scenario due to high tax revenues in sectors where they hold strong competitive advantages. This echoes the aforementioned theoretical prediction.

Our research connects to multiple strands of literature, primarily focusing on optimal trade policy within international trade models. Trade policy, as a specialized form of industrial policies, has long been a central topic in trade research (Ossa, 2011; Felbermayr et al., 2015; Haaland and Venables, 2016; Costinot et al., 2020). Previous literature frequently focuses on optimal trade policies restricted to a single or limited set of goods or examines trade policies that apply uniformly across sectors (Itoh and Kiyono, 1987; Opp, 2010; Ossa, 2014;

[2] See their footnote 8.

Bagwell and Lee, 2020). our paper introduces strategic interactions into a Ricardian model *à la* Costinot et al. (2015), analyzing both unilateral optimal and Nash Equilibrium trade policies across goods. A notable theoretical prediction—supported by quantitative simulations—is that a small country may prevail under a Nash Equilibrium of optimal trade policies compared to free trade, a result previously detected in the literature (Kennan and Riezman, 1988; Syropoulos, 2002; Ossa, 2014). Our paper also closely relates to Beshkar and Lashkaripour (2017), who develop a general equilibrium, multi-industry model that illustrates the interdependencies between import tariffs, export taxes, and non-revenue trade barriers. Notably, the Beshkar and Lashkaripour (2017) model converges to the framework of Costinot et al. (2015) when trade elasticity is uniform and approaches zero, which implies a Cobb–Douglas utility aggregator. By contrast, our model incorporates uniform and finite trade elasticity alongside a Constant Elasticity of Substitution (CES) utility aggregator, thus complementing the framework presented in Beshkar and Lashkaripour (2017).

Additionally, this paper contributes to the literature on quantitative approaches to industrial policies, with a focus on the agricultural sector. Many studies on industrial policies highlight the importance of scale effects (Bartelme et al., 2024). For example, Lashkaripour and Lugovskyy (2023) analyze optimal industrial and trade policies within a class of quantitative trade models featuring scale effects and profits, showing that industrial policies must work alongside trade policies to address sectoral misallocation. Similarly, Ju et al. (2024) quantify trade wars and industrial policy competition between China and the US by extending the Caliendo and Parro (2015) model to incorporate sectoral external economies of scale. The positive aggregate effects of industrial policy on a production network through input-output linkages are demonstrated by Liu (2019). The economic impacts of industrial policies are substantial, especially when considering specific industries within certain economies (Kalouptsidi, 2018; Itskhoki and Moll, 2019; Barwick et al., 2024b); notable examples include China's auto industry (Bai et al., 2020), the shipbuilding industry (Barwick et al., 2021), and the vehicle market (Barwick et al., 2024a). Our paper discusses optimal trade policies specifically within the agricultural sector, an arena historically significant in international trade negotiations and policy disputes (Ossa, 2014; Bagwell et al., 2016). Unlike the industries extensively studied in the aforementioned literature, agriculture is characterized by a Ricardian structure (Costinot and Donaldson, 2016), which warrants a focused investigation. We show that the optimal agricultural policy is to protect weaker sectors and provide a quantitative assessment of the welfare implications under various policy scenarios. Our numerical simulations suggest that even a small yet technologically competitive country may benefit more from strategic industry policies than from free trade, enriching our understanding of industrial policy effectiveness across a broader array of sectors.

This paper is organized as follows. Section 2 introduces the theoretical model and formulates the government's welfare optimization problem. In Section 3, we derive each country's unilateral optimal trade policy, given the trade taxes set by the other country, addressing the best response problem. Section 4 characterizes the Nash Equilibrium in which two strategic countries set their optimal trade policies noncooperatively. Section 5 discusses the welfare implications of different trade policies. Section 6 applies the theoretical predictions to real-world scenarios. Finally, Section 7 concludes the paper.

2 Model

2.1 Setup

We incorporate strategic interactions into a Ricardian model à la Costinot et al. (2015). In this framework, the world economy consists of two countries, Home and Foreign. The Home country has a population mass of L, while Foreign has a mass of L^*. For ease of reference, variables associated with the foreign country are marked with an asterisk. Consumer preferences span a continuum of non-numeraire goods, indexed by $i \in \Omega$, as well as a numeraire good priced at $p_0 = 1$. Preferences are represented by the following quasilinear utility function:

$$\mathcal{U}^l(c^l) \equiv \int_{i\in\Omega} u_i^l(c_i^l)di + c_0^l = \int_{i\in\Omega} \beta_i \frac{(c_i^l)^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} di + c_0^l,$$

where $c^l \equiv ((c_i^l)_{i\in\Omega}, c_0^l) \geq 0$ represents the individual consumption bundle, and $u_i^l(c_i^l) \equiv \beta_i \frac{(c_i^l)^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}$ denotes the Constant Elasticity of Substitution (CES) utility function for each good $i \in \Omega$. Here, $\sigma \geq 1$ is the elasticity of substitution between non-numeraire goods, and (β_i) are exogenous preference parameters, normalized such that $\int_i \beta_i di = 1$.

Labor is the sole factor of production, with a linear production technology across all sectors. The unit labor requirements for producing each good $i \in \Omega$ are constant, denoted by a_i for Home and a_i^* for Foreign. Firms operate under perfect competition, with labor perfectly mobile across sectors within each country but immobile internationally. Each consumer is assumed to be endowed with one unit of labor, resulting in labor endowments of L and L^* at Home and Foreign, respectively. The numeraire good is produced using a one-to-one technology in both countries, ensuring that it is always produced and consumed in each. Consequently, the wage rates are normalized to one: $w = w^* = 1$, which presupposes sufficiently large labor endowments. Additionally, the numeraire good is freely traded, with no taxes or subsidies imposed by either government.

We introduce strategic interactions between the two countries, with the domestic government imposing ad-valorem trade taxes or subsidies, denoted by $t \equiv (t_i)$, and the foreign government responding with its own ad-valorem trade taxes or subsidies, $t^* \equiv (t_i^*)$. When good i is imported, $t_i, t_i^* \geq 0$ represents an import tariff, while $t_i, t_i^* \leq 0$ represents an import subsidy. Conversely, if good i is exported, $t_i, t_i^* \geq 0$ indicates an export subsidy, and $t_i, t_i^* \leq 0$ an export tax. Tax revenues are rebated evenly to domestic consumers through lump-sum transfers, T and T^*, respectively.

In a competitive equilibrium with trade taxes for Home, utility optimization by domestic consumers, profit maximization by domestic firms, a balanced budget by the domestic government, and the clearing of the domestic labor market imply that:

$$c^l \in \text{argmax}_{\tilde{c}^l \geq 0}\{\int_i u_i^l(\tilde{c}_i^l)di + \tilde{c}_0^l | \int_i p_i(1+t_i)\tilde{c}_i^l di + \tilde{c}_0^l \leq 1 + T^l\}, \tag{1}$$

$$q_i \in \text{argmax}_{\tilde{q}_i \geq 0}\{p_i(1+t_i)\tilde{q}_i - a_i\tilde{q}_i\}, \tag{2}$$

$$T = T^l L = \int_i p_i t_i(c_i - q_i)di = \int_i p_i t_i(c_i^l L - q_i)di, \tag{3}$$

$$L = \int_i a_i q_i di + q_0, \tag{4}$$

where $p \equiv (p_i) \geq 0$, $q \equiv (q_i) \geq 0$, and $c \equiv (c_i) = (c_i^l L)$ represent the schedule of world prices, domestic output, and aggregate consumption in Home (H), respectively.

Similarly, the equilibrium conditions for Foreign imply:

$$c^{l*} \in \text{argmax}_{\tilde{c}^l \geq 0}\{\int_i u_i^{l*}(\tilde{c}_i^l)di + \tilde{c}_0^l | \int_i p_i(1+t_i^*)\tilde{c}_i^l di + \tilde{c}_0^l \leq 1 + T^{l*}\}, \tag{5}$$

$$q_i^* \in \text{argmax}_{\tilde{q}_i \geq 0}\{p_i(1+t_i^*)\tilde{q}_i - a_i^*\tilde{q}_i\}, \tag{6}$$

$$T^* = T^{l*}L^* = \int_i p_i t_i^*(c_i^* - q_i^*)di = \int_i p_i t_i^*(c_i^{l*}L^* - q_i^*)di, \tag{7}$$

$$L^* = \int_i a_i^* q_i^* di + q_0^*, \tag{8}$$

where $q^* \equiv (q_i^*) \geq 0$ denotes the foreign output, and $c^* \equiv (c_i^*) = (c_i^{l*}L^*)$ represents the aggregate consumption in Foreign (F).

Finally, good market clearing conditions imply:

$$q_0 + q_0^* = c_0 + c_0^* = c_0^l L + c_0^{l*}L^*, \tag{9}$$

$$q_i + q_i^* = c_i + c_i^* = c_i^l L + c_i^{l*}L^*, \forall i. \tag{10}$$

2.2 Consumers' Problem

As previously mentioned, we assume that labor endowments are sufficiently large to ensure that the production and consumption of the numeraire good remain active. In a quasilinear utility setting, this implies positive consumption for any non-numeraire good. The consumers' optimization problem can then be expressed as:

$$\max_{c^l > 0} \mathcal{U}^l(c^l) = \int_{i\in\Omega} \beta_i \frac{(c_i^l)^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} di + c_0^l$$

subject to

$$\int_i p_i(1+t_i)c_i^l di + c_0^l = 1 + T^l \equiv I^l.$$

Solving this optimization problem yields the individual consumption of the non-numeraire good i:

$$c_i^l = \left[\frac{p_i(1+t_i)}{\beta_i}\right]^{-\sigma}. \tag{11}$$

The resulting individual indirect utility function is given by:

$$\begin{aligned} \mathcal{U}^l &= 1 + T^l + \int_{i\in\Omega} \left[\beta_i \frac{(c_i^l)^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} - p_i(1+t_i)c_i^l\right] di \\ &= \underbrace{1}_{\text{wage}} + \underbrace{T^l}_{\text{tax revenue}} + \underbrace{\frac{1}{\sigma-1}\int_{i\in\Omega} \left\{[p_i(1+t_i)]^{1-\sigma}\beta_i^\sigma - \beta_i\sigma\right\} di}_{\text{consumer surplus}}. \end{aligned} \tag{12}$$

The welfare decomposition in (12) shows that, under quasilinear utility, consumers' gross utility consists of the consumer surplus from the consumption of non-numeraire goods, the producer surplus (which is zero under perfect competition), tax revenue, and wage income from one unit of endowed labor, as discussed in Furusawa and Konishi (2004). Given our assumption that the numeraire good is always consumed, this implies that $I^l \equiv 1 + T^l > \int_{i\in\Omega} [p_i(1+t_i)]^{1-\sigma} \beta_i^\sigma di$.

2.3 Government's Problem

Departing from the setup in Costinot et al. (2015), we assume that both Home and Foreign are strategic countries. Each government aims to maximize the aggregate utility of its domestic consumers by choosing an optimal trade policy t. Thus, the domestic government's optimization problem is defined as

$$\max_t \mathcal{U}(c) = \mathcal{U}^l(c^l)L = -\frac{\sigma L}{\sigma-1} + L + \int_{i\in\Omega} p_i t_i(c_i - q_i)di + \frac{L}{\sigma-1}\int_{i\in\Omega} [p_i(1+t_i)]^{1-\sigma} \beta_i^\sigma di, \tag{13}$$

where $q_i \in \text{argmax}_{\tilde{q}_i \geq 0}\{p_i(1+t_i)\tilde{q}_i - a_i\tilde{q}_i\}$, as shown in (2).

Note that the government's problem consists of two parts: the first is a constant, while the second is additively separable in i. This allows us to solve the government's problem on a good-by-good basis. Thus, the good-specific objective function in Home (H) is defined as:

$$\mathcal{W}_i = p_i t_i (c_i - q_i) + \frac{L}{\sigma - 1} \left[p_i (1 + t_i) \right]^{1-\sigma} \beta_i^{\sigma}. \tag{14}$$

Similarly, the foreign government's problem can be written as:

$$\max_{t^*} \mathcal{U}^*(c^*) = -\frac{\sigma^* L^*}{\sigma^* - 1} + L^* + \int_{i \in \Omega} p_i t_i^* (c_i^* - q_i^*) di + \frac{L^*}{\sigma^* - 1} \int_{i \in \Omega} \left[p_i (1 + t_i^*) \right]^{1-\sigma^*} (\beta_i^*)^{\sigma^*} di, \tag{15}$$

where $q_i^* \in \text{argmax}_{\tilde{q}_i \geq 0} \{ p_i(1 + t_i^*)\tilde{q}_i - a_i^* \tilde{q}_i \}$. The good-specific objective function in Foreign (F) is then:

$$\mathcal{W}_i^* = p_i t_i^* (c_i^* - q_i^*) + \frac{L^*}{\sigma^* - 1} \left[p_i (1 + t_i^*) \right]^{1-\sigma^*} (\beta_i^*)^{\sigma^*}. \tag{16}$$

3 Unilateral Optimal Trade Policy

As long as Home produces, perfect competition implies $p_i(1 + t_i) = a_i$. Home's aggregate consumption is then $c_i = L c_i^l = L(\frac{a_i}{\beta_i})^{-\sigma}$ from (11). We can thus express world prices, Home's output, and consumption as functions of its net imports, $m \equiv c - q$, and taxes as follows:

$$p_i(m_i, t_i, t_i^*) \equiv \min\{ \frac{a_i}{1 + t_i}, u_i'(m_i / L)/(1 + t_i) \}, \tag{17}$$

$$q_i(m_i, t_i, t_i^*) \equiv \max\{ L(a_i/\beta_i)^{-\sigma} - m_i, 0 \}, \tag{18}$$

$$c_i(m_i, t_i, t_i^*) \equiv \max\{ L(a_i/\beta_i)^{-\sigma}, m_i \}. \tag{19}$$

From (17)-(19), when Home's net imports are sufficiently high such that $L(\frac{a_i}{\beta_i})^{-\sigma} - m_i < 0$, Home's firms do not produce good i. In this case, Home's consumption equals its net imports, m_i, and the world price is determined by Home's marginal utility and taxes: $p_i(m_i, t_i, t_i^*) = u_i'(m_i/L)/(1 + t_i)$.

By similar reasoning, we find that foreign aggregate consumption is $c_i^* = L^* c_i^{l*} = L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}$, with the world price satisfying $p_i(1 + t_i^*) = a_i^*$ when Foreign produces. Consequently, world prices, Foreign's output, and consumption can be expressed as functions of Home's net imports and the respective taxes:

$$p_i(m_i, t_i, t_i^*) \equiv \min\{ u_i^{*\prime}(-m_i/L^*)/(1 + t_i^*), a_i^*/(1 + t_i^*) \}, \tag{20}$$

$$q_i^*(m_i, t_i, t_i^*) \equiv \max\{ 0, L^*(a_i^*/\beta_i^*)^{-\sigma^*} + m_i \}, \tag{21}$$

$$c_i^*(m_i, t_i, t_i^*) \equiv \max\{ -m_i, L^*(a_i^*/\beta_i^*)^{-\sigma^*} \}. \tag{22}$$

From (20)–(22), when Home's net imports are very low, i.e., $L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*} + m_i < 0$, Foreign's firms do not produce good i. In this case, Foreign's consumption equals Home's net exports, $-m_i$. The world price is then determined by Foreign's marginal utility and taxes: $p_i(m_i, t_i, t_i^*) = u_i^{*\prime}(-m_i/L^*)/(1+t_i^*)$.

From the analysis above, we identify two cut-off points for Home's net imports: $M_i^I \equiv -L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}$, above which Foreign must produce good i, and $M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$, below which Home must produce good i. Accordingly, we can divide Home's net imports of good i into three regions: Region I with $m_i \leq M_i^I \equiv -L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}$, Region II with $M_i^I \leq m_i \leq M_i^{II}$, and Region III with $m_i \geq M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$. Our objective in this section is to determine Home's optimal trade taxes (t_i), given Foreign's trade taxes $(t_i^*), \forall i \in \Omega$. In this scenario, the domestic government aims to $\max_{t_i} \mathcal{W}_i$, as defined in (14), subject to the conditions in (17)–(22). We will now solve the domestic government's welfare optimization problem region by region.

3.1 Region I: $m_i \leq M_i^I \equiv -L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}$

In the first region, where $m_i \leq M_i^I \equiv -L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}$, only Home produces good i. Under these conditions, equations (17)–(22) can be simplified as follows: $m_i \leq -L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}$, and $p_i = \frac{a_i}{1+t_i} = \frac{\beta_i^*}{1+t_i^*}\left(\frac{-m_i}{L^*}\right)^{-1/\sigma^*} \leq \frac{a_i^*}{1+t_i^*}$. Home's net imports can thus be expressed as $m_i = -L^*(\frac{a_i^*}{\beta_i^*}\frac{1+t_i^*}{1+t_i})^{-\sigma^*}$. Accordingly, the domestic government's objective function in (14) can be rewritten as:

$$\begin{aligned}
\mathcal{W}_i(m_i, t_i; t_i^*) &= p_i t_i (c_i - q_i) + \frac{L}{\sigma-1} a_i^{1-\sigma}\beta_i^{\sigma} \\
&= a_i m_i - p_i m_i + \frac{L}{\sigma-1} a_i^{1-\sigma}\beta_i^{\sigma} \\
&= a_i m_i - \frac{\beta_i^*}{1+t_i^*}\left(\frac{-m_i}{L^*}\right)^{-1/\sigma^*} m_i + \frac{L}{\sigma-1} a_i^{1-\sigma}\beta_i^{\sigma}. \qquad (23)
\end{aligned}$$

The function $\mathcal{W}_i(m_i, t_i; t_i^*)$ is strictly increasing for $m_i \in (-\infty, m_i^I)$ and strictly decreasing for $m_i \in (m_i^I, M_i^I)$, where $m_i^I \equiv -L^*[\frac{a_i}{\beta_i^*}\frac{\sigma^*}{\sigma^*-1}(1+t_i^*)]^{-\sigma^*}$. Consequently, the domestic government will select the optimized net imports of good i, denoted as $m_i^{Io} = m_i^I$, yielding a world price of $p_i = \frac{\beta_i^*}{1+t_i^*}(\frac{-m_i^I}{L^*})^{-1/\sigma^*} = \frac{\sigma^*}{\sigma^*-1}a_i$. Home's optimal trade tax for good i satisfies $1+t_i = a_i/p_i = \frac{\sigma^*-1}{\sigma^*}$, and the optimized welfare for Home is: $\mathcal{W}_i^{Io} = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^{\sigma} + \frac{L^* a_i}{\sigma^*-1}[\frac{a_i}{\beta_i^*}\frac{\sigma^*}{\sigma^*-1}(1+t_i^*)]^{-\sigma^*}$. Recalling that $M_i^I \equiv -L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}$, the interval $[m_i^I, M_i^I]$ is non-empty if $\frac{a_i}{a_i^*} \leq \frac{\sigma^*-1}{\sigma^*}\frac{1}{1+t_i^*} \equiv A_i^I$, or equivalently, $1+t_i^* \leq \frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i} \equiv B_i^I$.

However, if $m_i^I \geq M_i^I$, i.e., $1+t_i^* \geq \frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i}$, then $\mathcal{W}_i(m_i, t_i; t_i^*)$ is maximized at $m_i^{Io} = M_i^I$. From the condition $m_i = -L^*(\frac{a_i^*}{\beta_i^*}\frac{1+t_i^*}{1+t_i})^{-\sigma^*} = M_i^I = -L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}$, Home's optimal tax becomes $1+t_i = \frac{a_i}{a_i^*}(1+t_i^*) \geq \frac{\sigma^*-1}{\sigma^*}$. As a result, the world price satisfies $p_i = \frac{a_i}{1+t_i} = \frac{a_i^*}{1+t_i^*}$. Home's welfare, in this case, becomes $\mathcal{W}_i^{Io} = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + (a_i - \frac{a_i^*}{1+t_i^*}) \left[-L^* (\frac{a_i^*}{\beta_i^*})^{-\sigma^*} \right]$.

The intuition behind the varying values of $(1 + t_i)$ in response to $(1 + t_i^*)$ is as follows. In this region, only Home produces good i. When $\frac{a_i}{a_i^*} \le \frac{\sigma^*-1}{\sigma^*} \frac{1}{1+t_i^*} \equiv A_i^I$, or equivalently, $1 + t_i^* \le \frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} \equiv B_i^I$, Foreign's import tariff is relatively low, giving Home significant flexibility to influence the world price. In this case, Home sets a constant monopoly markup over its marginal cost, leading to a world price of $p_i = \frac{\sigma^*}{\sigma^*-1} a_i$. However, if $1 + t_i^* \ge \frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i}$—indicating that Foreign has raised its import tariff to shield its domestic producers—Home must lower its export tax to adjust to Foreign's tariff, ensuring that Foreign is indifferent between producing or not producing good i. As a result, the world price becomes $p_i = \frac{a_i}{1+t_i} = \frac{a_i^*}{1+t_i^*}$, reflecting a limit-pricing strategy by Home, as described by Costinot et al. (2015). If Foreign's tariff becomes exceedingly high, the world price could fall too low. To sustain domestic production and exports, the Home government might shift from an export tax to a positive export subsidy, with $1 + t_i > 1$. However, it's essential to note that this analysis assumes that only Home is producing. The trade taxes (t_i) determined here may not necessarily be the optimal ones overall, given the known values (a_i, a_i^*, t_i^*).

3.2 Region II: $-L^* (\frac{a_i^*}{\beta_i^*})^{-\sigma^*} \equiv M_i^I \le m_i \le M_i^{II} \equiv L (\frac{a_i}{\beta_i})^{-\sigma}$

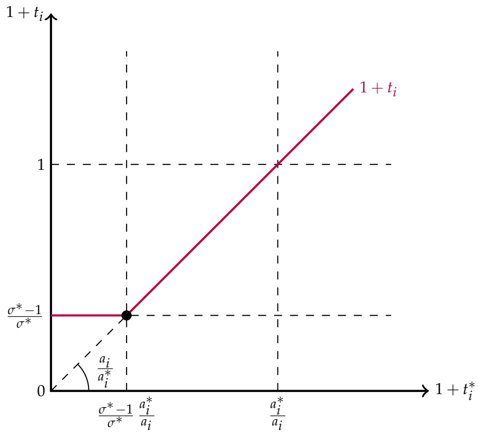
In the middle region, both countries are able to produce good i. Under these conditions, equations (17)–(22) can be simplified as follows: $-L^* (\frac{a_i^*}{\beta_i^*})^{-\sigma^*} \equiv M_i^I \le m_i \le M_i^{II} \equiv L (\frac{a_i}{\beta_i})^{-\sigma}$, and $p_i = \frac{a_i}{1+t_i} = \frac{a_i^*}{1+t_i^*}$. Accordingly, the domestic government's objective function in (14) can be rewritten as:

$$\mathcal{W}_i(m_i, t_i; t_i^*) = (a_i - p_i) m_i + \frac{L}{\sigma - 1} a_i^{1-\sigma} \beta_i^\sigma. \tag{24}$$

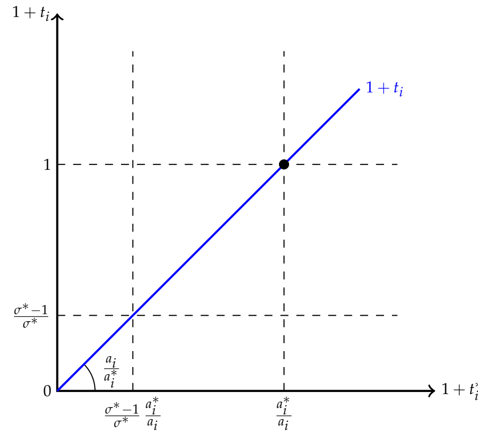
If $a_i \le \frac{a_i^*}{1+t_i^*} = p_i$, then $\mathcal{W}_i(m_i, t_i; t_i^*)$ is decreasing in m_i. Therefore, $\mathcal{W}_i(m_i, t_i; t_i^*)$ is maximized at $m_i^{IIo} = M_i^I$, where only Home produces. The optimal tax satisfies $1 + t_i = a_i / p_i = \frac{a_i}{a_i^*}(1 + t_i^*) \le 1$. Home's welfare in this case is: $\mathcal{W}_i^{IIo} = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + (a_i - \frac{a_i^*}{1+t_i^*}) \left[-L^* (\frac{a_i^*}{\beta_i^*})^{-\sigma^*} \right]$. We define this as Region IIA, where $\frac{a_i}{a_i^*} \le \frac{1}{1+t_i^*} \equiv A_i^{II}$ and $1 + t_i^* \le \frac{a_i^*}{a_i} \equiv B_i^{II}$.

In contrast, if $a_i \ge \frac{a_i^*}{1+t_i^*} = p_i$, then $\mathcal{W}_i(m_i, t_i; t_i^*)$ is increasing in m_i. Consequently, $\mathcal{W}_i(m_i, t_i; t_i^*)$ is maximized at $m_i^{IIo} = M_i^{II}$, where only Foreign produces. The optimal tax satisfies $1 + t_i = a_i / p_i = \frac{a_i}{a_i^*}(1 + t_i^*) \ge 1$. Home's welfare in this scenario is: $\mathcal{W}_i^{IIo} = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma + (a_i - \frac{a_i^*}{1+t_i^*}) \left[L (\frac{a_i}{\beta_i})^{-\sigma} \right]$. This region is defined as Region IIB, where $\frac{a_i}{a_i^*} \ge \frac{1}{1+t_i^*} \equiv A_i^{II}$ and $1 + t_i^* \ge \frac{a_i^*}{a_i} \equiv B_i^{II}$.

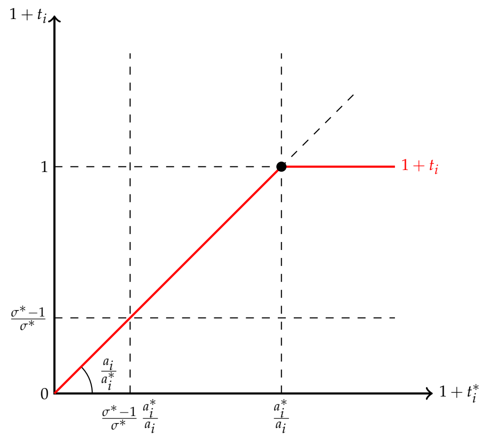
As noted, in Region II, both Home and Foreign are able to produce, but Home's capacity to influence world prices is significantly constrained by Foreign. Therefore, a limit-pricing strategy still applies in this region. In Region IIA, where $1 + t_i^* \le \frac{a_i^*}{a_i}$, Foreign's chosen trade



(a) Region I



(b) Region II



(c) Region III

Figure 1: Best Response in Region I, II, and III

taxes are relatively low, keeping world prices above Home's marginal production cost. This allows Home to take over the entire production of good i, export to Foreign, and impose positive export taxes. In contrast, when $1 + t_i^* \geq \frac{a_i^*}{a_i}$, Foreign's trade taxes are high enough to squeeze world prices below Home's marginal cost. Consequently, Home ceases production of these goods, relying instead on imports from Foreign and simultaneously imposing positive import tariffs.

3.3 Region III: $m_i \geq M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$

The argument presented for Region I similarly applies to Region III. In this third region, where $m_i \geq M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$, only Foreign produces good i. Under these conditions, equations (17)–(22) can be simplified as: $m_i \geq M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$, and $p_i = \frac{a_i^*}{1+t_i^*} = \frac{\beta_i}{1+t_i}\left(\frac{m_i}{L}\right)^{-1/\sigma} \leq \frac{a_i}{1+t_i}$.

From this, we obtain Home's net imports as $m_i = L\left[\frac{a_i^*(1+t_i)}{\beta_i(1+t_i^*)}\right]^{-\sigma}$ and the relationship $p_i(1+t_i) = \beta_i\left(\frac{m_i}{L}\right)^{-1/\sigma}$. Accordingly, the domestic government's objective function in (14) can be rewritten as:

$$\begin{aligned}\mathcal{W}_i(m_i, t_i; t_i^*) &= p_i t_i (c_i - q_i) + \frac{L}{\sigma-1}\left[p_i(1+t_i)\right]^{1-\sigma}\beta_i^{\sigma} \\ &= \left[\beta_i\left(\frac{m_i}{L}\right)^{-1/\sigma} - \frac{a_i^*}{1+t_i^*}\right] m_i + \frac{L}{\sigma-1}\left[\beta_i\left(\frac{m_i}{L}\right)^{-1/\sigma}\right]^{1-\sigma}\beta_i^{\sigma} \\ &= \frac{\sigma\beta_i}{\sigma-1}L^{1/\sigma}m_i^{1-1/\sigma} - \frac{a_i^*}{1+t_i^*}m_i \end{aligned} \tag{25}$$

which is strictly increasing if $m_i \in (M_i^{II}, m_i^{III})$, and strictly decreasing if $m_i \in (m_i^{III}, +\infty)$, where $m_i^{III} \equiv L[\frac{a_i^*}{\beta_i}\frac{1}{1+t_i^*}]^{-\sigma}$. Consequently, the domestic government will choose the optimized net imports of good i, $m_i^{IIIo} = m_i^{III}$, resulting in a world price of $p_i = \frac{\beta_i}{1+t_i}\left(\frac{m_i}{L}\right)^{-1/\sigma} = \frac{1}{1+t_i}\frac{a_i^*}{1+t_i^*} = \frac{a_i^*}{1+t_i^*}$. Thus, Home's optimal trade tax for good i must satisfy $1+t_i = 1$. Home's optimized welfare is then: $\mathcal{W}_i^{IIIo} = \frac{L}{\sigma-1}\left[\frac{a_i^*}{1+t_i^*}\right]^{1-\sigma}\beta_i^{\sigma}$. Recalling that $M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$, the interval $[M_i^{II}, m_i^{III}]$ is non-empty if $\frac{a_i}{a_i^*} \geq \frac{1}{1+t_i^*} \equiv A_i^{III} = A_i^{II}$ or $1+t_i^* \geq \frac{a_i^*}{a_i} \equiv B_i^{III} = B_i^{II}$.

However, if $m_i^{III} \leq M_i^{II}$, i.e., $0 < 1+t_i^* \leq \frac{a_i^*}{a_i}$, then $\mathcal{W}_i(m_i, t_i; t_i^*)$ is maximized at $m_i^{IIIo} = M_i^{II}$. Given $m_i = L\left[\frac{a_i^*(1+t_i)}{\beta_i(1+t_i^*)}\right]^{-\sigma} = M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$, Home's optimal tax becomes $1+t_i = \frac{a_i}{a_i^*}(1+t_i^*) \leq 1$. As a result, the world price is $p_i = \frac{a_i^*}{1+t_i^*} = \frac{a_i}{1+t_i}$. Home's welfare in this case becomes: $\mathcal{W}_i^{IIIo} = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^{\sigma} + (a_i - \frac{a_i^*}{1+t_i^*})\left[-L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}\right]$.

In this region, only Foreign produces good i. When $\frac{a_i}{a_i^*} \geq \frac{1}{1+t_i^*}$, or equivalently, $1+t_i^* \geq \frac{a_i^*}{a_i}$, Foreign implements a high export subsidy, enabling Home to import from Foreign at a low world price. In this scenario, Home effectively embraces free trade, and domestic firms cease production. However, when $0 < 1+t_i^* \leq \frac{a_i^*}{a_i}$—meaning that Foreign reduces its export subsidy for good i or even imposes a positive export tax, leading to a higher world price—Home must impose a positive import subsidy to ensure that its domestic firms remain indifferent to producing or not.

3.4 Best Response

We have determined Home's best responses in each region, as illustrated in Figure 1. According to the definition of optimal trade policy, (t_i) represents the domestic government's optimal choice given the foreign taxes (t_i^*) and the technologies (a_i, a_i^*) across all three regions. In other words,

$$1+t_i \equiv \text{argmax}_{(t_i^*; a_i, a_i^*)}\left\{\mathcal{W}_i^{Io}(t_i^*; a_i, a_i^*), \mathcal{W}_i^{IIo}(t_i^*; a_i, a_i^*), \mathcal{W}_i^{IIIo}(t_i^*; a_i, a_i^*)\right\}. \tag{26}$$

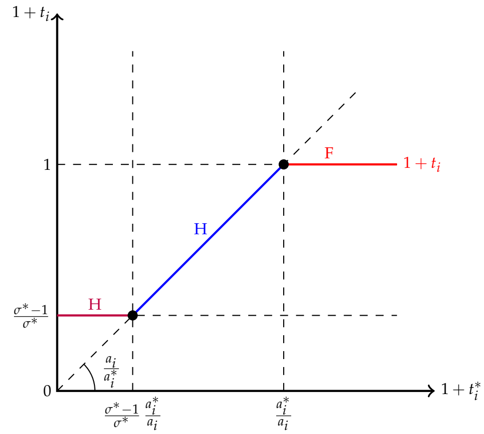


Figure 2: Home's Unilateral Optimal Trade Policy

Note that there are two cut-off values for $(1+t_i^*)$: $B_i^I \equiv \frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i}$ and $B_i^{II} \equiv \frac{a_i^*}{a_i} \equiv B_i^{III}$. Accordingly, Home's optimal trade policy $(1+t_i)$ can be divided into five scenarios: Scenario 1 with $0 < 1+t_i^* < \frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i}$, Scenario 2 with $1+t_i^* = \frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i}$, Scenario 3 with $\frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i} < 1+t_i^* < \frac{a_i^*}{a_i}$, Scenario 4 with $1+t_i^* = \frac{a_i^*}{a_i}$, and Scenario 5 with $1+t_i^* > \frac{a_i^*}{a_i}$. In each scenario, Home selects the trade policy that maximizes welfare among $\mathcal{W}_i^{Io}(t_i^*;a_i,a_i^*)$, $\mathcal{W}_i^{IIo}(t_i^*;a_i,a_i^*)$, and $\mathcal{W}_i^{IIIo}(t_i^*;a_i,a_i^*)$.

After comparing the optimized welfare across the three regions in each of the five scenarios, we can summarize Home's unilateral optimal trade policy in the following proposition, as shown in Figure 2.

Proposition 1 *If the two strategic countries set their optimal trade policies noncooperatively, given the foreign trade taxes (t_i^*) and technology parameters (a_i, a_i^*), Home's unilateral optimal trade policy is characterized as follows. The producer, optimal trade taxes (t_i), world prices (p_i), Home's net imports (m_i), and Home's optimized welfare $(\mathcal{W}_i)$ satisfy:*

(a) only Home produces, $1+t_i = \frac{\sigma^*-1}{\sigma^*}$, $p_i = \frac{\sigma^*}{\sigma^*-1}a_i$, $m_i = m_i^I$, *and* $\mathcal{W}_i = \mathcal{W}_i^{Io}$, *if* $1+t_i^* \in (0, B_i^I)$;

(b) only Home produces, $1+t_i = \frac{\sigma^*-1}{\sigma^*} = \frac{a_i}{a_i^*}(1+t_i^*)$, $p_i = \frac{\sigma^*}{\sigma^*-1}a_i = \frac{a_i^*}{1+t_i^*} = \frac{a_i}{1+t_i}$, $m_i = m_i^I = M_i^I$, *and* $\mathcal{W}_i = \mathcal{W}_i^{Io} = \mathcal{W}_i^{IIo}$, *if* $1+t_i^* = B_i^I$;

(c) only Home produces, $1+t_i = \frac{a_i}{a_i^*}(1+t_i^*)$, $p_i = \frac{a_i}{1+t_i} = \frac{a_i^*}{1+t_i^*}$, $m_i = M_i^I$, *and* $\mathcal{W}_i = \mathcal{W}_i^{IIo}$, *if* $1+t_i^* \in (B_i^I, B_i^{II})$;

(d) either Home or Foreign produces, $1+t_i = 1$, $p_i = a_i = \frac{a_i^*}{1+t_i^*}$, $m_i \in [M_i^I, M_i^{II}]$, *and* $\mathcal{W}_i = \mathcal{W}_i^{IIo} = \mathcal{W}_i^{IIIo}$, *if* $1+t_i^* = B_i^{II}$;

(e) only Foreign produces, $1+t_i = 1$, $p_i = \frac{a_i^*}{1+t_i^*}$, $m_i = m_i^{III}$, *and* $\mathcal{W}_i = \mathcal{W}_i^{IIIo}$, *if* $1+t_i^* \in (B_i^{II}, +\infty)$; *where* $B_i^I \equiv \frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i}, B_i^{II} \equiv \frac{a_i^*}{a_i} \equiv B_i^{III}; M_i^I \equiv -L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}, M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}; m_i^I \equiv -L^*[\frac{a_i}{\beta_i^*}\frac{\sigma^*}{\sigma^*-1}(1+t_i^*)]^{-\sigma^*}, m_i^{III} \equiv L[\frac{a_i^*}{\beta_i}\frac{1}{1+t_i^*}]^{-\sigma}; \mathcal{W}_i^{Io} \equiv \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^{\sigma} + \frac{L^* a_i}{\sigma^*-1}[\frac{a_i}{\beta_i^*}\frac{\sigma^*}{\sigma^*-1}(1+t_i^*)]^{-\sigma^*}, \mathcal{W}_i^{IIo} \equiv$

$\frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^{\sigma} + (a_i - \frac{a_i^*}{1+t_i^*})\left[-L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}\right], \mathcal{W}_i^{IIIo} \equiv \frac{L}{\sigma-1}\left[\frac{a_i^*}{1+t_i^*}\right]^{1-\sigma}\beta_i^{\sigma}$.

Proposition 1 underscores the significant influence of fundamental comparative advantage ($\frac{a_i^*}{a_i}$) and Foreign's trade policy (t_i^*) on production decisions, optimal import levels, and the imposition of optimal trade taxes, with strategic interactions accounted for. If Foreign sets taxes $(1+t_i^*)$ above its fundamental comparative advantage ($\frac{a_i^*}{a_i}$), thereby overly protecting its domestic firms, it becomes optimal for Home to adopt a free trade policy and rely solely on imports from Foreign. Conversely, if Foreign's production is not excessively protected, i.e., $1+t_i^* < \frac{a_i^*}{a_i}$, Home can take over the entire production of good i and impose a positive export tax. However, Home must employ a limit-pricing strategy that leaves foreign firms indifferent to producing good i. If the foreign tariff is exceptionally low—meaning Home has a strong comparative advantage in producing good i—then Home can apply a monopoly markup over its marginal cost, effectively behaving like a monopolist.

An interesting case arises when Foreign sets $t_i^* = 0$. From the preceding discussion, a specialization cut-off exists at $\frac{a_i^*}{a_i} = 1$, which is related to absolute advantage. This cut-off stems from the equalized wage rates in both countries, a result of quasilinear preferences. As long as Home has an absolute advantage over Foreign in producing good i, it will take over the entire production and can impose a positive export tax. Otherwise, Home will adopt a free trade policy without producing good i, reallocating its labor toward the numeraire good.

The unilateral optimal trade policy can also be examined in the presence of exogenous iceberg trade costs. In Appendix A, we demonstrate that the unilateral optimal trade policy with trade costs remains qualitatively similar to the case without trade costs, with the primary difference being the emergence of a range of nontraded goods.

4 Nash Equilibrium

Based on Home's best response above, we can similarly characterize Foreign's best response, as depicted in Figure 3. Recall that Home's optimal trade policy introduced two cut-offs for $(1+t_i^*)$: $\frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i}$ and $\frac{a_i^*}{a_i}$. Now, Foreign's best response adds two additional cut-offs: $\frac{\sigma-1}{\sigma}$ and 1. Combining these four cut-offs, we identify seven possible cases for $\frac{a_i}{a_i^*}$: Case 1 with $0 < \frac{a_i}{a_i^*} < \frac{\sigma^*-1}{\sigma^*}$, Case 2 with $\frac{a_i}{a_i^*} = \frac{\sigma^*-1}{\sigma^*}$, Case 3 with $\frac{\sigma^*-1}{\sigma^*} < \frac{a_i}{a_i^*} < 1$, Case 4 with $\frac{a_i}{a_i^*} = 1$, Case 5 with $1 < \frac{a_i}{a_i^*} < \frac{\sigma-1}{\sigma}$, Case 6 with $\frac{a_i}{a_i^*} = \frac{\sigma-1}{\sigma}$, and Case 7 with $\frac{a_i}{a_i^*} > \frac{\sigma-1}{\sigma}$. We will discuss each of these cases individually. Figure 4 illustrates the Nash Equilibrium in each scenario, with the blue curve representing Home's best response and the red curve representing Foreign's.

Case 1: $0 < \frac{a_i}{a_i^*} < \frac{\sigma^*-1}{\sigma^*}$. In this case, the ranking of the four cut-offs is $\frac{\sigma-1}{\sigma} < 1 < \frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i} < \frac{a_i^*}{a_i}$. Based on this ranking, we can plot the best responses of both countries together, as shown in

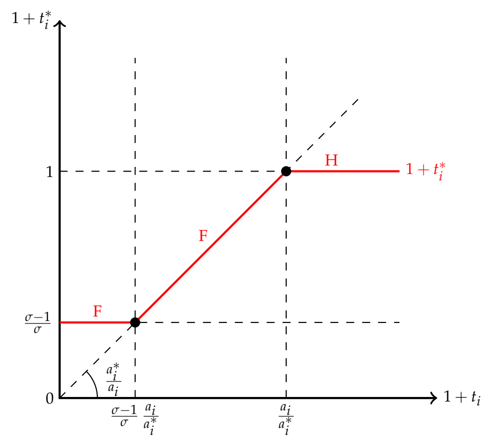


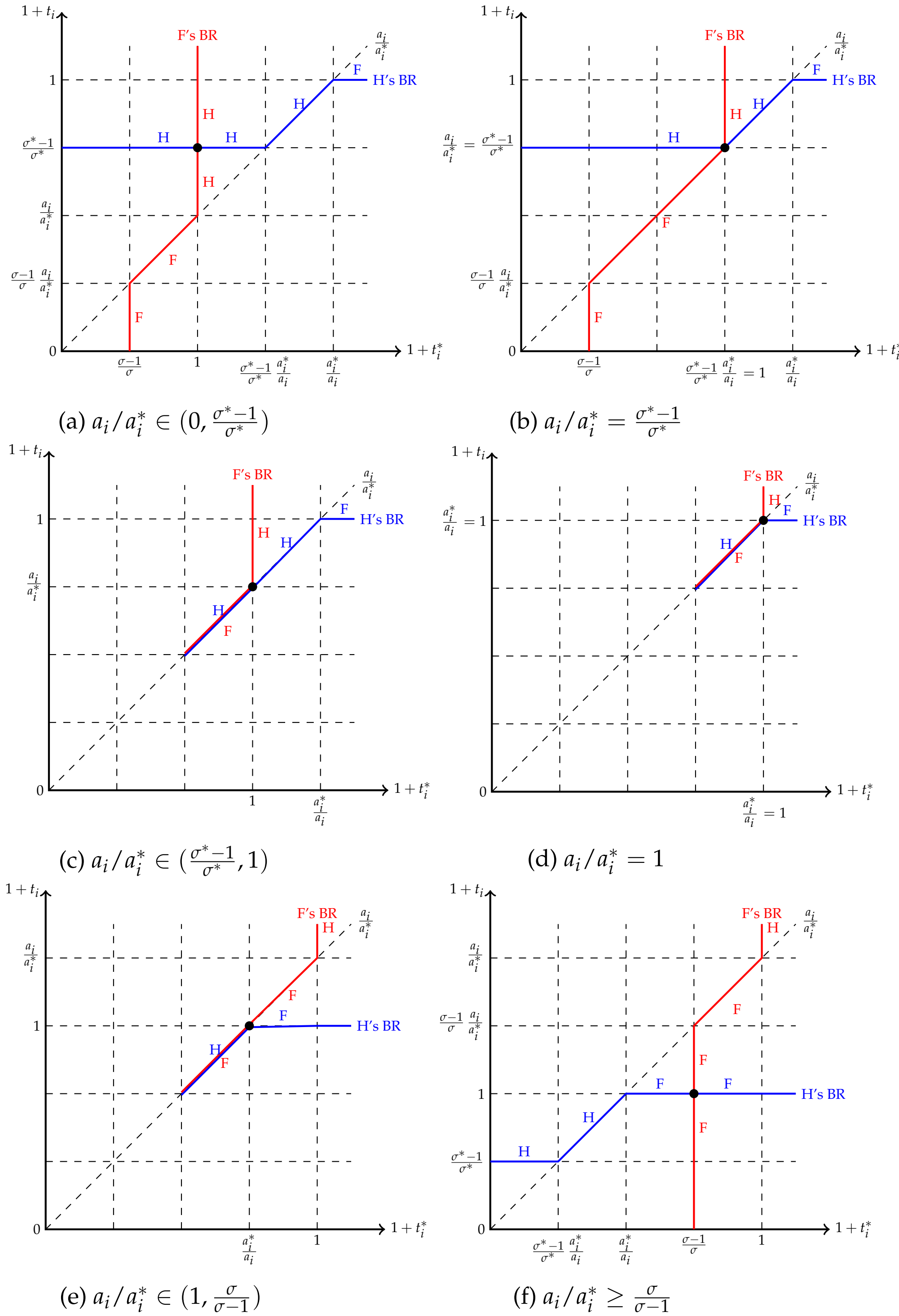
Figure 3: Foreign's Unilateral Optimal Trade Policy

Sub-figure 4a of Figure 4. The Nash Equilibrium policy is $1 + t_i = \frac{\sigma^*-1}{\sigma^*}$ and $1 + t_i^* = 1$. In equilibrium, only Home produces good i, and the world price is $p_i = \frac{\sigma^*}{\sigma^*-1}a_i$. Home's optimal imports are $m_i^o = -L^*\left[\frac{a_i}{\beta_i^*}\frac{\sigma^*}{\sigma^*-1}\right]^{-\sigma^*}$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^\sigma + \frac{L^*a_i}{\sigma^*-1}\left[\frac{a_i}{\beta_i^*}\frac{\sigma^*}{\sigma^*-1}\right]^{-\sigma^*}$ and domestic price $p_i^d = p_i(1 + t_i) = a_i$.

Case 2: $\frac{a_i}{a_i^*} = \frac{\sigma^*-1}{\sigma^*}$**.** In this case, the ranking of the four cut-offs is $\frac{\sigma-1}{\sigma} < 1 = \frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i} < \frac{a_i^*}{a_i}$. Based on this ranking, we can plot both countries' best responses together, as shown in Sub-figure 4b of Figure 4. The Nash Equilibrium policy is $1 + t_i = \frac{\sigma^*-1}{\sigma^*}$ and $1 + t_i^* = 1$. In equilibrium, only Home produces good i, and the world price is $p_i = \frac{\sigma^*}{\sigma^*-1}a_i = a_i^*$. Home's optimal imports are $m_i^o = -L^*\left[\frac{a_i}{\beta_i^*}\frac{\sigma^*}{\sigma^*-1}\right]^{-\sigma^*} = -L^*\left[\frac{a_i^*}{\beta_i^*}\right]^{-\sigma^*} = M_i^I$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^\sigma + \frac{L^*a_i}{\sigma^*-1}\left[\frac{a_i}{\beta_i^*}\frac{\sigma^*}{\sigma^*-1}\right]^{-\sigma^*} = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^\sigma + (a_i^* - a_i)L^*\left[\frac{a_i^*}{\beta_i^*}\right]^{-\sigma^*}$ and domestic price $p_i^d = a_i$.

Case 3: $\frac{\sigma^*-1}{\sigma^*} < \frac{a_i}{a_i^*} < 1$**.** In this case, the ranking of the four cut-offs is $\frac{\sigma-1}{\sigma} < 1 < \frac{a_i^*}{a_i}$ and $\frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i} < 1 < \frac{a_i^*}{a_i}$. Based on this ranking, we can plot both countries' best responses together, as shown in Sub-figure 4c of Figure 4. The Nash Equilibrium policy is $1 + t_i = \frac{a_i}{a_i^*}$ and $1 + t_i^* = 1$. In equilibrium, only Home produces good i, and the world price is $p_i = a_i^*$. Home's optimal imports are $m_i^o = -L^*\left[\frac{a_i^*}{\beta_i^*}\right]^{-\sigma^*} = M_i^I$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^\sigma + (a_i^* - a_i)L^*\left[\frac{a_i^*}{\beta_i^*}\right]^{-\sigma^*}$ and domestic price $p_i^d = a_i$.

Case 4: $\frac{a_i}{a_i^*} = 1$**.** In this case, the ranking of the four cut-offs is $\frac{\sigma-1}{\sigma} < 1 = \frac{a_i^*}{a_i}$ and $\frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i} < 1 = \frac{a_i^*}{a_i}$. Based on this ranking, we can plot both countries' best responses together, as shown



(a) $a_i/a_i^* \in (0, \frac{\sigma^*-1}{\sigma^*})$

(b) $a_i/a_i^* = \frac{\sigma^*-1}{\sigma^*}$

(c) $a_i/a_i^* \in (\frac{\sigma^*-1}{\sigma^*}, 1)$

(d) $a_i/a_i^* = 1$

(e) $a_i/a_i^* \in (1, \frac{\sigma}{\sigma-1})$

(f) $a_i/a_i^* \geq \frac{\sigma}{\sigma-1}$

Figure 4: Nash Equilibrium in Each Case

in Sub-figure 4d of Figure 4. The Nash Equilibrium policy is $1+t_i = 1$ and $1+t_i^* = 1$. In equilibrium, either Home or Foreign can produce good i, and the world price is $p_i = a_i^* = a_i$. Home's optimal imports m_i^o can take any value in $[M_i^I, M_i^{II}]$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma$ and domestic price $p_i^d = a_i = a_i^*$.

Case 5: $1 < \frac{a_i}{a_i^*} < \frac{\sigma}{\sigma-1}$. In this case, the ranking of the four cut-offs is $\frac{\sigma-1}{\sigma} < \frac{a_i^*}{a_i} < 1$ and $\frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} < \frac{a_i^*}{a_i} < 1$. Based on this ranking, we can plot both countries' best responses together, as shown in Sub-figure 4e of Figure 4. The Nash Equilibrium policy is $1+t_i = 1$ and $1+t_i^* = \frac{a_i^*}{a_i}$. In equilibrium, only Foreign produces good i, and the world price is $p_i = a_i$. Home's optimal imports are $m_i^o = L\left[\frac{a_i}{\beta_i}\right]^{-\sigma} = M_i^{II}$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma$ and domestic price $p_i^d = a_i$.

Case 6: $\frac{a_i}{a_i^*} = \frac{\sigma}{\sigma-1}$. In this case, the ranking of the four cut-offs is $\frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} < \frac{a_i^*}{a_i} = \frac{\sigma-1}{\sigma} < 1$. The Nash Equilibrium policy is $1+t_i = 1$ and $1+t_i^* = \frac{a_i^*}{a_i} = \frac{\sigma-1}{\sigma}$. In equilibrium, only Foreign produces good i, and the world price is $p_i = a_i = \frac{\sigma}{\sigma-1} a_i^*$. Home's optimal imports are $m_i^o = M_i^{II} = L\left[\frac{a_i}{\beta_i}\right]^{-\sigma} = L\left[\frac{a_i^*}{\beta_i} \frac{\sigma}{\sigma-1}\right]^{-\sigma}$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1} a_i^{1-\sigma} \beta_i^\sigma = \frac{L}{\sigma-1}\left[a_i^* \frac{\sigma}{\sigma-1}\right]^{1-\sigma} \beta_i^\sigma$ and domestic price $p_i^d = a_i = \frac{\sigma}{\sigma-1} a_i^*$.

Case 7: $\frac{a_i}{a_i^*} > \frac{\sigma}{\sigma-1}$. In this case, the ranking of the four cut-offs is $\frac{\sigma^*-1}{\sigma^*} \frac{a_i^*}{a_i} < \frac{a_i^*}{a_i} < \frac{\sigma-1}{\sigma} < 1$. Based on this ranking, we can plot both countries' best responses together, as shown in Sub-figure 4f of Figure 4. The Nash Equilibrium policy is $1+t_i = 1$ and $1+t_i^* = \frac{\sigma-1}{\sigma}$. In equilibrium, only Foreign produces good i, and the world price is $p_i = \frac{\sigma}{\sigma-1} a_i^*$. Home's optimal imports are $m_i^o = L\left[\frac{a_i^*}{\beta_i} \frac{\sigma}{\sigma-1}\right]^{-\sigma}$, with optimized welfare $\mathcal{W}_i^o = \frac{L}{\sigma-1}\left[a_i^* \frac{\sigma}{\sigma-1}\right]^{1-\sigma} \beta_i^\sigma$ and domestic price $p_i^d = \frac{\sigma}{\sigma-1} a_i^*$.

Combining the previous discussion, we summarize the Nash Equilibrium in noncooperative trade policy with respect to each country's comparative advantage in the following proposition, as illustrated in Figure 5.

Proposition 2 *If the two strategic countries set their optimal trade policy noncooperatively, the Nash Equilibrium taxes are (a)* $1+t_i = \frac{\sigma^*-1}{\sigma^*}$ *and* $1+t_i^* = 1$*, if* $\frac{a_i}{a_i^*} \leq \frac{\sigma^*-1}{\sigma^*}$*; (b)* $1+t_i = \frac{a_i}{a_i^*}$ *and* $1+t_i^* = 1$*, if* $\frac{a_i}{a_i^*} \in (\frac{\sigma^*-1}{\sigma^*}, 1]$*; (c)* $1+t_i = 1$ *and* $1+t_i^* = \frac{a_i^*}{a_i}$*, if* $\frac{a_i}{a_i^*} \in (1, \frac{\sigma}{\sigma-1}]$*; (d)* $1+t_i = 1$ *and* $1+t_i^* = \frac{\sigma-1}{\sigma}$*, if* $\frac{a_i}{a_i^*} > \frac{\sigma}{\sigma-1}$ *.*

Proposition 2 underscores the significant role of comparative advantage in determining the Nash trade taxes, especially with the introduction of strategic interactions. As noted in Costinot et al. (2015), when a country has a very strong comparative advantage in producing

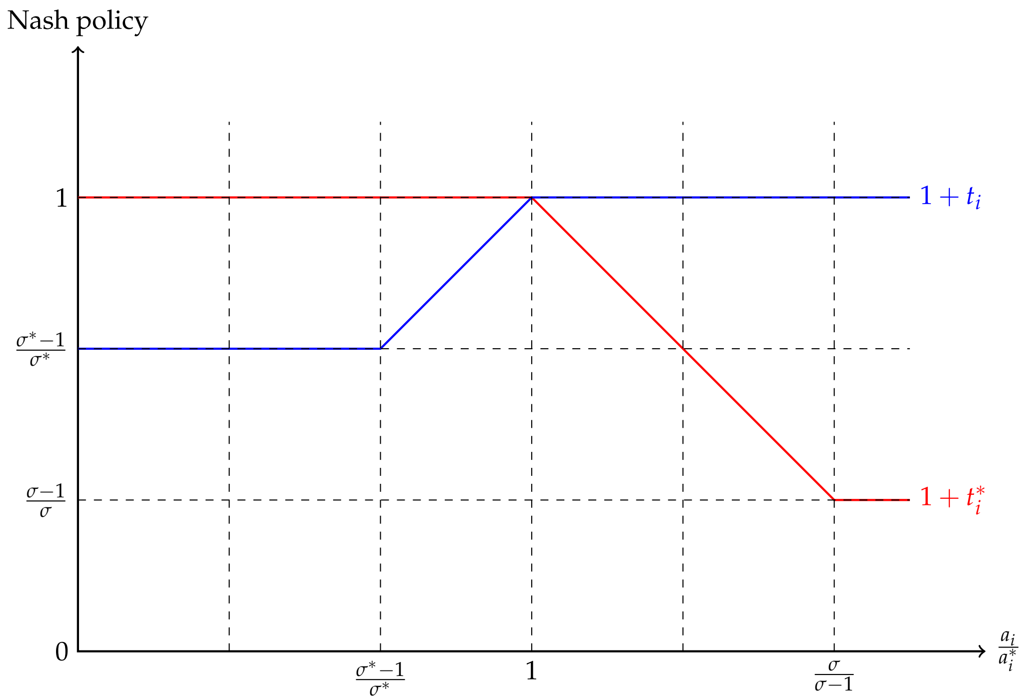


Figure 5: Nash Equilibrium in Noncooperative Trade Policy

good i, it can manipulate world prices by charging a monopoly markup over its marginal cost. In contrast, the other country, with much lower productivity in producing this good, is forced to adopt free trade, unable to constrain its partner from exploiting its monopoly power.

In the intermediate range, where neither country's technology dominates the other's, each country's ability to manipulate the world price is constrained by its partner's unit labor requirement. In equilibrium, a specialization cut-off for comparative advantage, $\frac{a_i}{a_i^*} = 1$, emerges, closely linked to absolute advantage. If Home has an absolute disadvantage in producing good i compared to Foreign (i.e., $a_i > a_i^*$), Home will opt for free trade and cease production of good i. In this scenario, Foreign takes over the entire production of good i and imposes a positive export tax that increases with its comparative advantage. This pricing strategy, known as limit-pricing as described by Costinot et al. (2015), is designed to make Home's firms exactly indifferent to producing or not.

Notably, there is never a situation where an import tariff arises. As emphasized, the quasi-linear preferences imply equalized wages between the two countries.[3] If a country has an absolute disadvantage in producing good i, it will abandon production of that good entirely and adopt free trade, reallocating the labor previously employed in producing good i to the nu-

[3]Differentiated wages across countries could easily be introduced by assigning different unit labor requirements for the numeraire good, which would yield different comparative advantage cut-offs in the Nash Equilibrium characterization. However, the conclusions regarding Nash policies remain qualitatively unchanged.

meraire good. Alternatively, if the country were to subsidize domestic firms to produce this good, it would increase the domestic price, thereby reducing consumer surplus, regardless of whether positive or negative tax revenue is generated.

Let's compare Home's policy in the Nash Equilibrium with its trade policy when Foreign is passive (i.e., $t_i^* = 0$). The structure of trade taxes in relation to comparative advantage remains identical in both scenarios. However, Home is strictly worse off in the Nash Equilibrium, where Foreign is also strategic. The reasoning is straightforward: for goods in which Home has an absolute disadvantage, if Foreign is passive, Home will adopt free trade, allowing production to be taken over by Foreign's lower-cost firms. Consequently, the world price and Home's domestic price are determined solely by Foreign's marginal cost, enabling Home to gain a large consumer surplus. In contrast, if Foreign is strategic, it imposes strictly positive export taxes on these goods, resulting in higher world and domestic prices for Home. This reduces Home's consumer surplus. This finding provides theoretical support for Costinot et al.'s conjecture that Home's welfare may be substantially lower in the Nash Equilibrium.

5 Welfare Implication

Given the Nash policies described above, we can now compare the welfare implications under four trade settings: autarky, free trade, unilateral optimal trade taxes, and Nash Equilibrium with optimal trade taxes. Recall that under a quasilinear utility function, the consumer's objective is to maximize $\mathcal{U}^l(c^l) \equiv \int_{i\in\Omega} u_i^l(c_i^l)di + c_0^l = \int_{i\in\Omega} \beta_i \frac{(c_i^l)^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}di + c_0^l$, subject to the budget constraint $\int_i p_i^d c_i^l di + c_0^l = I^l = 1 + T^l$, where the individual's total income I^l consists of wage income and tax revenue T^l, and p_i^d represents domestic prices. The maximized aggregate utility, or domestic welfare, can be expressed as:

$$\mathcal{W} = -\frac{\sigma L}{\sigma - 1} + \underbrace{L}_{\text{wage}} + \underbrace{T}_{\text{tax revenue}} + \underbrace{\frac{L}{\sigma - 1}\int_{i\in\Omega}(p_i^d)^{1-\sigma}\beta_i^\sigma di}_{\text{consumer surplus}}. \tag{27}$$

5.1 Autarky

In autarky, all goods are produced domestically with no ad-valorem taxes or subsidies, so $p_i^d = a_i, \forall i$. From (27), the autarky welfare at home is given by:

$$\mathcal{W}^{Autarky} = -\frac{\sigma L}{\sigma - 1} + \underbrace{L}_{\text{wage}} + \underbrace{0}_{\text{tax revenue}} + \underbrace{\frac{L}{\sigma - 1}\int_{i\in\Omega} a_i^{1-\sigma}\beta_i^\sigma di}_{\text{consumer surplus}}. \tag{28}$$

5.2 Free Trade

If both countries adopt a free trade policy, then $p_i^d = p_i = \min\{a_i, a_i^*\}$, as shown in Dornbusch et al. (1977). From (27), the domestic welfare under free trade is:

$$\mathcal{W}^{FreeTrade} = -\frac{\sigma L}{\sigma - 1} + \underbrace{L}_{\text{wage}} + \underbrace{0}_{\text{tax revenue}} + \underbrace{\frac{L}{\sigma-1}\int_{i\in\{a_i\le a_i^*\}} a_i^{1-\sigma}\beta_i^{\sigma} di + \frac{L}{\sigma-1}\int_{i\in\{a_i>a_i^*\}} (a_i^*)^{1-\sigma}\beta_i^{\sigma} di}_{\text{consumer surplus}}. \tag{29}$$

It is evident that both countries are better off under free trade compared to autarky, as shown by comparing (28) with (29):

$$\mathcal{W}^{FreeTrade} - \mathcal{W}^{Autarky} = \underbrace{\frac{L}{\sigma-1}\int_{i\in\{a_i>a_i^*\}} \left[(a_i^*)^{1-\sigma} - a_i^{1-\sigma}\right]\beta_i^{\sigma} di}_{\text{positive consumer surplus}}. \tag{30}$$

Each country gains from trade by specializing in sectors where it has a comparative advantage, generating higher consumer surplus in sectors where it has a comparative disadvantage by consuming imports from the other country.

5.3 Unilateral Optimal

If only the Home country sets trade policies while Foreign remains passive (i.e., $t_i^* = 0$), then $p_i^d = p_i(1+t_i) = \min\{a_i, a_i^*\}$. From (27), the domestic welfare under the unilateral optimal policy is:

$$\begin{aligned}\mathcal{W}^{Unilateral} &= -\frac{\sigma L}{\sigma - 1} + \underbrace{L}_{\text{wage}} + \underbrace{\frac{L}{\sigma-1}\int_{i\in\{a_i\le a_i^*\}} a_i^{1-\sigma}\beta_i^{\sigma} di + \frac{L}{\sigma-1}\int_{i\in\{a_i>a_i^*\}} (a_i^*)^{1-\sigma}\beta_i^{\sigma} di}_{\text{consumer surplus}} \\ &+ \underbrace{\frac{L^*}{\sigma^*-1}\int_{i\in\{\frac{a_i}{a_i^*}<\frac{\sigma^*-1}{\sigma^*}\}} \left(\frac{\sigma^*-1}{\sigma^*}\right)^{\sigma^*} a_i^{1-\sigma^*}(\beta_i^*)^{\sigma^*} di + \int_{i\in\{\frac{\sigma^*-1}{\sigma^*}\le\frac{a_i}{a_i^*}<1\}} L^*\left(\frac{a_i^*}{\beta_i^*}\right)^{-\sigma^*}(a_i^*-a_i)di}_{\text{tax revenue}}.\end{aligned} \tag{31}$$

Comparing (29) with (31), we find that the Home country is strictly better off under the unilateral optimal policy than under free trade:

$$\mathcal{W}^{Unilateral} - \mathcal{W}^{FreeTrade} = \underbrace{\frac{L^*}{\sigma^*-1}\int_{i\in\{\frac{a_i}{a_i^*}<\frac{\sigma^*-1}{\sigma^*}\}} \left(\frac{\sigma^*-1}{\sigma^*}\right)^{\sigma^*} a_i^{1-\sigma^*}(\beta_i^*)^{\sigma^*} di + \int_{i\in\{\frac{\sigma^*-1}{\sigma^*}\le\frac{a_i}{a_i^*}<1\}} L^*\left(\frac{a_i^*}{\beta_i^*}\right)^{-\sigma^*}(a_i^*-a_i)di}_{\text{positive tax revenue}}.$$

The intuition here is that, for goods in which Home has an absolute advantage, it can generate positive tax revenue by imposing strictly positive export taxes on these products.

5.4 Nash Equilibrium

When both countries set trade policies noncooperatively, the world prices p_i and domestic optimal trade taxes t_i are determined in Nash Equilibrium, as shown in Proposition 2. Thus, Home's prices are:

$$p_i^d = p_i(1+t_i) = \begin{cases} a_i, & \text{otherwise;} \\ \frac{\sigma}{\sigma-1}a_i^*, & \text{if } \frac{a_i}{a_i^*} > \frac{\sigma}{\sigma-1}. \end{cases} \tag{32}$$

Using (32), (27), and Proposition 2, we can express Home's welfare under noncooperative trade policies as follows:

$$\begin{aligned} \mathcal{W}^{NE} = & -\frac{\sigma L}{\sigma-1} + \underbrace{L}_{\text{wage}} + \underbrace{\frac{L}{\sigma-1}\int_{i\in\{\frac{a_i}{a_i^*}\le\frac{\sigma}{\sigma-1}\}} a_i^{1-\sigma}\beta_i^\sigma di + \frac{L}{\sigma-1}\int_{i\in\{\frac{a_i}{a_i^*}>\frac{\sigma}{\sigma-1}\}}\left(\frac{\sigma}{\sigma-1}a_i^*\right)^{1-\sigma}\beta_i^\sigma di}_{\text{consumer surplus}} \\ & + \underbrace{\frac{L^*}{\sigma^*-1}\int_{i\in\{\frac{a_i}{a_i^*}<\frac{\sigma^*-1}{\sigma^*}\}}\left(\frac{\sigma^*-1}{\sigma^*}\right)^{\sigma^*} a_i^{1-\sigma^*}(\beta_i^*)^{\sigma^*}di + \int_{i\in\{\frac{\sigma^*-1}{\sigma^*}\le\frac{a_i}{a_i^*}<1\}} L^*\left(\frac{a_i^*}{\beta_i^*}\right)^{-\sigma^*}(a_i^*-a_i)di}_{\text{tax revenue}}. \end{aligned} \tag{33}$$

We compare Home's welfare under Nash Equilibrium with that under autarky:

$$\begin{aligned} \mathcal{W}^{NE} - \mathcal{W}^{Autarky} = & \underbrace{\frac{L}{\sigma-1}\int_{i\in\{\frac{a_i}{a_i^*}>\frac{\sigma}{\sigma-1}\}}\left[\left(\frac{\sigma}{\sigma-1}a_i^*\right)^{1-\sigma} - a_i^{1-\sigma}\right]\beta_i^\sigma di}_{\text{positive consumer surplus}} \\ & + \underbrace{\frac{L^*}{\sigma^*-1}\int_{i\in\{\frac{a_i}{a_i^*}<\frac{\sigma^*-1}{\sigma^*}\}}\left(\frac{\sigma^*-1}{\sigma^*}\right)^{\sigma^*} a_i^{1-\sigma^*}(\beta_i^*)^{\sigma^*}di + \int_{i\in\{\frac{\sigma^*-1}{\sigma^*}\le\frac{a_i}{a_i^*}<1\}} L^*\left(\frac{a_i^*}{\beta_i^*}\right)^{-\sigma^*}(a_i^*-a_i)di}_{\text{positive tax revenue}}. \end{aligned} \tag{34}$$

We find that the Home country is strictly better off under noncooperative trade policies than under autarky. This is due to two reasons. First, Home can generate larger consumer surplus by opting for free trade on its least productive goods, despite Foreign imposing a constant monopoly export tax. Second, strictly positive tax revenue can be obtained from goods where Home has a comparative advantage in production.

Next, we compare Home's welfare under Nash Equilibrium with its welfare under a unilateral optimal trade policy:

$$\mathcal{W}^{NE} - \mathcal{W}^{Unilateral} = \underbrace{\frac{L}{\sigma-1}\int_{i\in\{\frac{\sigma}{\sigma-1}<\frac{a_i}{a_i^*}\le 1\}}\left[\left(\frac{\sigma}{\sigma-1}a_i^*\right)^{1-\sigma} - a_i^{1-\sigma}\right]\beta_i^\sigma di + \frac{L}{\sigma-1}\int_{i\in\{\frac{a_i}{a_i^*}>1\}}\left[\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}-1\right](a_i^*)^{1-\sigma}\beta_i^\sigma di}_{\text{negative consumer surplus}}.$$

We explicitly demonstrate that Home's welfare is lower under Nash Equilibrium than under the unilateral optimal trade policy. In the unilateral optimal trade policy setting, where Foreign remains passive, Home can obtain higher consumer surplus for goods in which it has an absolute disadvantage while still generating the same tax revenue as it would under Nash Equilibrium.

Furthermore, we compare Home's welfare under Nash Equilibrium with that under free trade:

$$\mathcal{W}^{NE} - \mathcal{W}^{FreeTrade} = \underbrace{\frac{L}{\sigma-1}\int_{i\in\{1<\frac{a_i}{a_i^*}\leq\frac{\sigma}{\sigma-1}\}}\left[a_i^{1-\sigma}-(a_i^*)^{1-\sigma}\right]\beta_i^{\sigma}di + \frac{L}{\sigma-1}\int_{i\in\{\frac{a_i}{a_i^*}>\frac{\sigma}{\sigma-1}\}}\left[\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}-1\right](a_i^*)^{1-\sigma}\beta_i^{\sigma}di}_{\text{negative consumer surplus}}$$
$$+\underbrace{\frac{L^*}{\sigma^*-1}\int_{i\in\{\frac{a_i}{a_i^*}<\frac{\sigma^*-1}{\sigma^*}\}}\left(\frac{\sigma^*-1}{\sigma^*}\right)^{\sigma^*}a_i^{1-\sigma^*}(\beta_i^*)^{\sigma^*}di + \int_{i\in\{\frac{\sigma^*-1}{\sigma^*}\leq\frac{a_i}{a_i^*}<1\}}L^*\left(\frac{a_i^*}{\beta_i^*}\right)^{-\sigma^*}(a_i^*-a_i)di.}_{\text{positive tax revenue}} \tag{35}$$

The sign of $(\mathcal{W}^{NE} - \mathcal{W}^{FreeTrade})$ depends on two factors: the relative size of the labor endowment L/L^* and the distribution of comparative advantage, or technology, $G(a_i/a_i^*)$. Simple algebra leads us to the following proposition:

Proposition 3 *All else constant, Home's welfare under Nash trade policies relative to free trade* $(\mathcal{W}^{NE} - \mathcal{W}^{FreeTrade})$ *is higher if Home has a smaller relative labor endowment size* L/L^**, and if the technology distribution* $G(a_i/a_i^*)$ *is more left-skewed compared to* $F(a_i/a_i^*)$*; in other words, if* $F(a_i/a_i^*)$ *first-order stochastically dominates* $G(a_i/a_i^*)$*.*

Proposition 3 suggests that Home may even be strictly better off under noncooperative trade policies compared to free trade, i.e., $\mathcal{W}^{NE} - \mathcal{W}^{FreeTrade} > 0$. An extreme example illustrating this outcome occurs when $G(a_i/a_i^*)$ is sufficiently left-skewed such that only the first region remains non-empty. In this scenario, Home can collect strictly positive tax revenues while maintaining its consumer surplus at the same level as under free trade. Naturally, this advantage for Home comes at the expense of a welfare loss for Foreign.

Home may also benefit from having a smaller relative size L/L^*, due to welfare gains from relatively larger positive tax revenue in its competitive sectors. This result is somewhat counter-intuitive, as most literature generally suggests that larger countries tend to gain more from trade policies or "win" in trade wars.[4] The reasoning behind this outcome is as follows: with quasilinear preferences, wages are fixed, eliminating the usual terms-of-trade considerations associated with wage adjustments. Instead, in this context, the optimal tax policy is driven by demand or relative market size. In sectors where Home has a stronger comparative advantage, it produces, exports, and imposes taxes. As shown in (35), the larger the size of the foreign country, the more tax revenue Home can generate, leading to higher domestic

[4] See, for example, Kennan and Riezman (1988), Syropoulos (2002), and Ossa (2011, 2014).

welfare.

6 Application

In this section, we apply our theoretical findings to two empirical scenarios. Each scenario focuses on two key players: the US and the Rest of the World (ROW), and China and the US. For each scenario, we quantitatively assess welfare, tax revenue, and consumer surplus across different trade situations—namely, autarky, free trade, Nash Equilibrium, and unilateral optimal trade policy. Our objective is to analyze how comparative advantage and trade scenarios drive quantitative changes in welfare. Furthermore, we aim to validate Proposition 3 and highlight several intriguing insights from our findings.

Our quantitative analysis focuses on the agricultural sector for several compelling reasons. First, from a theoretical perspective, the agricultural market structure aligns well with the assumptions of our quasilinear model. Second, from a policy perspective, agricultural security and trade policy are highly debated topics of global interest, as demonstrated by the lengthy and significant Doha Round negotiations within the World Trade Organization (WTO). Lastly, from a measurement standpoint, comparative advantage in agriculture tends to be stable over time, driven primarily by natural conditions and the cumulative scientific expertise of agronomists.

6.1 Data

Our data primarily come from the Global Agro-Ecological Zones (GAEZ) version 4 project, developed by the Food and Agriculture Organization (FAO). To compute domestic welfare as shown in Section 5, we use the following parameters: (1) the unit factor requirements in each area for each of the 49 crops,[5] denoted by a_i and a_i^*; (2) exogenous preference parameters, $\beta_i = \beta_i^* = 1$, under the assumption that all crops enter utility symmetrically in both countries; (3) the elasticity of substitution, $\sigma = \sigma^* = 2.9$, assuming a constant elasticity of substitution across all non-numeraire goods, consistent with the median estimate for the elasticity of substitution across our 49 crops as reported in Broda and Weinstein (2006);[6] (4) the sizes of the two regions, L and L^*, determined by the hectare devoted to the crops considered and vary according to specific scenarios. With these parameters defined, we calculate welfare, tax revenue, and consumer surplus for each trade scenario.

[5] In the original GAEZ v4 database, we consolidate the crop listings into 49 categories by grouping sub-varieties under their main varieties. For example, dryland rice and wetland rice are combined into a single category, "combined rice," using the maximum predicted GAEZ yield as the representative yield for this category. Other crops requiring similar consolidation include sorghum, cocoa, coffee, maize, millet, barley, wheat, yam, rye, rice, and phaseolus beans and grams.

[6] We restrict our analysis to five-digit SITC codes that match the raw versions of the 49 FAO crops.

The single factor of production in our model is agricultural land. Consequently, the size of a country is measured by its amount of agricultural land, data for which is available from the GAEZ project. The unit factor requirements for each crop also come from the GAEZ project (Costinot et al., 2015; Costinot and Donaldson, 2016). Using data on local conditions such as soil, elevation, topography, and climate, the GAEZ agronomic model computes the yield that parcels of land worldwide could achieve for each of the 49 crops we consider, assuming "high" levels of inputs and "rain-fed" water supply. We interpret a_i and a_i^* as the average hectare per ton of output across lands in Home and Foreign, respectively.[7]

6.2 The US and the Rest of the World

In the first scenario, we examine the welfare changes when the home country is the US and the foreign country represents the rest of the world (ROW). Accordingly, we set $L = 408,947,000$ and $L^* = 4,375,900,180$, matching the hectare devoted to the crops in our study. Table 1 presents the welfare, tax revenue, and consumer surplus for the US and ROW across four trade situations: autarky, free trade, Nash Equilibrium, and unilateral optimal trade policy. For clarity, we normalize each region's welfare to 100% under autarky when reporting all numerical results.

Table 1: The US and the Rest of the World

Region	**Trade Situation**	**Welfare**	**tax revenue**	**Consumer Surplus**
US	Autarky	100.00%	0.00%	100.30%
	Free trade	105.20%	0.00%	105.49%
	Nash Equilibrium	402.05%	300.36%	101.99%
	Unilateral optimal	405.56%	300.36%	105.49%
ROW	Autarky	100.00%	0.00%	100.65%
	Free trade	231.50%	0.00%	232.16%
	Nash Equilibrium	123.82%	0.39%	124.08%
	Unilateral optimal	231.90%	0.39%	232.16%

Notes: This table presents welfare, tax revenue, and consumer surplus for the US and ROW under four trade situations: autarky, free trade, Nash Equilibrium, and unilateral optimal trade policy. Welfare in each region is normalized to 100% under autarky for consistent numerical comparison.

[7] We conduct robustness checks using the average hectare per kilogram of output across lands. The welfare results are nearly identical, differing only in decimal points as a percentage. Alternative tables are available upon request.

As shown in Table 1, both welfare and consumer surplus reach their lowest levels under autarky. Both regions experience welfare gains when moving from autarky to free trade. As expected, each region achieves maximum welfare when it can set trade policy unilaterally while the other remains passive. These quantitative results align with the conclusions in Section 5. Additionally, we observe that the welfare of ROW under Nash Equilibrium is lower than under either free trade or unilateral optimal conditions.

In contrast to the ROW, the US experiences a substantial welfare increase in Nash Equilibrium compared to free trade, as shown in Table 1. This welfare boost for the US stems primarily from significant tax revenue in its competitive sectors, which compensates for the decline in consumer surplus. As outlined in Proposition 3 of Section 5, this difference arises because the US, while relatively small compared to the ROW, possesses a technological advantage in agricultural sectors. The Nash outcome between the US and ROW thus validates Proposition 3, suggesting that a smaller country can prevail in a trade war if it holds a technological edge.

6.3 China and the US

In the second scenario, we examine the welfare implications of different trade settings between China and the US. Here, we designate China as the home country and the US as the foreign country. We set $L = 529,038,600$ and $L^* = 408,947,000$ to reflect the hectare dedicated to the crops under consideration. Table 2 presents the welfare, tax revenue, and consumer surplus for China and the US across four trade situations: autarky, free trade, Nash Equilibrium, and unilateral optimal trade policy.

Table 2 presents findings under autarky, free trade, and unilateral optimal trade policy for both China and the US, consistent with the results discussed in Section 6.2. The most notable insight lies in the welfare comparison between free trade and Nash Equilibrium. For China, domestic welfare under Nash Equilibrium is significantly lower than under free trade. In contrast, the US experiences a substantial welfare gain in Nash Equilibrium relative to free trade.

The trade conflict between China and the US highlights the impact of relative country size and technological distribution. Here, country size is gauged by agricultural land, positioning the US as the smaller country relative to China. Starting from a free trade baseline, a shift to a noncooperative Nash Equilibrium introduces a notable dynamic: the smaller country, the US, faces a reduction in consumer surplus due to higher prices in sectors where it lacks a competitive edge, while increasing production and exports in sectors where it has a competitive advantage. Given the strong demand from China, the US's superior agricultural productivity enables it to generate substantial tax revenue, offsetting its consumer surplus loss. Thus, the trade conflict results in a welfare increase for the US and a decrease for China. This finding

Table 2: China and the US

Region	Trade Situation	Welfare	tax revenue	Consumer Surplus
China	Autarky	100.00%	0.00%	100.39%
	Free trade	133.56%	0.00%	133.95%
	Nash Equilibrium	100.76%	0.73%	100.42%
	Unilateral optimal	134.29%	0.73%	133.95%
US	Autarky	100.00%	0.00%	100.30%
	Free trade	101.14%	0.00%	101.44%
	Nash Equilibrium	124.74%	24.71%	100.33%
	Unilateral optimal	125.85%	24.71%	101.44%

Notes: This table presents welfare, tax revenue, and consumer surplus for China and the US under four trade situations: autarky, free trade, Nash Equilibrium, and unilateral optimal trade policy. Welfare in each region is normalized to 100% under autarky for consistent numerical comparison.

further supports our proposition that smaller countries, under certain conditions, can prevail in a trade war.

7 Conclusion

In this paper, we examine the optimal trade policy within a Ricardian model framework where both countries act strategically. Our findings reveal that, in a Nash Equilibrium, a country's optimal import tariffs are zero, while optimal export taxes increase with its comparative advantage. This outcome aligns structurally with the optimal trade policy model of Costinot et al. (2015), where the Foreign country is passive; however, the welfare gains from trade policy are lower in the strategic context. We further validate our theoretical predictions through empirical simulations in the agricultural sector, finding that the numerical results support the model's predictions. A particularly intriguing insight is that, under certain conditions, a smaller country may achieve greater welfare under a Nash Equilibrium of optimal trade policies compared to a free trade scenario. This result holds across simulations, including policy interactions between the US and the rest of the world (ROW) and between China and the US.

Several limitations in this study point to future research directions. First, we employ a numeraire good to fix the wage, which precludes consideration of the Lerner Symmetry Theorem (cf. Costinot and Werning, 2019). Second, extending the model to incorporate multiple sectors with sector-specific elasticities of substitution could further enrich the conclusions, as shown in Ossa (2014) and Beshkar and Lashkaripour (2017). Finally, incorporating dynamic decision-making into this Nash policy game could offer new insights into the protracted WTO negotiations. We leave these avenues open for future research.

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A Unilateral Optimal Trade Policy with Trade Costs

We extend the model to include exogenous iceberg trade costs, which are asymmetric. Specifically, if one unit of good i arrives at Home, $\delta \geq 1$ units must be shipped from Foreign; similarly, if one unit of good i arrives at Foreign, $\delta^* \geq 1$ units must be shipped from Home. With these trade costs, two distinct world prices emerge: p_i, the price at Home, and p_i^* the price in Foreign. The relationship between p_i and p_i^* can be expressed as:

$$\phi(m_i) \equiv \begin{cases} \delta, & \text{if } m_i \geq 0, \\ 1/\delta^*, & \text{if } m_i < 0. \end{cases} \tag{36}$$

Thus, the trade balance condition implies that Foreign's net imports are given by:

$$m_i^* = -m_i\phi(m_i) = \begin{cases} -\delta m_i, & \text{if } m_i \geq 0, \\ -\frac{1}{\delta^*} m_i, & \text{if } m_i < 0. \end{cases} \tag{37}$$

Accordingly, Equations (20)-(22) in Section 3 are modified as follows:

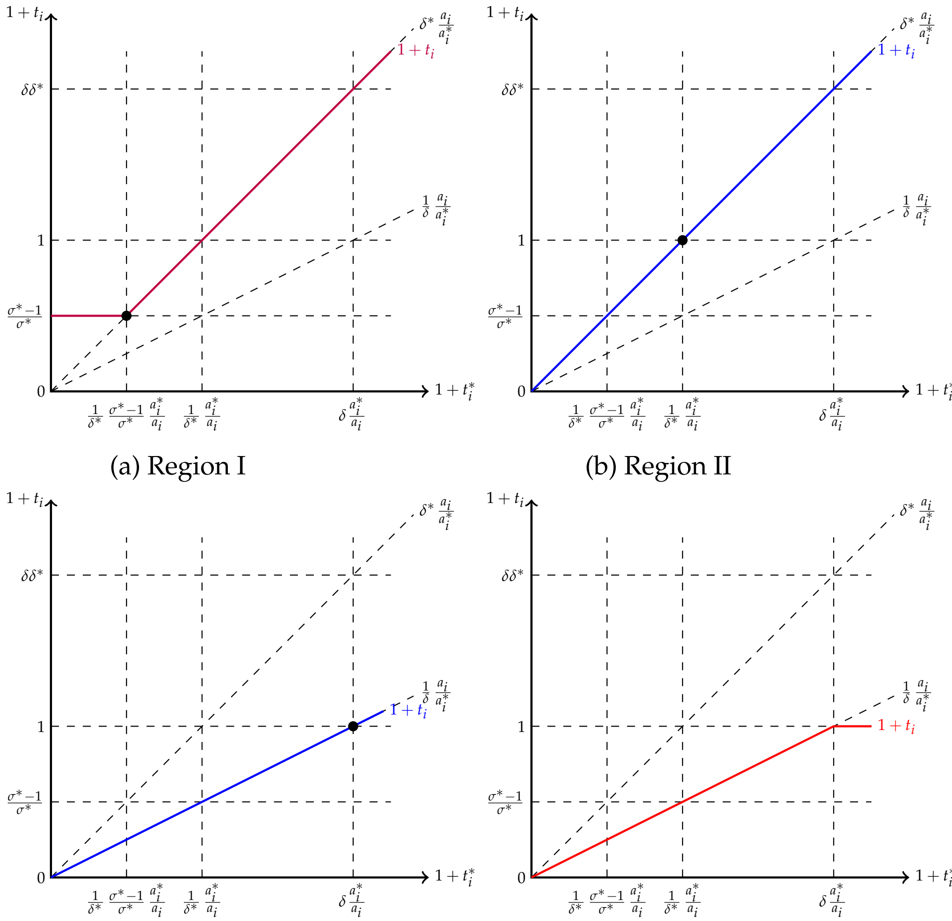
$$p_i^*(m_i, t_i, t_i^*) \equiv \min\{u_i^{*\prime}(-(m_i\phi(m_i))/L^*)/(1+t_i^*), a_i^*/(1+t_i^*)\}, \tag{38}$$

$$q_i^*(m_i, t_i, t_i^*) \equiv \max\{0, L^*(a_i^*/\beta_i^*)^{-\sigma^*} + m_i\phi(m_i)\}, \tag{39}$$

$$c_i^*(m_i, t_i, t_i^*) \equiv \max\{-m_i\phi(m_i), L^*(a_i^*/\beta_i^*)^{-\sigma^*}\}. \tag{40}$$

Equations (20)-(22) indicate that the threshold of net imports, above which Foreign is required to produce good i, now adjusts to $\delta^* M_i^I = -\delta^* L^* \left(\frac{a_i^*}{\beta_i^*}\right)^{-\sigma^*}$ due to the inclusion of iceberg trade costs. Additionally, as discussed in Dornbusch et al. (1977) and Costinot et al. (2015), these trade costs result in the emergence of a range of nontraded goods, identified by a new cut-off at $m_i = 0$. Together with the original cut-off $M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}$, below which Home must produce good i, we now have three cut-offs defining the boundaries of Home's net imports. Consequently, the range of Home's net imports for good i can be divided into four distinct regions: Region I with $m_i \leq \delta^* M_i^I$, Region II with $\delta^* M_i^I \leq m_i \leq 0$, Region III with $0 \leq m_i \leq M_i^{II}$, and Region IV with $m_i \geq M_i^{II}$. Each region represents a different production and trade scenario based on the relative size of Home's net imports and the trade cost-adjusted thresholds.

The introduction of iceberg trade costs does not alter the qualitative findings presented in Sections 3 and 4. Accordingly, we can extend the previous analysis to generalize Propositions 1 and 2. To proceed, let us examine the domestic government's welfare optimization problem on a region-by-region basis.



(a) Region I (b) Region II

(c) Region III (d) Region IV

Figure A.1: Best Response with Trade Costs in Regions I, II, III, and IV

Region I: $m_i \leq \delta^* M_i^I$**.** In this region, only Home produces good i. If $1 + t_i^* \leq \frac{1}{\delta^*}\frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i} = \frac{1}{\delta^*}B_i^I$, then optimal net imports are $(\delta^*)^{1-\sigma^*}m_i^I = -L^*[\frac{a_i}{\beta_i^*}\frac{\sigma^*}{\sigma^*-1}(1+t_i^*)]^{-\sigma^*}(\delta^*)^{1-\sigma^*}$, with $p_i = \frac{a_i}{1+t_i} = \frac{\sigma^*}{\sigma^*-1}a_i$ and $1 + t_i = \frac{\sigma^*-1}{\sigma^*}$. Thus, Home's optimized welfare is $\mathcal{W}_i^{Io} = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^\sigma + \frac{L^* a_i}{\sigma^*-1}[\frac{a_i}{\beta_i^*}\frac{\sigma^*}{\sigma^*-1}(1+t_i^*)]^{-\sigma^*}(\delta^*)^{1-\sigma^*}$. In contrast, if $1 + t_i^* \geq \frac{1}{\delta^*}\frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i}$, then optimal net imports are $\delta^* M_i^I$, with $p_i = \frac{a_i}{1+t_i} = \frac{1}{\delta^*}\frac{a_i^*}{1+t_i^*}$ and $1 + t_i = \frac{a_i}{a_i^*}(1+t_i^*)\delta^* \geq \frac{\sigma^*-1}{\sigma^*}$. Thus, Home's optimized welfare is $\mathcal{W}_i^{Io} = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^\sigma + (a_i - \frac{1}{\delta^*}\frac{a_i^*}{1+t_i^*})\left[-L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}\delta^*\right]$.

Region II: $\delta^* M_i^I \leq m_i \leq 0$**.** In this region, both Home and Foreign can produce good i, but only Home exports, or no trade occurs. If $1 + t_i^* \leq \frac{1}{\delta^*}\frac{a_i^*}{a_i} = \frac{1}{\delta^*}B_i^{II}$, then optimal net imports are $\delta^* M_i^I$, with $p_i = \frac{a_i}{1+t_i} = \frac{1}{\delta^*}\frac{a_i^*}{1+t_i^*}$ and $1 + t_i = \frac{a_i}{a_i^*}(1+t_i^*)\delta^*$. Thus, Home's optimized welfare is $\mathcal{W}_i^{IIo} = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^\sigma + (a_i - \frac{1}{\delta^*}\frac{a_i^*}{1+t_i^*})\left[-L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}\delta^*\right]$. In contrast, if $1 + t_i^* \geq \frac{1}{\delta^*}\frac{a_i^*}{a_i}$, then optimal net imports are zero, with $p_i = \frac{a_i}{1+t_i} = \frac{1}{\delta^*}\frac{a_i^*}{1+t_i^*}$ and $1+t_i = \frac{a_i}{a_i^*}(1+t_i^*)\delta^* \geq 1$. Thus, Home's optimized welfare is $\mathcal{W}_i^{IIo} = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^\sigma$.

Region III: $0 \leq m_i \leq M_i^{II}$**.** In this region, both Home and Foreign can produce good i, but only Home imports, or no trade occurs. If $1+t_i^* \leq \delta\frac{a_i^*}{a_i} = \delta B_i^{II}$, then optimal net imports are zero, with $p_i = \frac{a_i}{1+t_i} = \delta\frac{a_i^*}{1+t_i^*}$ and $1+t_i = \frac{1}{\delta}\frac{a_i}{a_i^*}(1+t_i^*) \leq 1$. Thus, Home's optimized welfare is $\mathcal{W}_i^{IIIo} = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^\sigma$. In contrast, if $1+t_i^* \geq \delta\frac{a_i^*}{a_i}$, then optimal net imports are M_i^{II}, with $p_i = \frac{a_i}{1+t_i} = \delta\frac{a_i^*}{1+t_i^*}$ and $1+t_i = \frac{1}{\delta}\frac{a_i}{a_i^*}(1+t_i^*) \geq 1$. Thus, Home's optimized welfare is $\mathcal{W}_i^{IIIo} = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^\sigma + (a_i - \delta\frac{a_i^*}{1+t_i^*})\left[L(\frac{a_i}{\beta_i})^{-\sigma}\right]$.

Region IV: $m_i \geq M_i^{II}$**.** In this region, only Foreign produces good i. If $1+t_i^* \geq \delta\frac{a_i^*}{a_i} = \delta B_i^{II}$, then optimal net imports are $\delta^{-\sigma}L[\frac{a_i^*}{\beta_i}\frac{1}{1+t_i^*}]^{-\sigma} = \delta^{-\sigma}m_i^{III}$, with $p_i = \delta p_i^* = \delta\frac{a_i^*}{1+t_i^*}$ and $1+t_i = 1$. Thus, Home's optimized welfare is $\mathcal{W}_i^{IVo} = \frac{L}{\sigma-1}[p_i(1+t_i)]^{1-\sigma}\beta_i^\sigma = \frac{L}{\sigma-1}\left[\delta\frac{a_i^*}{1+t_i^*}\right]^{1-\sigma}\beta_i^\sigma$. In contrast, if $0 < 1+t_i^* \leq \delta\frac{a_i^*}{a_i}$, then optimal net imports are M_i^{II}, with $p_i = \delta p_i^* = \delta\frac{a_i^*}{1+t_i^*} = \frac{a_i}{1+t_i}$ and $1+t_i = \frac{1}{\delta}\frac{a_i}{a_i^*}(1+t_i^*) \leq 1$. Thus, Home's optimized welfare is $\mathcal{W}_i^{IVo} = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^\sigma + (a_i - \delta\frac{a_i^*}{1+t_i^*})\left[L(\frac{a_i}{\beta_i})^{-\sigma}\right]$.

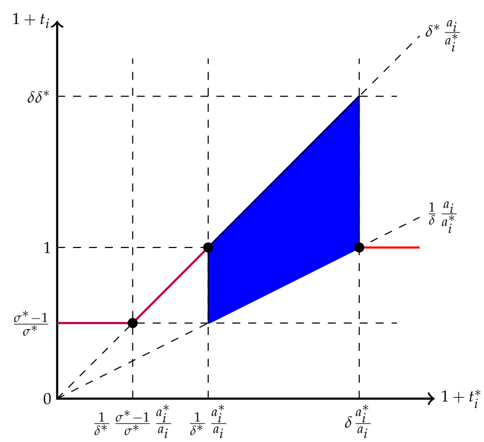


Figure A.2: Home's Unilateral Optimal Trade Policy with Trade Costs

Home's optimal responses across the four regions are illustrated in Figure A.1. Notably, there are three critical cut-offs for $(1+t_i^*)$: $\frac{1}{\delta^*}\frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i} = \frac{1}{\delta^*}B_i^I$, $\frac{1}{\delta^*}\frac{a_i^*}{a_i} = \frac{1}{\delta^*}B_i^{II}$, and $\delta\frac{a_i^*}{a_i} = \delta B_i^{II}$. Based on these cut-offs, Home's optimal trade policy $(1+t_i)$ can be categorized into seven scenarios, determined by foreign trade taxes (t_i^*), technology parameters (a_i, a_i^*), and asymmetric iceberg trade costs (δ, δ^*). By comparing the optimized welfare across the four regions within each scenario, we can summarize Home's optimal trade policy with trade costs as outlined in the following proposition, presented in Figure A.2.

Proposition 4 *If the two strategic countries set their optimal trade policies noncooperatively, given the foreign trade taxes* (t_i^*)*, technology parameters* (a_i, a_i^*)*, and asymmetric iceberg trade costs* (δ, δ^*)*, Home's unilateral optimal trade policy is characterized as follows. The producer, optimal trade taxes* (t_i)*, prevailing world prices at Home* (p_i)*, Home's net imports* (m_i)*, and Home's optimized welfare* ($\mathcal{W}_i$) *satisfy:*

(a) only Home produces, $1+t_i = \frac{\sigma^*-1}{\sigma^*}$, $p_i = \frac{\sigma^*}{\sigma^*-1}a_i$, $m_i = (\delta^*)^{1-\sigma^*}m_i^I$, *and* $\mathcal{W}_i = \mathcal{W}_i^{Io}$, *if* $1+t_i^* \in (0, \frac{1}{\delta^*}B_i^I)$;

(b) only Home produces, $1+t_i = \frac{\sigma^*-1}{\sigma^*} = \frac{a_i}{a_i^*}(1+t_i^*)\delta^*$, $p_i = \frac{\sigma^*}{\sigma^*-1}a_i = \frac{1}{\delta^*}\frac{a_i^*}{1+t_i^*} = \frac{a_i}{1+t_i}$, $m_i = \delta^* m_i^I = \delta^* M_i^I$, *and* $\mathcal{W}_i = \mathcal{W}_i^{Io} = \mathcal{W}_i^{IIo}$, *if* $1+t_i^* = \frac{1}{\delta^*}B_i^I$;

(c) only Home produces, $1+t_i = \frac{a_i}{a_i^*}(1+t_i^*)\delta^*$, $p_i = \frac{a_i}{1+t_i} = \frac{1}{\delta^*}\frac{a_i^*}{1+t_i^*}$, $m_i = \delta^* M_i^I$, *and* $\mathcal{W}_i = \mathcal{W}_i^{IIo}$, *if* $1+t_i^* \in (\frac{1}{\delta^*}B_i^I, \frac{1}{\delta^*}B_i^{II})$;

(d) case (A) only Home produces, $1+t_i = 1$, $p_i = a_i$, $m_i = \delta^* M_i^I$, *and* $\mathcal{W}_i = \mathcal{W}_i^{IIo} = \mathcal{W}_i^{IIIo}$*; case (B) both produce,* $1+t_i = \frac{a_i}{p_i} \in [\frac{1}{\delta\delta^*}, 1]$, $p_i = \frac{a_i}{1+t_i} \in [a_i, a_i\delta\delta^*]$, $m_i \in [\delta^* M_i^I, 0]$, *and* $\mathcal{W}_i = \mathcal{W}_i^{IIIo}$, *if* $1+t_i^* = \frac{1}{\delta^*}B_i^{II}$;

(e) both produce, $1+t_i = \frac{a_i}{p_i} \in [\frac{1}{\delta}\frac{a_i}{a_i^*}(1+t_i^*), \frac{a_i}{a_i^*}(1+t_i^*)\delta^*]$, $p_i = \frac{a_i}{1+t_i} \in [\frac{1}{\delta^*}\frac{a_i^*}{1+t_i^*}, \delta\frac{a_i^*}{1+t_i^*}]$, $m_i = 0$, *and* $\mathcal{W}_i = \mathcal{W}_i^{IIIo}$, *if* $1+t_i^* \in (\frac{1}{\delta^*}B_i^{II}, \delta B_i^{II})$;

(f) case (A) both produce, $1+t_i = \frac{a_i}{p_i} \in [1, \delta\delta^*]$, $p_i = \frac{a_i}{1+t_i} \in [\frac{a_i}{\delta\delta^*}, a_i]$, $m_i = 0$, *and* $\mathcal{W}_i = \mathcal{W}_i^{IIIo}$*; case (B) only Foreign produces,* $1+t_i = 1$, $p_i = a_i$, $m_i = M_i^{II} = \delta^{-\sigma}m_i^{III}$, *and* $\mathcal{W}_i = \mathcal{W}_i^{IIIo} = \mathcal{W}_i^{IVo}$, *if* $1+t_i^* = \delta B_i^{II}$;

(g) only Foreign produces, $1+t_i = 1$, $p_i = \delta p_i^* = \delta\frac{a_i^*}{1+t_i^*}$, $m_i = \delta^{-\sigma}m_i^{III}$, *and* $\mathcal{W}_i = \mathcal{W}_i^{IVo}$, *if* $1+t_i^* > \delta B_i^{II}$, *where* $B_i^I \equiv \frac{\sigma^*-1}{\sigma^*}\frac{a_i^*}{a_i}, B_i^{II} \equiv \frac{a_i^*}{a_i}; M_i^I \equiv -L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}, M_i^{II} \equiv L(\frac{a_i}{\beta_i})^{-\sigma}; m_i^I \equiv -L^*[\frac{a_i}{\beta_i^*}\frac{\sigma^*}{\sigma^*-1}(1+t_i^*)]^{-\sigma^*}, m_i^{III} \equiv L[\frac{a_i^*}{\beta_i}\frac{1}{1+t_i^*}]^{-\sigma}; \mathcal{W}_i^{Io} = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^{\sigma} + \frac{L^* a_i}{\sigma^*-1}[\frac{a_i}{\beta_i^*}\frac{\sigma^*}{\sigma^*-1}(1+t_i^*)]^{-\sigma^*}(\delta^*)^{1-\sigma^*}, \mathcal{W}_i^{IIo} = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^{\sigma} + (a_i - \frac{1}{\delta^*}\frac{a_i^*}{1+t_i^*})\left[-L^*(\frac{a_i^*}{\beta_i^*})^{-\sigma^*}\delta^*\right], \mathcal{W}_i^{IIo} = \frac{L}{\sigma-1}a_i^{1-\sigma}\beta_i^{\sigma}, \mathcal{W}_i^{IVo} \equiv \frac{L}{\sigma-1}\left[\frac{a_i^*}{1+t_i^*}\right]^{1-\sigma}\beta_i^{\sigma}.$